On the Validity of Simulating Stagewise Development by Means of PDP Networks: Application of Catastrophe Analysis and an Experimental Test of Rule-Like Network Performance

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This article addresses the ability of Parallel Distributed Processing (PDP) networks to generate stagewise cognitive development in accordance with Piaget's theory of cognitive epigenesis. We carried out a replication study of the simulation experiments by McClelland (1989) and McClelland and Jenkins (1991) in which a PDP network learns to solve balance scale problems. In objective tests motivated from catastrophe theory, a mathematical theory of transitions in epigenetical systems, no evidence for stage transitions in network performance was found. It is concluded that PDP networks lack the ability to recover the positive outcomes of analogous catastrophe analyses of real cognitive developmental data. In an attempt to further characterize the learning behaviour of PDP networks, we carried out a second simulation study using the discrimination-shift paradigm. The results thus obtained indicate that PDP learning is compatible with the learning of stimulus-response relationships, not with the acquisition of mediating rules such as conceived in (neo-)Piagetian theory. In closing, we speculate about the feasibility of simulating stagewise development with alternative network architectures.

INTRODUCTION

Authors, in both the area of experimental research and the area of computer simulation in developmental psychology, have considered neural networks (including connectionist PDP networks) as important process models of...
cognitive development (Bates & Elman, 1993; Grossberg, 1980; McClelland, 1989; Plunkett & Sinha, 1992; Siegler, 1989). Contrary to the relatively fixed architectures of the symbol manipulating production systems commonly employed in cognitive science, neural networks develop their knowledge base in adaptive interaction with the environment. In this respect, neural networks are compatible with epigenetical theory, according to which such interactions are the prime source of the emergence of more powerful cognitive structures (cf. Molenaar, 1986b). PDP networks, introduced by McClelland, Rumelhart, and the PDP research group (1986), constitute a distinct subset of neural network models. They are thought to be capable of both acquiring symbolic systems up to the level of natural language and modeling specific developmental phenomena like the accommodation process, which lies at the heart of Piaget's theory of developmental change (McClelland & Jenkins, 1991, p. 69). The applicability of those models to developmental processes is mainly studied by comparing their performance with empirical data, pertaining to, for example, English verb morphology, concept formation and vocabulary growth, and stagewise cognitive development (Plunkett & Sinha, 1992).

In this article, we will take a closer look at the latter application: the simulation of stagewise cognitive development. McClelland and others (e.g., McClelland & Jenkins, 1991) have drawn two main conclusions from their study of a PDP network that learns the balance scale task. First, the learning behaviour can be described as the acquisition of increasingly complex rules. This conclusion is based on an application of Siegler's (1981) rule-assessment methodology, in which observed response patterns are classified as being generated by one of four distinct rules. Second, the acquisition of more complex rules by the network appears to proceed in a stagewise manner. That is, the performance of the network is for a while consistent with a particular rule and then suddenly shifts to another rule.

However, questions have been raised about the evidence on which these conclusions are based. The four increasingly complex rules in Siegler's (1981) scheme constitute a measurement scale with discrete values. Hence, an application of this scheme to the learning behaviour of the network implies that the observed response patterns, which can vary continuously along multiple dimensions, are collapsed into a few ordered classes. It then follows that a sudden shift in assigned class membership of network responses is not sufficient evidence for the presence of a stage transition, because such a shift is quite compatible with gradual changes in the observed response patterns (Molenaar, 1986a). In order to ascertain the presence of a sudden jump in learning performance, one needs to analyse the values of raw response patterns. Furthermore, even if sudden jumps are detected in the raw network responses, this only constitutes necessary, but not sufficient, evidence for the occurrence of a stage transition.
Additional evidence is required to distinguish between fast continuous changes on the one hand and genuine discontinuities characterising transitions on the other hand. This can be obtained by testing observed network behaviour against a set of criteria which are motivated from catastrophe theory (van der Maas & Molenaar, 1992). Therefore, we carried out an exact replication of the simulation experiment in which a PDP network learns to solve the balance scale task, but in addition to the original analyses of McClelland et al. (1986), we tested the raw network responses against several catastrophe criteria.

There are also questions concerning the other conclusion drawn from the results of the original simulation study, namely, that the learning behaviour of the network can be described as the acquisition of increasingly complex rules. According to both Inhelder & Piaget (1958) and Siegler (1978), such rules are not merely convenient descriptions of cognitive performance, but constitute the essence of a child's growing competence underlying performance. Both included verbal explanations ("methode clinique") in their experiments to assess this rule-governed competence. Siegler (1976), however, used the verbal explanations only to perform a reliability analysis of his rule-assessment methodology. Obviously, verbal explanations are problematic with neural networks. There is, however, a more suitable procedure to assess whether a PDP network indeed learns rules in the sense as intended by Piaget and Siegler. Assuring that rules are related to competence, it is expected that a network that has learned these rules will distinguish between stimuli belonging to functionally distinct categories by constructing mediating concepts instead of direct stimulus-response (SR) relationships. The question of whether PDP networks are capable of learning mediating concepts has been mainly dealt with on a theoretical level (see Fodor & Pylyshyn, 1988; Pinker & Prince, 1988; Smolensky, 1988).

The same issue was the subject of discussion between behaviourist theories of discrimination learning (Spence, 1936) and concept-mediation theory of discrimination learning (H.H. Kendler & Kendler, 1969). These theories are also referred to as continuity and discontinuity theories of discrimination learning, respectively. The discrimination-shift task is the associated experimental paradigm with which the learning behaviour of animals, children, and older humans are extensively studied from the fifties until the seventies (reviews are given by Esposito, 1975; H.H. Kendler & Kendler, 1975; Slamencka, 1968; Wolff, 1967). Generally, it is concluded that the discrimination-shift behaviour of animals is consistent with learning SR relations. In contrast, older humans seem to construct mediating concepts. To study the nature of rules in a PDP network experimentally, we applied the discrimination-shift task, with different experimental designs, to PDP networks with different architectures and compared their behaviour with empirical studies from the literature.
This article is organized as follows: The second section concerns theoretical aspects of stagewise cognitive development, including a method to test the discontinuity hypothesis based on catastrophe theory, and Siegler’s rule-assessment methodology. The third section shows empirical evidence for the presence of catastrophe flags, which is partly based on results reported in the literature and newly performed analyses of existing data. In the fourth section, the behaviour of the PDP network is compared to empirical findings. The fifth section deals with the discrimination-shift behaviour of PDP networks, which is compared to discrimination-shift behaviour of animals, children, and older humans. Section 6 summarises the findings of the simulation studies with the PDP network. Moreover, we discuss the possibility of constructing networks that show stagewise development that accords with both the discontinuity hypothesis and the construction of mediating rules.

STAGES IN COGNITIVE DEVELOPMENT

The equilibration theory of Piaget introduces the concept of stages in the cognitive development of the child. Piaget described stages as relatively stable periods of cognitive development during which only minor progress is made. These stages are alternated with relatively short transitional periods in which knowledge increases discontinuously; qualitative new (i.e., more powerful) knowledge is acquired. The general idea is that the system reaches a distinct equilibrium with a different dynamic regime. But the concept of stages is a major subject of discussion in the field of cognitive development (Campell & Bickhard, 1986; Emde & Harmon, 1984; Levin, 1986; Pinard, 1981). To test this stage-hypothesis, Piaget and Inhelder (1969) introduced several criteria to establish the presence of stages. Later on, others introduced criteria to establish the presence of transitions. Although empirical evidence was found, these criteria have met major criticism (Brainerd, 1978; Fisher & Silvern, 1985). One point of criticism concerns the impossibility of distinguishing between a discontinuous change and a continuous acceleration in the development on the basis of the criteria. Another criticism concerns the lack of a theoretical model within which the criteria are integrated.

Catastrophe Theory

Recently, van der Maas and Molenaar (1992) proposed a method to test the discontinuity hypothesis based on catastrophe theory. Catastrophe theory (Thom, 1975) is part of nonlinear dynamic systems theory. It provides elementary mathematical models of equilibrium behaviour (i.e., dependent variables) which undergoes a discontinuous transition due to continuous change of control variables (i.e., independent variables). Several authors have noticed the connection between the stage-hypothesis of Piaget and catastrophe theory (e.g., Freedle, 1977; Klahr & Wallace, 1976; Preece, 1980;
van der Maas and Molenaar proposed a cusp model for the acquisition of conservation. The cusp model is one of the elementary catastrophes. This means that all discontinuous transitions of one behaviour variable which is guided by two control variables can be modelled by a cusp.

