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Cognitively active externalization for situated reflection

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Abstract

This paper offers an explanation of how collaboration leads to abstract and flexible problem solving. We asked the individual and paired subjects to indicate 3/4 of 2/3 of the area of a square sheet of paper and found that (1) they primarily folded or partitioned the paper rather than algorithmically calculating the answer, (2) they strongly tendened to backtrack and confirm their proto-plans on externalized traces such as creases on the paper, and (3) only the paired subjects shifted to the mathematical strategy in their second trials. Based on these results, we propose that two factors, individuals' activeness in choosing and confirming the initial strategies and the frequent role exchange between task-doing and monitoring in collaborative situations, interact in collaboration to generate various solutions differing in the degree of abstraction, which are then reflected upon by the participants to lead them to abstraction.

Keywords: Externalization; Situated reflection; Collaboration; Role exchange

1. Introduction

1.1. Overview

In this paper, we offer an explanation of how collaboration can lead to abstract and flexible problem solving. We base our arguments on two factors. First, human beings actively use external resources; they are active in the sense of finding ways to utilize such resources rather than passively reacting to the physical features of those resources. They actively leave *traces* while solving a problem, such as written notations and folded pages, which enable them to retrace the path to their present point. Because of the same activeness, however, they may only

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see what they expect to find in such traces and are not always able to assume objective perspectives. We present ample empirical evidence to support this view in this paper. Second, the roles of performing and monitoring tasks are often frequently exchanged during collaborative joint problem solving, which enables the paired subjects to produce solutions varying not just in kind but in degree of abstraction. The monitor has opportunities to observe emerging events in the situation from a perspective slightly different from that of the doer. We will demonstrate that in a collaborative situation these two factors, the active use of external resources and frequent role exchanges, interact and produce solutions that vary from highly individualistic and situated to abstract and flexible alternatives. By experiencing this set of various solutions, our paired subjects adopted the most abstract solution to the subsequent transfer problem more often than the solos did.

The basic motivation of our research is our desire to understand the mechanisms that support flexible problem solving, an indispensable factor in creative learning. We believe that the basic form of human higher order cognition is the interaction among inner and outer resources; the inner being the accumulated and structured knowledge that humans have and the outer the physical and social environments in which humans live. Initial solutions to problems in the outer world may depend heavily on (or may take full advantage of) what is available in the external environment at the given time and space. Different solutions may evolve for the same problem being solved in slightly different situations. People then sort these solutions, to integrate those that are most promising into a "schema," or some abstracted representation of the solutions. Starting from externally bound problem-solving attempts, humans accumulate various solutions with different degrees of abstraction, and then organize them together so that the new solution-schema may be tested with yet newer problems at a different time and in a different space. The key factor in this development is to obtain various solutions. Collaboration with co-problem-solvers is a very promising source of such variability. This is a highly plausible course of learning (though not necessarily an easy one), and studies on the interaction of inner and outer resources should lead us to a better design of environments that will facilitate such a productive learning process.

We used a simple task that can be solved with external materials and many different strategies, one of which relies solely on internal resources (algorithmic knowledge), to empirically justify the above claim. The task we used is a simple fraction calculation, obtaining three-fourths of two-thirds, following the classic example of de la Rocha's cottage cheese problem (de la Rocha, 1986; Lave, Murtaugh, & de la Rocha, 1984). We did not use cheese but rather a square sheet of origami paper and other materials. The task was solved in many different ways, the variability of which offered us an opportunity to observe the subjects actively using external materials and their situated, yet interactively flexible, reflections in a collaborative setting.

Human beings are very active external resource users. They actively found ways to use external resources, such as a sheet of origami paper, more than 90% of the time, creating external traces of their trials. They showed a strong tendency (65% of those whose paths could be analyzed) to backtrack the process on externalized traces as if to confirm that they were proceeding as planned. They actively (or subjectively) perceived the final solution as the outcome of their own trial path and were not always able to take a more objective viewpoint. We interpreted this as the subjects having some proto-plan based on the initial judgement they made upon taking on the problem and pursuing it as far as they could see it worked.