Catastrophe theory seems preeminently appropriate for the analysis of stage transitions, because it incorporates a basic formal model of transition criteria. According to catastrophe theory, a system in transition meets eight criteria, the so-called catastrophe flags: bimodal distribution of the behaviour variable, inaccessible region, sudden jump, anomalous variance, hysteresis, divergence, divergence of linear response, and critical slowing down (Gilmore, 1981). The catastrophe flags can be interpreted as empirical criteria for testing the discontinuity of development. Moreover, these empirical criteria can be applied without defining the control variables of the model, hysteresis being an exception. Some of the criteria have been mentioned before in literature on stagewise development: sudden jumps (e.g., Fischer & Silvern, 1985), bimodality of test scores (e.g., Fischer et al. 1984; Tabor & Kendler, 1981; Wohlwill, 1973), and increased variability of responses during transitional periods (e.g., Flavell & Wohlwill, 1969). All catastrophe flags are necessary conditions for a discontinuous transition, hysteresis being the exception. But, not all catastrophe flags are sufficient evidence for a discontinuous transition. For instance, a bimodal distribution of test score is an indication that a discontinuous transition is present. But, a continuous acceleration can also result in a bimodal distribution of the behaviour variable. Absence of a bimodal distribution, however, excludes the presence of a transition. Hence, the analysis of this criterion is essential to establish transitions. Presence of some catastrophe flags, like hysteresis, is sufficient evidence for discontinuous transition. Although all catastrophe flags are well defined mathematically, the interpretation of some of these flags in the context of cognitive development is not straightforward; van der Maas and Molenaar (1992) made several proposals for all of them. Those for which empirical evidence exists are explained in the section entitled “Empirical Evidence for Catastrophe Flags” in this article.

The Rule-Assessment Methodology

Piaget proposed a sequence of developmental stages in cognitive development. These stages are investigated by means of specially designed problems. The task for liquid conservation, for instance, should discriminate between the pre-operational stage and the concrete-operational stage. The balance scale task examines the formal-operational stage. In addition to these stages, Piaget proposed a general within-concept developmental sequence that describes the manner in which concrete-operational and formal-operational tasks are learned (described in Siegler, 1981). Siegler (1976, 1981) introduced the rule-assessment methodology to elucidate the increasingly powerful rules for solving problems which children acquire during cognitive
development. According to this method, rules can be studied by means of response patterns on specially designed items.

Many concrete-operational and formal-operational problems pertain to the judgment of the equality of two features (e.g., the volume of the liquid in two glasses of water or the torque of the two sides of a balance scale) that can differ on two perceptual dimensions (viz., height and width in the conservation of liquid task and weight and distance in the balance scale task). To make a correct judgment, both dimensions should be taken into consideration. To integrate both dimensions appears to be particularly difficult for young children: Often one dimension dominates (height in the conservation of liquid task and weight in the balance scale task). Siegler (1981) distinguished four rules in his analysis of children's within-concept learning behaviour.¹

Rule I: only consider the dominant dimension.
Rule II: consider the subordinate dimension if and only if the values on the dominant dimension are equal.
Rule III: consider both dimensions; in case of a conflict, muddle through.
Rule IV: consider both dimensions in a proper way.

To investigate the application of these rules in children, Siegler used six types of items: equal (values on both dimensions are equal), dominant (only the values of the dominant dimension differ), subordinate (only the values of the subordinate dimension differ), conflict dominant (values of the two dimensions are conflicting, but the correct judgment can be based on the dominant dimension), conflict subordinate (conflicting values, the judgment can be based on the subordinate dimension), and conflict equal problems (conflicting dimensions, but the result is equal). Judgments consistent with one of the four rules result in a characteristic response pattern. Table 1 shows item types, rules, and expected response patterns for the balance scale. The responses of a child are compared with the patterns yielded by simple rules. If the match is sufficient (i.e., 87% agreement in Siegler, 1981), the child is classified as responding according to that rule. A drawback of Siegler's methodology is that no statistical test is used to establish whether the response patterns fit the rules. Recently, van der Maas (1995) suggested that latent class analysis can test the presence of rules in response

¹ Although we have stressed the similarity between liquid conservation and judgment of the balance scale, many differences have been mentioned in the literature, for example, Siegler (1981, p. 35). One important difference is the number of stages that is involved. Learning the balance scale may involve two stage transitions: from the pre-operational stage to the concrete operational stage and from the concrete operational stage to the formal operational stage. Liquid conservation, on the contrary, concerns only one stage transition: from the pre-operational stage to the concrete operational stage.
TABLE 1
Siegler’s Rules on the Balance Scale Task

<table>
<thead>
<tr>
<th>Problem Type</th>
<th>Values at Both Sides</th>
<th>% Correct According to Rules</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Weight</td>
<td>Distance</td>
</tr>
<tr>
<td>Balance</td>
<td>=</td>
<td>=</td>
</tr>
<tr>
<td>Weight</td>
<td>≠</td>
<td>=</td>
</tr>
<tr>
<td>Distance</td>
<td>=</td>
<td>≠</td>
</tr>
<tr>
<td>Conflict-w</td>
<td>≠</td>
<td>≠</td>
</tr>
<tr>
<td>Conflict-D</td>
<td>≠</td>
<td>≠</td>
</tr>
<tr>
<td>Conflict-B</td>
<td>≠</td>
<td>≠</td>
</tr>
</tbody>
</table>

Note: The names of the items (first column) coincide with the dimension on which a correct decision can be based (e.g., for weight as well as for conflict-weight items, the side with more weight goes down). The second and third columns indicate whether weight and distance are equal on both sides (=) or unequal (≠). The last four columns show the expected response patterns on the item types according to Siegler’s (1981) rules.

Empirical Evidence for Catastrophe Flags

The balance scale is one of the Piagetian tasks that has been investigated extensively by Siegler. On a commonly used balance scale, several blocks of equal weight can be placed on both sides of the fulcrum at various fixed distances. The balance scale can be blockaded such that it remains level irrespective of the distribution of the blocks that are placed. Children are asked to judge whether the scale will remain level when the blockade is removed, and, if not, which side goes down. According to Siegler’s (1981) model, the score on four out of six item types changes during the acquisition of Rules II through IV (Table 1). Of all item types, the score on the distance items reveal the most dramatic developmental increment: It is expected to increase from 0 to 100% when Rule II is acquired. Moreover, the transition from Rule I to Rule II is completely characterized by the score on these items. Therefore, the following analysis of the presence of the first three catastrophe flags (bimodality, inaccessibility, and a sudden jump) is based on these items. That is, the score on the distance items is taken as the behaviour variable in the analyses.
Bimodality and Inaccessibility

The first catastrophe flag is bimodality, which implies that if the transition from Rule I to Rule II is discontinuous, the score on the distance items should have a bimodal distribution with one top around 0% correct and the other top around 100% correct. The second flag is inaccessibility and prescribes an empty score range between the means of the two components of the bimodal distribution.

Our analyses are based on data from two studies on the balance scale task: one was performed by Siegler (1981, Experiment 2), the other one was performed by van Maanen, Been, and Sijtsma (1989). In both studies, balance scale items were used which were constructed according to Siegler's rule-assessment methodology. In Siegler's study, there was a total of 59 participants (3 years to 13 years of age, and a group of college students). The test included four distance items ($\mu_{obs} = 2.034$, $s^2 = 3.17$). In van Maanen et al.'s study a total of 484 children participated (235 were in Grade 7, 249 were in Grade 8). The test included five distance items ($\mu_{obs} = 3.510$, $s^2 = 3.85$). Figure 1 shows the frequency distributions of the score on distance items of children in both studies.

For both data sets, unimodal and bimodal mixture distributions of binomials are fitted by means of maximum likelihood estimators. A bimodal mixture distribution, which we will call a mixture distribution with two components, is defined by the following equation:

$$P(X) = \pi_1 p_1(x; \mu_1) + \pi_2 p_2(x; \mu_2)$$

where $\pi_i$ is a proportion between 0 and 1 ($\pi_1 + \pi_2 = 1$), and $\mu_i$ is the probability parameter of a binomial probability function $p_i$. In general, a mixture distribution is defined by a probability ($\mu_i$) and an associated proportion ($\pi_i$) for each component $i$. If the transition from Rule I to Rule II is discontinuous, the estimated models for both data sets are expected to be bimodal.
with $\mu_1 = 0$ and $\mu_2 = 1$ (with a maximum deviation of .15). In order to select the simplest, best model for the data, four measures are calculated. The first measure is the Pearson goodness-of-fit statistic: chi-square ($\chi^2$). A small $\chi^2$ produces a large $p$ value, which indicates that the estimated distribution does not differ significantly from the experimental data. The second measure is the ratio of the variance accounted for under the model to the total variance: VAF. VAF is defined as the quotient of the variance under the mixture model and the variance of the sample. The variance of the estimated model can only accidentally exceed the variance of the sample; when it does, the ratio is truncated at 1. The third measure is the model selection criterion AIC (Akaike’s Information Criterion). The AIC is defined as $-2 \log$-likelihood plus twice the number of parameters to estimate (i.e., twice the number of components minus 1). The model with the lowest AIC is considered to be the simplest, best model. The fourth measure is the difference on $\chi^2$ tests of two hierarchic nested models ($\Delta \chi^2$). Because this measure is itself $\chi^2$ distributed (with degrees of freedom ($df$) equal to the difference between the $dfs$ of the two models), it can be tested whether or not the fit of the data increases significantly with a larger number of components in the model (Wilks, 1938). The last three statistics should be interpreted with care. The procedure to find the best-fitting model and the limitations of the statistics are described in Thomas (1989) and Thomas and Turner (1991).

In the following, we will present the results of fitting mixture distributions on both data sets. The data from Siegler’s (1981) study fit a bimodal distribution, $\chi^2 (1, N=59) = 1.75, p = .18$, with probabilities of both components that agree with expected values ($\mu_1 = .046, \mu_2 = .881, \pi_1 = .446; SE = .03$). Although the frequency distribution of the data set of van Maanen et al. (1989) suggests two peaks (Figure 1), it differs significantly from the best-fitting bimodal model, $\chi^2 (2, N=249) = 45.99, p < .001$. However, the probabilities of the fitted bimodal model are as expected ($\mu_1 = .091, \mu_2 = .921, \pi_1 = .264; SE = .02$). In both data sets, only a limited number of items were used. This small number of items is associated with a small number of degrees of freedom ($df = \#items - 2 \times \#comp. + 1$). Therefore, we could not fit trimodal models. To decide whether a bimodal model describes the data better than a unimodal model, we calculated the described statistics. The differences on $\chi^2$ tests indicates that the bimodal models fit significantly better than the unimodal models for both data sets (Siegler, 1981: $\Delta \chi^2(2, N=59) = 197.65, p < .001$; van Maanen et al., 1989: $\Delta \chi^2 (2, N=249) = 7150.55, p < .001$). The AICs of the bimodal models are lower than the AICs of the unimodal models (Siegler: 169.5 and 281.5; van Maanen et al.: 1380.4 and 2422.7, respectively). The VAFs of the bimodal models are by far the largest (Siegler: .32 and .97; van Maanen et al.: .27 and .97, respectively).

From these statistics, we can conclude that the bimodal models fit the data better than the unimodal models. However, conclusions concerning
the bimodality of both data sets are only preliminary. First, data from van Maanen et al. (1989) do not fit a biomodal model significantly. Second, trimodal models could not be fitted. These problems can be solved with further experiments by extending the number of distance items in the test. Empirical results reported in literature concerning several other Piagetian tasks show strong evidence for bimodality of the score distribution (Field, 1987; Tabor & Kendler, 1981; van der Maas, 1993; Wohlwill, 1973).

Consistent with the inaccessibility flag, both score distributions contain a region between the two components with only few cases. In both data sets, a test score around 50% is rare, as shown in Figure 1. The probabilities of the fitted bimodal models are also in accordance with these conclusions.

The Sudden Jump
The most intuitive flag, the sudden jump, implies that the raw test score increases suddenly at the moment of the possible jump. Because each child is not expected to make a transition at exactly the same age, longitudinal data are required to analyse the presence of this flag. The following empirical data were gathered in a longitudinal experiment with children who performed a nonverbal computer test of conservation (van der Maas, 1993). The test consisted of eight items of the conservation of liquid task. One-hundred and one children between the ages of 6 and 11 participated in a longitudinal study of conservation of liquid. The experiment consisted of 11 consecutive sessions over a period of 8 months. During this period, 24 children underwent a transition from nonconserver to conserver. Figure 2 shows the development of the mean of the score of these children. Only the raw data, that is, the number of correct items, of these children are included in the analysis. The individual time series are shifted with regard to each other, in that for each child, Session 11 is the first time they were classified as using conservation strategies. This implies that, by definition, Session 11 is the transition point.

Figure 2 shows a very clear jump in the test score between sessions 10 and 11, rather than a continuous progress. This observation is tested with the following analysis. Multiple-regression analysis with two predictor variables, time (i.e., session) and position with regard to the possible jump (0 if session < 11, 1 otherwise), is carried out. If the progress can be explained by a continuous development, a significant contribution of the time variable to the variance accounted for under the model is expected. It appears that the contribution of the jump variable is significant, \( p < .001 \). In contrast, the contribution of the time variable is not significant, \( p > .1 \). The total variance accounted for under the model, \( R^2 \), is .88. In the acquisition of conservation by children, the third catastrophe flag, the sudden jump, appears to be extant. It is important to note that this conclusion does not result from the discrete classification of response patterns in rules, because raw test scores are used in the analysis.
Figure 2. The sudden jump at Session 10 in the raw score on the conservation of liquid task of 24 participants. The individual scores are corrected for latency of transition points which enlarges the horizontal axis to 19 sessions. Vertical bars indicate standard errors. Extended explanation of these results is presented by van der Maas (1993).

Other Catastrophe Flags
Apart from bimodality, inaccessibility, and a sudden jump, only circumstantial evidence for catastrophe flags is gathered. As mentioned before, the interpretation of the catastrophe flags in terms of a developing child is not straightforward for all of them. In addition, some of the flags, hysteresis and divergence, require longitudinal data with a high frequency of measuring and very careful experimentation. Besides the previously discussed flags, van der Maas (1993) reported some evidence for anomalous variance and hysteresis. Anomalous variance can be interpreted as an increased variance of the behaviour variable in the transitional period. The data of the longitudinal study of van der Maas (1993) show anomalous strategies during the transitional period. Although at this moment, the information about the presence of catastrophe flags in empirical data is incomplete, the evidence suffices for our purposes (the investigation of discontinuity of development of the PDP network in comparison with children).

A PDP NETWORK FOR STAGEWISE DEVELOPMENT?

The Network
McClelland (1989) and McClelland and Jenkins (1991) simulated the learning of the balance scale task with a three-layer feedforward neural network
with a backpropagation error-correction algorithm. The units are arranged in three layers: an input layer, a hidden layer, and an output layer. Units are connected in a feedforward way between layers only, that is, from input units to hidden units to output units (Figure 3). The activity of each unit is a number between 0 and 1 and is a sigmoidal function of the weighted sum of its inputs. During the learning process, the weights, which correspond to the proportions of the transferred activations, are chosen so as to minimize an error signal arising from the desired response of the network. So far, the network is a classical feedforward PDP network with error-backpropagation learning as described in Rumelhart, Hinton, and Williams (1986). By constraining the connection structure, the "architecture assumption", McClelland adjusted the network: Input units that represent different dimensions of the problem, that is, weight and distance, are connected to distinct hidden units. To let the network judge balance scale problems, stimuli are translated into activation patterns. The activation patterns of the input units represent the stimuli, and the activation patterns of the output units represent the judgment. The network handles a balance scale problem by the following procedure:

1. Represent values of weight and distance of the blocks at both sides of the fulcrum on the input layer by activating units.
2. Compute the activation of the hidden units and output units.
3. Translate the activation of the output units into a judgment (i.e., left down, right down, or balance).

* Precise description of both the dynamics of the network and the learn and test procedures are given in McClelland (1989), McClelland and Jenkins (1991) and Plunkett & Sinha (1992).
The network undergoes learning procedures and test procedures which are repeated for many epochs. During the learning phase, the network obtains feedback, after each judgment, in the form of the correct response. Learning consists of adjusting the connection strengths, in that the judgment error (i.e., the difference between the given response and correct answer) minimises. Because of random variation in the initial connection strengths, the network develops differently in separate runs. During each learning phase, the network judges 100 problems, whereupon it receives feedback so that connection strengths can be adjusted. The problems are chosen randomly from the set of all possible problems, but in such a way that weight (the dominant dimension) is more frequently available as a basis for predicting outcome than distance (the subordinate dimension). McClelland (1989) called this the "environment assumption". In the following experiments, this ratio is 10 to 1. At both sides of the fulcrum, the maximum values of weight and distance are 5, which implies that there exist $5^4 = 625$ different problems. Each learning phase is followed by a test phase consisting of 24 problems, four of each of the six problem types. The sequence of responses on the test items is classified as being consistent with the initial rate (initially, the network judges always balance), one of Siegler's four rules (i.e., 87% agreement), Rule I and Rule II simultaneously, or no rule at all.

**Simulation Results**

McClelland (1989) reported that in 85% of the epochs, in an average of four runs excluding the initial phases, the model responds consistently with one of Siegler's four rules. Classifications are made according to the previously mentioned criteria. In comparison, Siegler (1981) showed that 90% of the children in his study fit the criteria. The network passes through the rules in the expected sequence, from Rule I to Rule IV by way of Rule II and Rule III. Unlike a group of 12-year-olds (15%) and a group of adults (30%), the network never attains the final rule without regression to Rule III. During the development of the network, the classification remains equal for some period, whereupon changes (e.g., from Rule I to Rule II) appear suddenly. Meanwhile, both the environment assumption and the architecture assumption remain constant. From these observations, it is concluded that the network shows a stagewise development. "It (i.e., the model) captures its stage-like character, while at the same time exhibiting an underlying continuity which accounts for gradual change in readiness to move on to the next stage" (McClelland & Jenkins, 1991, p. 69).

As referred to previously, catastrophe theory specifies a set of criteria for testing the discontinuity hypothesis proposed by Piaget. Therefore, we performed a full replication study and applied catastrophe flags to the simulation data. Both the network architecture and the dynamic rules are the same
as in McClelland's (1989) simulations, except that the adjustment of connection strengths takes place after each item instead at the end of the learning phase. These learning dynamics are very common (cf. Rumelhart et al. 1986) and seem more natural to us. It appeared that this change does not effect the reported results. We also extended the number of test problems: We used 72 test problems (instead of 24), 12 of each problem type. The latter is important with regard to the fit of mixture distributions. A network with the slightly different dynamics learns the 24 test items used by McClelland equivalently, that is, 85.2% of the epochs of 5 runs agrees with a rule (the probability of the score from Epochs 91 to 100 is .74, SD = .08). Concerning the test with 72 items, 78.4% of the response patterns, averaged over 11 runs, is consistent with one of Siegler's four rules (the probability of the score from Epochs 91 to 100 is .80, SD = .045). The score patterns that are classified as a particular rule are also analysed separately. For each rule, the score on the distinct problem types shows the same pattern as McClelland (1989) reported. For example, with respect to Rule III and Rule IV, the score on conflict distance items appears to be worse than conflict weight problems.

**Bimodality and Inaccessibility**

In a process that includes a discontinuous change, catastrophe flags are present during or near the transitional period. Furthermore, if the analysis focuses on the moment of transition, that is, it considers only the behaviour variable just around the jump, the behaviour still meets the criteria. The behaviour variable is the score on distance items during the test phase of an epoch. In Figure 4, the frequency diagrams of two score distributions are shown. Counted are the number of epochs a certain score has reached. The left panel displays only the scores of the epochs around the possible jump. That is, only the scores of the first epoch the network has a score pattern according to Rule I up to the last epoch the network has a score pattern according to Rule II are included. The right panel displays the scores of all epochs up to 100, excluding the initial phase. Scores of epochs of 11 runs are taken together. In the first run, none of the response patterns was classified as Rule II. This run was therefore excluded from the first analysis. According to the catastrophe flag biomodality, both frequency distributions

\[^3\] McClelland reported no progress of the model after 100 epochs. Siegler's test contains only a few items that discriminate between Rule IV (torque rule) and the so-called addition rule (Anderson & Cuneo, 1978). Therefore, we presented the network an additional test containing items that discriminate between Rule IV and the addition rule. It turns out that the network performs the former significantly more after 1,000 than after 100 epochs (M = 78% and 60% correct, respectively, on items discriminating between the rules, p = .0001), although the score on the standard test remains equal.
Figure 4. Frequency distributions of 12 possible scores on distance items originating from 11 runs of the network. Counted are the number of epochs that a certain score has reached. Score 0 corresponds with 0% correct, score 12 corresponds with 100% correct. Left panel: Only the epochs between the first time the network responds according to Rule 1 up to the last time the network responds according to Rule II are counted. Right panel: All epochs (up to 100) except those in the initial phase are counted.

shown in Figure 4 should be bimodal. Moreover, according to Siegler’s model, the means of the two components are expected to be 0 and 1 with a maximum deviation of 0.15 (which is the criterion of the response pattern to fit a rule). Statistical analyses show that, even if the number of parameters are taken into account, a trimodal model fits better than a bimodal model and a unimodal model on both score distributions. According to the chi-square tests, the bimodal model does not fit in either case (left: $\chi^2 (9)=36.19$, $p<.001$; right: $\chi^2 (9)=1979.13$, $p<.001$). A trimodal model only fits the distribution displayed in the left panel (left: $\chi^2 (7)=13.40$, $p>.05$; right: $\chi^2 (7)=48.41$, $p<.001$). The differences in chi-squares between the bimodal and trimodal models are strongly significant (left: $\Delta \chi^2 (2)=22.79$, $p<.001$; right: $\Delta \chi^2 (2)=1930.72$, $p<.001$). The AICs of the estimated models of the left score distribution are as follows: unimodal model 1269.61, bimodal model 911.26, and trimodal model 895.20. Concerning the right score distribution, the AIC is 6716.74 for the unimodal model, 3303.81 for the bimodal model, and 1995.19 for the trimodal model. Moreover, the probabilities of the estimated bimodal models differ from the probabilities that are expected from Siegler’s methodology (left: $\pi_1 = .146$, $\mu_1 = .657$, $\mu_2 = .382$; right: $\mu_1 = .359$, $\mu_2 = .974$, $\pi_1 = .154$).

In addition to bimodality, the score distribution should contain an inaccessible region: the second catastrophe flag. This means that the frequency of a score of 6 should be very low. However, no significant difference is found between the left score distribution and the estimated trimodal model. The probabilities of the components with associated proportions show that the distribution does not contain an inaccessible region around 0.5 ($\mu_1 = .677$, $\mu_2 = .016$, $\mu_3 = .245$, $\pi_1 = .569$, $\pi_2 = .125$, $\pi_3 = .306$).
The Sudden Jump

The network fails to satisfy bimodality and inaccessibility which is in itself enough evidence to conclude that the network lacks a discontinuous transition. Nevertheless, we examined the presence of a sudden jump, because the application of the third flag is very illustrative with regard to the problems of analysing the abruptness of developmental changes in conceptual behaviour. The domain under consideration of the empirical data analysed in the section “Sudden Jump”, the conservation of liquid task, differs from the balance scale. However, both are tasks in which two dimensions, height (dominant) and width (subordinate), should be learned in combination. Therefore, McClelland and Jenkins (1991, p. 69) claimed that the network is not only a model for learning the balance scale problem, but it is also a model for learning compensation and conservation problems in general. The latter implies that the model should account for the acquisition of liquid conservation as well.

The analysis of the presence of a jump in the learning behaviour is based on the score on distance items. Only the epochs from the first Rule I up to the last Rule II classification are included in the following analysis. Eleven runs are taken together (i.e., 10 runs because 1 is excluded). The 10 time series are shifted with regard to each other in such a way that the possible transition points (the first time responses are consistent with Rule II) coincide with \( t = 15 \). Figure 5 shows the means of the proportion of correct responses on the distance items (y-axis) and standard errors (vertical bars) at each moment in time (x-axis). The peak at Time 15 is an artefact, because if the score is consistent with Rule II (this counts for all networks at Time 15 by construction, but not at time > 15), the proportion correct on distance items is relatively high. In the presented analysis, Time 15 is removed from the time series. In order to test the presence of a jump at Time 15, we performed (analogue to the “Sudden Jump” section of “Empirical Evidence for Catastrophe Flags”) a multiple regression with two predictive variables: time (measurement 1 to 20) and position with regard to the possible jump (0 = before, 1 = after). The analysis shows that only the time factor, \( p = .0001 \), and not the jump variable, \( p = .43 \), counts for a significant proportion of variance (total \( R^2 = .68 \)). Furthermore, the residuals with regard to a simple regression model with only time as predictive variable are not autocorrelated (Durbin-Watson = 1.543, \( p > .05 \)). From this we can conclude that during the transition from Rule I to II, the network shows an increase in score on

\[ \text{It can be shown formally that the high score at Time 15 is an artefact by including a third predictive variable, that is 1 at Time 15 and 0 elsewhere, in a multiple-regression analysis. Now, Time 15 is not removed from the time series. It appears that the contribution of both time (measurement 1 to 20) and the artefact independent variable contribute significantly, } p = .0001 \text{ and } p = .005, \text{ respectively. Again, however, like in the original analysis, the jump variable does not account for a significant proportion of variance, } p = .42; \text{ total } p \text{ value } = .0001, R^2 = .71. \]
distance items which is accounted for by a linear improvement in time, and not by a discontinuous jump.

**Conclusions**

Empirical results based on real developmental data obtained in various Piagetian tasks, including the balance scale, show strong evidence for the presence of at least three catastrophe flags: bimodality, inaccessibility, and sudden jump, and some evidence for anomalous variance. In contrast, the behaviour of the PDP network that learns the balance scale task does not obey these criteria for the transition from Rule I to Rule II. According to McClelland (1989), McClelland and Jenkins (1991), and Plunkett and Sinha (1992), this transition is abrupt. However, the sudden jumps of the behaviour variable, namely, the fitted rule on which they based their claim, are due to the discrete classification of the raw test scores. In contrast, our analyses were carried out on raw scores. To us, the reported negative results concerning the feedforward PDP networks with error backpropagation do not imply that neural networks in general are not appropriate as models for stagewise development. As mentioned before, the examined networks are only a subclass of neural networks. Consequently, we only conclude that the architecture and the dynamics under consideration are not appropriate for simulating stagewise development. The possibility of defining other networks that can show stagewise development is discussed at the end of this article. At this point, however, a second aspect of Piaget’s theory and Siegler’s approach concerned with the use of concepts and rules in cognitive development is investigated with regard to the feedforward PDP networks with error backpropagation.
McClelland’s (1989) conclusions concerning the stagewise development of the network performing the balance scale task depend largely on the conclusion that during the learning process the network complies with Siegler’s four rules in the predicted sequence. This conclusion is justifiable because, according to Siegler’s (1981) criterion, the response patterns appeared to be consistent with the rules, as in children. However, as we mentioned before, the rule-assessment methodology does not include a statistical test for the presence of rules in response patterns. As a reliability analysis, Siegler (1976) asked children to tell what rule they used, in addition to the rule-assessment methodology, in judging balance scale problems. Both measures, the verbal justification and the rule-assessment methodology, appear to correlate with each other. According to Inhelder & Piaget (1958) and Siegler (1978, p. 119), the rules that are acquired in a developmental sequence not only constitute a consistent description of response patterns but also generate the responses.

To what extent the behaviour of the PDP network can be called rule based is questionable. On the one hand, obviously, explicit rules are not available, although the responses of the above described network are mostly consistent with rules. McClelland (1989) stated that the lack of explicit rules could be the reason the network does not solve conflict balance items without failure because he believes that Rule IV can only be mastered as an explicit (i.e., arithmetic) rule. On the other hand, networks are not thought to learn fully in accordance with behaviourist theories and are believed to have particular cognitive processing properties (Plunkett & Sinha, 1992) such as selective encoding of input patterns. The hidden units should function as mediating concepts. However, if a task can be learned both by constructing rules and by forming stimulus–response (SR) associations, in the end, given responses are the same and the learning processes are no longer distinguishable on the basis of the response patterns.

The latter can be the reason for a pragmatic point of view. Reber (1993) argued that if the knowledge is held explicitly and is in accordance with verbal descriptions, we can speak of a rule. In other cases, this issue is tricky and nothing can be said for certain. Nevertheless, Reber speculated, “that connectionist models will do a better job at simulating the implicit processes, whereas the production systems will be more successful in capturing the explicit” (p. 110). The question whether learned associations of networks can be called rules is mostly dealt with on a theoretical level. According to Smolensky (1988), connectionist models process knowledge on a subsymbolic

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3 From the 120 children who participated, 84 were classified to the same rule by their item responses and by their explanations and 15 children were classified to different rules. The other children could not be classified by their item responses or by their explanation.
level, but at the same time, under ideal circumstances, admit good descriptions on a conceptual level. This does not make symbolic cognitive research redundant but should enrich the existing cognitive science. Pinker and Prince (1988) argued, on the basis of language research, that there exist much more evidence for symbolic, rule-based systems than for connectionist models, in particular the RM-model (Rumelhart & McClelland, 1987). They did not exclude, however, that more powerful connectionist mechanisms might revise our understanding of language. Fodor and Pylyshyn (1988) emphasized the combinatorial character of syntactic and semantic structures which are lacking in connectionist models.

**Discrimination-Shift Behaviour: Theoretical Predictions**

We will discuss the issue of whether the learning behaviour of networks is congruent with forming SR relations or with constructing rules from an empirical perspective. The discrimination-shift paradigm, which is applied to both humans and infrahuman subjects, seems rather appropriate for this purpose. The paradigm is extensively used in the continuity–noncontinuity controversy: the applicability of behaviourist models (e.g., Spence, 1936; Spiker & Cantor, 1973) versus concept-mediating models (e.g., H.H. Kendler & Kendler, 1962, 1975; Zaeman & House, 1963). In later work, T.S. Kendler (1979, 1983), in her levels-of-functioning theory, distinguished between two more or less separated issues concerning the continuity–noncontinuity controversy. The first is whether the encoding of the stimulus is selective or nonselective. The second is whether learning of correct responses is incremental or based on hypothesis testing. Selective encoding means that an object is coded by means of relevant characteristics only. According to Kendler, nonselective encoding implies forming SR relations. Selective encoding is consistent with hypotheses testing, but does not imply it necessarily. This is supported by the findings of T.S. Kendler (1979) that many selective encoders are classified as incremental learners, whereas nonselective encoders appear to be hypothesis testers in only a few cases.

Below, we describe several experimental designs and analyses of the discrimination-shift task: reversal shift versus extradimensional shift, trial-by-trial analysis, optional shift, and win-stay, lose-shift strategy, including their theoretical and empirical implications. All experimental designs of the task will be applied to a class of feedforward PDP networks with error backpropagation which resemble the PDP network applied to the balance scale task. A detailed description of the configuration of these networks is given below.

**Reversal Shift Versus Extradimensional Shift**

In the standard discrimination-shift task, participants learn to discriminate on the basis of reinforcement contingencies between four stimuli which are presented in two distinct pairs (Figure 6). The stimuli are distinguishable on
two dimensions: shape (round/triangle) and colour (white/black). Each stimulus pair appears in two configurations of which only the positions of the stimuli differ. The task comprises three phases: the pre-shift phase, the reversal-shift (RS) phase, and the extradimensional-shift (EDS) phase. The pre-shift phase continues until the number of correct responses in a sequence of adjacent trials meets a given criterion. After the pre-shift phase, learning continues, but the reinforcement is changed by either a RS or an EDS. A RS implies that all stimuli that received positive reinforcement get negative reinforcement, and vice versa. An EDS means that the dimension upon which the reinforcement is based, shape or colour, is shifted. The main difference between the two shifts is the number of SR relations that change: After a RS, all relations change, whereas an EDS changes only half of the relations. On the basis of this distinction, H.H. Kendler and Kendler (1962) concluded that behaviourist models (e.g., Spence, 1936) predict that EDSs are learned faster than RSs (i.e., the EDS needs fewer trials before criterion). In contrast, a model that presumes the use of a mediating concept (selective encoding) is expected to learn the RS faster because only the link between the mediating concept and the response should be changed.

**Trial-by-Trial Analysis**

Behaviourist models and models that presume mediating concepts also make contrasting predictions concerning the learning curves of the separate stimulus pairs in the EDS phase (Estes, 1983; Tighe, Glick, & Cole, 1971). Behaviourist models predict that the performance on the stimulus pair with unchanged reinforcement remains high after an EDS. In contrast, the performance on the stimulus pair with changed reinforcement is predicted to start low and to increase gradually. The learning curves of both stimulus pairs after a RS are predicted to be equivalent to the curve of the changed
EDS pair. On the contrary, concept-mediating models predict that the performance on all stimulus pairs decreases after an EDS.

**Optional Shift**
The optional-shift procedure is related to the first experimental design. It includes the same pre-shift phase, which is followed by a shift-discrimination phase and test series. During the shift-discrimination phase, only one of the two stimulus pairs is learned with reversed reinforcement. The reinforcement of this stimulus pair agrees with both shifts (Pair I of both RS and EDS in Figure 6). During the test series, that is, after the attainment of criterion in the shift-discrimination phase, all stimulus pairs are presented again, but without reinforcement. The test series show how the subject learned the shift-discrimination phase: either by reversing the attribution of the relevant dimension or by changing only the SR relation of the presented stimulus pair. Obviously, the former is predicted by concept-mediating models, and the latter is predicted by behaviourist models. In contrast to the first experimental design, the optional-shift procedure is designed to determine directly the discrimination-shift behaviour of each individual. According to T.S. Kendler (1979), people making a RS are selective encoders, people making an EDS are nonselective encoders.

**Win-Stay, Lose-Shift Strategy**
The preceding experimental designs of the discrimination-shift task are mainly dealing with Kendler’s first distinction: selective encoding versus nonselective encoding. The discrimination-shift task is, however, also informative with regard to her second distinction: incremental learning versus hypothesis testing. Extended experimental procedures, like the blank-trials method and the pre-solution reversals are developed to elaborate hypothesis-testing behaviour in humans (e.g., Bower & Trabasso, 1963; Levine, 1959, 1966; Offenbach, 1974). However, in order to discriminate merely between incremental learners and hypothesis testers, the pre-shift phase of the discrimination-shift task is sufficiently informative. T.S. Kendler (1979) examined whether participants followed the win-stay strategy, that is, whether once a participant made a correct choice he or she stuck to it until criterion. In the same manner, the lose-shift strategy is an indicator of hypothesis-testing behaviour (Levine, 1959). Both rules also follow from the all-or-nothing theory of Bower and Trabasso (1963), according to which participants sample at random from the set of possible hypotheses when they make an error.

**Discussion**
Four experimental designs concerning discrimination-shift learning have been described. Many empirical studies included extra manipulations (e.g., verbalization, perceptual pretraining, overtraining, words instead of geo-
metric stimuli, changing values of the dimensions after the shift) which are harder to perform with artificial networks. Those manipulations are often introduced because of methodological criticism (Brier & Jacobs, 1972; Esposito, 1975; Slamencka, 1968). The main criticism concerns individual differences (participants appear to prefer particular stimulus dimensions), instability of the optional-shift behaviour, and alternative explanations for preferring a RS instead of an EDS. These aspects are important for interpreting experimental results, mainly those concerning children. The Kendlers concentrated on the developmental aspect of the discrimination-shift behaviour. T. Kendler's levels of functioning theory concerns the development of the two mentioned hypothetical processes, in that children up to 6 years of age are nonselective encoders and incremental learners, and adults are selective encoders and hypothesis testers. According to her, the development consists of a continuous increase of the probability that humans will solve the discrimination task by selective encoding and hypothesis testing. Although a lot of evidence is gathered in favour of this theory, criticism like the existence of a preference dimension applies mainly to these studies (Wolff, 1967).

However, the criticism seems to apply less to discrimination-shift learning of artificial networks. First, PDP networks are more controllable: Representations of dimensions and values of dimensions can be made equally salient. Second, it is clear that test series without feedback, as in the optional-shift task, directly reflect the learning process of a PDP network because meta-cognitive considerations of the model are not under discussion. Third, after the pre-shift phase (i.e., with the weight matrix formed during the pre-shift phase), each network can learn both a RS and an EDS without learning effects coming in. This implies that, also on the basis of the RS-versus-EDS task, the learning behaviour of each individual network can be characterized: namely by the difference of the learning rapidity after a RS and an EDS. Results of the simulation studies will be compared both with theoretical predictions and empirical results from human and infrahuman research.

**Discrimination-Shift Behaviour: Empirical Results**

These experimental designs are so elementary that they have been used with animals, children, and older humans. Esposito (1975), H.H. Kendler and Kendler (1975), Slamencka (1968), and Wolff (1967) gave extended reviews of the large body of existing literature.

**Reversal Shift Versus Extradimensional Shift**

Research on discrimination-shift behaviour with the first described empirical design shows a clear difference between animals and adults. Repeatedly, it was found that college students execute a RS more rapidly than an EDS (Buss, 1956; Harrow & Friedman, 1958; T.S. Kendler & D'Amato, 1955).
In contrast, rats, pigeons, fish, and monkeys are found to execute an EDS faster than a RS (Kelleher, 1956; Schade & Bitterman, 1966; Tighe, 1964). H.H. Kendler and Kendler (1962) reported a study with kindergarten children, in which half of those children who performed the pre-shift phase above median executed RS faster than EDS (6.0 and 15.5 trials to criterion, respectively), and the other half of the children performed the other way around (24.4 and 9.0 trials, respectively).

**Trial-by-Trial Analysis**

Trial-by-trial analysis applied to the data of the EDS phase shows a difference between 4-year-old and 10-year-old children (Tighe, Glick, & Cole, 1971; Tighe, 1973). Four-year-old children appear to learn, during the EDS phase, the stimulus pair with changed reinforcement much slower than the pair with unchanged reinforcement, which agrees with predictions of behaviourist models. In contrast, learning curves of 10-year-old children show a drop in performance in the first trials of both EDS stimulus pairs as well as both stimulus pairs during the RS phase. But the RS curve surpasses the EDS curves. In addition, the younger children perform the EDS faster than the RS, whereas the older children perform the RS faster than the EDS.

**Optional Shift**

The performance of animals on the optional-shift task agrees with the idea that the discrimination-shift behaviour of animals, unlike human adults, is in accordance with behaviourist theories. It appears that rats execute more frequently an EDS than a RS (H.H. Kendler, Kendler, & Silfen, 1964; Tighe & Tighe, 1966). Whereas human adults are more likely to make a RS instead of an EDS, which agrees with selective encoding models (T.S. Kendler, 1983: adults 92% RS; H.H. Kendler, Kendler, & Learnard, 1962: 10-year-olds 62.5%). According the H.H. Kendler and Kendler (1975), the proportion of children who execute a RS faster than an EDS increases linearly with the logarithm of age. Tighe and Tighe (1966) found an additional difference between humans and animals: Overtraining, which means that the number of learning trials is increased, results in an increase of the proportion of children performing a RS. The latter does not hold for the population of rats.

**Win-Stay, Lose-Shift Strategy**

Most studies concerning hypothesis-testing behaviour in humans are performed with the blank-trials method by which trials with reinforcement are alternated with sets of trials without reinforcement (blank trials). The responses on blank trials should indicate the hypothesis of the subject. Generally, these studies show that hypothesis-testing behaviour increases with age (Eimas, 1969; Gholson, Levine, & Phillips, 1972; Ingalls & Dickerson, 1969). A few studies were based on the reversals-prior-to-solution method
(according to which reinforcement is shifted after an error on a critical trial), such as the studies of Bower and Trabasso (1963). They found that adults learn according to a hypothesis-testing method. Tighe and Tighe (1972) used the latter method with humans of different ages and found that fifth-grade children apply hypothesis testing, whereas the learning method of younger children (kindergarten and first grade) is congruent with incremental learning. T.S. Kendler (1979) tested whether children from different ages follow the win-stay strategy during the pre-shift phase of the discrimination-shift task. It appeared that the proportion of children who follow the win-stay strategy increases over age.

**Discussion**

The main conclusion that can be drawn from empirical studies is that the learning behaviour of animals agrees with behaviourist theories, whereas the learning behaviour of human adults agrees with concept-mediating models. In terms of T.S. Kendler (1979, 1983), animals are nonselective encoders and incremental learners, whereas older humans are selective encoders and hypothesis testers. Furthermore, Kendler stated that children continuously shift from a behaviourist learning to a concept-mediation learning. Tabor & Kendler (1981) gave evidence for a continuous transition from nonselective encoder to selective encoder by showing that the distribution of test scores of an optional-shift task is unimodal. Note that this transition is not a transition from applying one rule to applying another rule, as with the balance scale task. The question of how these two transitions relate to each other is complicated.

**Discrimination-Shift Behaviour: Simulation Results of PDP Networks**

We will now describe simulation experiments concerning the discrimination-shift behaviour of a class of feedforward PDP networks with error backpropagation and one hidden layer. The architecture of the network and the dynamics equal those of the PDP network solving the balance scale (dynamics are described in Rumelhart et al., 1986). We performed simulations with several different network architectures by varying the number of hidden nodes (4 and 2) and the connectivity (either constrained or unconstrained). In the unconstrained network (UC), each hidden node is connected to all input nodes. A constrained connection structure (C) implies that each hidden node is connected to the input nodes of one stimulus dimension (either shape or colour) as in McClelland’s (1989) model (the architecture assumption). The networks are denoted with 8-2-2 C, 8-4-2 C, 8-2-2 UC, and 8-4-2 UC.

The representation of the input of the network is similar to the representation of balance scale problems. One input pattern consists of two stimuli as in most empirical studies. The input layer consists of eight input nodes: four nodes represent one stimulus. Each stimulus is represented by values
SIMULATING STAGEWISE DEVELOPMENT OF RULES

Stimulus

<table>
<thead>
<tr>
<th>Colour</th>
<th>Shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>white</td>
<td>circle</td>
</tr>
<tr>
<td>black</td>
<td>triangle</td>
</tr>
</tbody>
</table>

**Figure 7.** Representation of a pair of stimuli on the input layer. The input layer consists of eight nodes, denoted by small circles, which can be activated, black, or inactivated, white. The left stimulus is represented by the upper four nodes, the right stimulus is represented by the lower four nodes. The left four nodes represent the colour of the stimulus, the right four nodes represent the shape of the stimulus.

**TABLE 2**

Reversal Shift Versus Extradimensional Shift in PDP Networks

<table>
<thead>
<tr>
<th>Network Architect</th>
<th>Learning Cycles RS</th>
<th>Learning Cycles EDS</th>
<th>Two-Way ANOVA</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>SD</td>
<td>M</td>
</tr>
<tr>
<td>8-4-2 UC</td>
<td>183.130</td>
<td>29.415</td>
<td>78.130</td>
</tr>
<tr>
<td>8-4-2 C</td>
<td>151.510</td>
<td>25.255</td>
<td>142.160</td>
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<tr>
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<td>103.620</td>
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<tr>
<td>8-2-2 C</td>
<td>180.040</td>
<td>24.195</td>
<td>183.130</td>
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</table>

Two-Way ANOVA

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<tr>
<th>F Value</th>
<th>p</th>
<th>df</th>
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</thead>
<tbody>
<tr>
<td>990.44</td>
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</tr>
<tr>
<td>6.75</td>
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<td>198</td>
</tr>
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<td>482.76</td>
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</tr>
<tr>
<td>0.46</td>
<td>.4556</td>
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</table>

on two dimensions (Figure 7). The output layer consists of two nodes. If the left node is active and the right node is inactive, this is interpreted as a choice for the left stimulus, and vice versa. As in the empirical designs, the first task consists of three phases: the pre-shift phase, the RS phase and the EDS phase. The pre-shift phase consists of learning the first SR relations correctly for some trials. After the pre-shift phase, the network learns both the RS and the EDS. The network starts learning in both phases with the same connection strengths, namely, the matrix that resulted from the pre-shift phase. Variability in learning rapidity within an experimental condition is caused by the random adjustment of the initial values of connection strengths.

**Reversal Shift Versus Extradimensional Shift**

Most network architectures, that is, 8-4-2 networks (both C and UC) and 8-2-2 UC networks, learn an EDS faster than a RS. This accords with non-selective encoding. The 8-2-2 C networks show no difference between the number of learning cycles in the RS phase and the EDS phase (Table 2). This gives no clear indication for the learning mode of these networks. Therefore, the 8-2-2 C networks are examined by means of a trial-by-trial analysis.

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6 Pre-shift phase ends when the network responds 10 times in sequence correctly. That is, the activity of the output node is above 0.55 if the correct responses are 1 and below 0.45 otherwise.
Figure 8. Time courses of the averages (of 10 runs) of the proportions of correct responses of a 8-2-2 C network in the RS and the EDS phases. Responses on both stimulus pairs in the RS phase are represented by a thick solid line. Proportion correct on stimulus pairs in the EDS phase with unchanged reinforcement are represented by a thin solid line. Proportion correct on stimulus pairs in the EDS phase with changed reinforcement are represented by a thin dashed line.

**Trial-by-Trial Analysis**

Because the 8-2-2 C network is the only network which deviates from the predictions of behaviourist theories, we performed a trial-by-trial analysis of the discrimination-shift task. Figure 8 displays the scores on the stimulus pairs with unchanged reinforcement during the EDS phase, the scores on the stimulus pairs with changed reinforcement during the EDS phase, and the scores on both stimulus pairs during the RS phase. Scores are averaged over 10 runs. The responses on stimulus pairs with unchanged reinforcement during the EDS phase remain high after the shift. In contrast, the stimulus pairs with changed reinforcement appear to have a learning curve that resembles the responses during the RS phase: The score starts low and increases gradually. Figure 8 agrees fully with predictions from behaviourist theories. Networks with the other architectures are analysed by comparing separately the learning rapidity concerning the stimulus pairs with changed and unchanged reinforcement of the EDS phase with the learning rapidity of the RS phase. For all three network architectures, it holds that after the EDS, the stimulus pairs with unchanged reinforcement need significantly less trials than the stimulus pairs with changed reinforcement (Scheffe's S: p < .001 for all comparisons). As expected, within both the changed and the unchanged stimulus pairs, no difference is found (Scheffe's S: p > .9 for all comparisons).

**Optional Shift**

The third experimental design of the discrimination-shift task applied to the four PDP networks concerns the optional shift. After a pre-shift, the net-
work learns only one stimulus pair, consisting of two input patterns, with reversed feedback. After the network reaches criterion, the remaining input patterns are judged by the network without learning taking place. The responses on these test trials indicate a RS, indicate an EDS, are both right, are both left, or indicate no choice (i.e., both output nodes are either above .5 or below .5). Figure 9 shows the results of the four network architectures in two conditions: with and without increasing the number of learning trials during the pre-shift and the shift-discrimination phases. Without extra learning trials, the criterion for an output node being active or inactive is less severe (> .55 = 1, < .45 = 0). In this condition, networks of all tested configurations respond most of the time congruently with an EDS. Only in a few cases, a RS is performed. These results agree with the predictions of behaviourist theories and the empirical results concerning animals. If the number of learning trials is increased, that is, the criterion for an output node being active or inactive is more severe (> .9 = 1, < .1 = 0), the responses agree more often with an EDS. These results agree with behaviour found in rats. Children, however, do not show this effect as a result of increasing the number of learning trials (Tighe & Tighe, 1966).

Win-Stay, Lose-Shift
The last performed analysis concerns the win-stay, lose-shift behaviour. T.S. Kendler (1979, 1983) used this criterion as an indication of hypothesis-testing behaviour. At the outset, it is expected that the network performs a kind of incremental learning (McClelland & Jenkins, 1991, p. 69). The learning behaviour of the network confirms this expectancy: The lose-shift strategy
is never performed consistently during the pre-shift phase (four configurations each completed 100 runs); win-stay is performed consistently very often (96% of 8-4-2 C and 8-2-2 C networks, 89% of 8-2-2 UC networks, and 88% of the 8-4-2 UC network followed a win-stay strategy). T.S. Kendler (1979, 1983) only examined the win-stay behaviour as an indicator of hypothesis testing. These simulation results show, however, that this might be insufficient evidence for hypothesis testing.

Summary
Simulation results of the tested feedforward PDP networks with error backpropagation performing the discrimination-shift tasks show that the learning behaviour of these networks agree with both predictions of behaviourist theories and discrimination-shift behaviour of animals. T.S. Kendler (1979, 1983) distinguished two components of discrimination-learning behaviour: encoding (selective and nonselective) and behaviour regulation (incremental learning and hypothesis testing). It was expected beforehand that the network would perform in an incremental learning mode (McClelland & Jenkins, 1991, p. 69), which is confirmed in that the network does not perform lose-shift behaviour. However, selective encoding of input stimuli was at least expected for networks with constrained connections. The latter could not be confirmed. Although the 8-2-2 C network, in contrast to the others, did not need more learning trials for a RS than an EDS, the analyses of the other experimental designs (trial-by-trial analysis and optional shift) yielded clear evidence for the nonselective encoding mode. The optional-shift and trial-by-trial analysis show that the learned SR relations are stimulus specific and not concept specific. The learning behaviour of feedforward PDP networks with error backpropagation in all tested configurations appears to be better described as making direct connections between stimulus and response rather than making connections by way of mediating concepts.  

An equivalent conclusion was drawn in a study by Jansen and van der Maas (1995), who analysed balance scale data of both the empirical study of van Maanen et al. (1989) and the neural network described in the first part of this article by means of latent class analysis (LCA). In contrast to Siegler's (1981) rule-assessment methodology, LCA provides a statistical

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7 Apart from the reported results, the RS versus EDS design is also performed by presenting only one stimulus at a time (the input layer consists of four nodes) and either one or two output nodes and either four or two hidden nodes. Results of these simulations agree completely with the above reported results in that the EDS is learned faster than the RS.

8 At the 24th annual symposium of the Jean-Piaget Society, we were introduced to the article by Kruschke (1992) in which the connectionist model ALCOVE was applied to the discrimination-shift task. It appeared that ALCOVE, like the PDP networks described earlier, learns extradimensional shifts (interdimensional relevance shifts) faster than reversal shifts (intradimensional feedback reversals).
test for the presence of latent classes of response patterns that agree with rules. The latent class models do not fit the PDP data indicating that the behaviour of the PDP network, in contrast to the empirical data, could not be characterized by rules. Hence, the results of the discrimination-shift simulations do not seem to depend fully on the simplicity of the task. Nevertheless, it would be interesting to perform a simulation experiment with a discrimination-shift task in which the discrimination of the stimuli is more complicated to learn.

From this we can conclude that the rules detected with the rule-assessment methodology in the behaviour of the PDP network do not satisfy the rule systems Piaget and Siegler had in mind. McClelland (1989, p. 39) speculated that the available representations "might serve as the raw material used by the more explicit reasoning processes that appear to play a role." However, if explicit rules are not only a reflection of the learning behaviour but play a prominent role in it, what is suggested by both Piaget's theoretical ideas and the empirical results of the discrimination-shift task, the scope of the network is too small to be appropriate for modelling the process of stagewise development.

CONCLUSION AND DISCUSSION

McClelland (1989), McClelland and Jenkins (1991), and Plunkett and Sinha (1992) proposed feedforward PDP networks with error backpropagation for modelling specific developmental phenomena, including stagewise development. We investigated this claim with regard to two important properties of Piaget's and Siegler's theory: the discontinuity hypothesis and rule-governed behaviour.

The first aspect, which concerns the stagewise character of cognitive development, is investigated with the use of catastrophe flags. Catastrophe flags provide precise criteria for detecting discontinuous transitions due to continuous changes of control variables. Empirical studies of learning Piagetian concrete-operational tasks and formal-operational tasks in children give strong evidence for the presence of at least three catastrophe flags. In order to apply these catastrophe flags to the behaviour of the PDP network that learns the balance scale task, a full replication study of McClelland's (1989) simulations was carried out. It appears that the learning behaviour of the network does not meet the tested criteria: bimodality, inaccessible region, and sudden jump. Because these criteria define necessary conditions, this implies that the network does not show discontinuous transitions. The contrasting results between our study and that of McClelland can probably be attributed to the fact that McClelland classified responses before analyses. Whereas we used the raw score in the analyses. In detecting discontinuities,
raw scores are preferred, because the classification in rules is already discrete in itself.\footnote{One other connectionist model of simulating stages of cognitive development is proposed: the cascade-correlational model of Shultz and Schmidt (1991) and Shultz (1991). According to the authors, qualitative changes occur through the recruitment of new hidden units. The latter seems to be a discontinuous event in itself. If that is the case, we cannot speak of a genuine discontinuity by means of continuous or gradual change of a control parameter. However, we did not replicate their studies in order to examine the occurrence of discontinuities by means of catastrophe flags.}

The second aspect, that is, the nature of concepts and rules that are learned, was investigated by means of the discrimination-shift task. This task is thought to discriminate between learning behaviour based on SR relations, on the one hand, and learning behaviour based on concept mediation, on the other hand (H.H. Kendler & Kendler, 1975). Inhelder and Piaget (1958) and Siegler (1978) argued that rules are not only a consistent description of the behaviour, but they govern the behaviour as well. This implies that during development on for example the balance scale task, children acquire mediating concepts instead of simple SR relations. Empirical results from the literature show that animals and children up to about 6 years of age behave as predicted by behaviourist models, but older humans do not. Simulation studies with feedforward PDP networks with error backpropagation in several configurations reveal that the learning behaviour of these models agrees with that of animals and not with older humans. This implies that these networks are better described as systems that form SR relations than systems that learn rules.

Do these negative results imply that neural networks cannot contribute considerably to developmental theory in general or stagewise development in particular? We do not think so for two reasons. First, we agree with McClelland, Jenkins, Plunkett, and Sinha that it is essential in developmental theory that neural networks include learning behaviour, which does not hold for most cognitive models. Moreover, the learning paradox seems to be a serious problem for cognitive models. That is, cognitive models cannot explain the learning of qualitative new behaviour. Second, we think that solutions exist for both these problems observed in this article.

A solution of the first problem requires neural networks which show phase transitions in their learning behaviour. Phase transitions have been observed several times in neural networks with feedback recall, that is, a recurrent connection structure. Mostly, those phase transitions have no positive effect on the network (van der Maas, Verschure, & Molemaar, 1990) and are sometimes described as serious problems for recurrent networks with error-backpropagation learning (Doya, 1992; Pineda, 1989). However, in the so-called neural oscillators (Grossberg & Somers, 1991; Shuster &
Wagner, 1990) they appear to be of great significance in linking visual objects (phase locking) which agree in one or more properties like position, orientation, movement direction, and velocity. However, the discontinuities in neural oscillators have little to do with learning. For now, neural networks that show phase transitions which are functional in the learning process are very rare. There might be one example: the dynamical recognizer of Pollack (1991). Although this network seems very promising, further examination is needed to establish genuine phase transitions in the learning process. From a biological perspective, phase transitions are more common in neural networks. The central nervous system appears to be inherently nonlinear at several different processing levels like biochemical regulation (King, Raese, & Barchas, 1981), neural activity (Meunier, 1992), and higher levels of neural mass action (Freeman, 1975; Gray, 1982; Grossberg, 1982). Therefore, specific nonlinear properties, for instance, phase transitions, are expected to occur in neural implementations that agree with biological structure, instead of being an exception.

The second problem, the behaviourist character of the learning process, seems partly due to the manner in which concepts are formed in the supervised feedforward PDP networks with error backpropagation. In order to classify input, the correct answer is required. This is the reason for calling these networks supervised. Feedforward PDP networks with error-backpropagation can learn an extremely broad range of input-output functions. However, the need of supervision constrains the applicability of these models. On the one hand, empirical research on, for instance, implicit memory (Cleeremans, 1983; Reber, 1993) shows that learning can occur without specific feedback being available. On the other hand, research on language acquisition (Pinker, 1989) shows that even if feedback is available, it is not always effective. Networks in which processing is only based on local information without an external teacher being incorporated (instead of global information and supervised learning) are often called self-organizing. These networks usually function as associative memories and are able to make classifications without an external teacher. Often, they are proposed as mechanisms of implicit learning (Cleeremans, 1993; Reber, 1993). Consult Simpson (1990) for an extended overview of both supervised and unsupervised networks. However, in order to learn concepts that are appropriate for solving a specific task, categorisation should participate on failure and success. Carpenter, Grossberg, and Reynolds (1991) showed that both properties, supervision and concept formation by self-organization, can be combined in one network, ARTMAP. In an ART network (Carpenter & Grossberg, 1987; Grossberg, 1980), which is the main building block of ARTMAP, forming categories consists of hypotheses formation and confirmation, whereas the implementation is consistent with neurology. Although it is not clear whether these or other networks can meet the criteria applied
in this article, there exist several alternatives for the specific properties of feedforward PDP networks with error backpropagation.

We mentioned two problematic aspects of a feedforward PDP network with error backpropagation in modelling stagewise development. Those shortcomings of the network became clear because precise criteria were used (a) for detecting transitions, namely, catastrophe flags, and (b) to distinguish concept-mediating learning behaviour from learning SR relations, namely, the discrimination-shift task. To our view, we did not come to inherent difficulties of neural networks. On the contrary, we think neural networks can contribute greatly to developmental psychology, on the condition that extended comparisons are made between neural networks, on the one hand, and both cognitive psychology and neurobiology, on the other hand.

REFERENCES


SIMULATING STAGENWISE DEVELOPMENT OF RULES