We also demonstrate that paired subjects took advantage of a collaborative situation in a highly interactive, reflectable way. The solution paths externalized by each subject in a paired collaborative situation provided the pair with chances to objectively re-interpret their solutions. One member's solution was immediately followed by the other's provision of a different interpretation of the trial, one only slightly less externally bound but sufficiently different to trigger the first member's shift of perspective toward a more broadly oriented, abstract view of the entire problem situation. When asked to solve an isomorphic but apparently different transfer task, this situated reflection often led our paired subjects to shift the solution from an externally bound to more algorithmically oriented one.

The issues we address in this paper are not new. We focus on the nature of

1. external resources in everyday problem solving,
2. reflection of external traces of the cognitive process, and
3. collaborative and interactive problem solving.

Recent literature describes interesting contrasts on these topics, and we wish to contribute our results. We first summarize such contrasting views in the following subsections.

1.2. Nature of external resources

The contrasting views of interpreting the human use of external resources consist of whether it is *active* or *passive*. For example, two contrary reactions were invoked when Brown, Collins, and Duguid (1989a) adopted de la Rocha's case as supporting evidence for a situated perspective of cognition. A dieter in de la Rocha's case responded to a recipe calling for "3/4 of 2/3 cup of cottage cheese" by taking two-thirds cup of the cheese, flattening it into a uniformly thick circular disk on a cutting board, and drawing a cross on it with his finger so that he could save the desired amount by discarding the quarter. He never verified his procedure with a written algorithm, which would have produced $(3/4) \times (2/3)$ cup $= 1/2$ cup.

This dieter's action can be viewed as a human actively using and constructing external resources that include the question, the available utensils, and the circular spatial object in the situation (Brown et al., 1989a; Pea, 1993). In contrast, it reveals human passiveness to available methodology, suppressing retrieval of even simple mathematical knowledge (Palinscar, 1989; Salomon, 1990; Wineburg, 1989). The latter researchers stated that the dieter's solution was constrained strongly by the situation and could not be transferred to other situations with imaginary examples. Wineburg (1989) challenged the dieter by giving him an imaginary Thanksgiving recipe for his extended family of 16 or using molasses instead of cheese, while Salomon (1990) requested conditions without a cutting board. They expected this dieter to fail in these extended situations, and thus, considered him to be a passive responder to the given situation, at the same time implying the advantage of the algorithmic solution that uses internal, abstract resources. Brown, Collins, and Duguid (1989b) responded by proposing that we should consider activeness in this use of external resources and understand its merit to identify the nature of everyday cognition, stating that "[the dieter] did what all of us do all the time . . . . We need to understand the initial impulse to try a situated approach." Reconsidering this debate, it is clear that Brown et al. assumed some internal tendency toward the use of external resources that is inherent in humans, not the environment. This is a driving cognitive mechanism over the internal–external interaction, which is also different from an internal resource such as algorithmic knowledge, computationally able to be simulated without relying on any external factors. The first aim of this paper is to clarify the nature of this tendency, what Brown et al. called the "initial impulse" without any specification.

A closer inspection of the dieter's solution process itself suggests that this "initial impulse" could be considered to be a proto-plan for the solver to initiate his solution path without failure, given the availability of the external world. The dieter first measured 2/3 of cheese, and then laboriously removed it from the cup and made a circular disk so that he could divide it into four equal parts. The representation of the cheese as 2/3 of the whole cup was transformed to a "one as a whole" disk. This indicates that he utilized the intermediate result and re-represented it in the external world, which made completion of his first solution step clearer as well as making it easier for him to initiate creation of the second step and verify its progress externally. In this sense, the dieter was an active user of external resources, with the proto-plan of dividing the overall task into simpler subtasks to obtain the secondary amount of "3/4 of 2/3 cup" and by interactively actualizing the plan in the external world.

Proto-plans can also be identified in other literature. Kirsh (1996) demonstrated how people arrange their external world to make their environment more hospitable by using the task of identifying the longest stick from among a scatter of 20 sticks of different lengths. His subjects tended to manage the spatial organization by partitioning the sticks into "discarded" regions and "candidate" regions, which helped them to keep track of the sticks they had already sorted and those that remained. The proto-plan here was interpreted as the subdivision of the overall task into a selection task of the longest one from the candidate region that the subjects interactively created in the external world. Hutchins (1995), in a very different context of ship navigation, talks about "propagation of representational state," to show how externalizations and their re-interpretations of routine procedures externalized forms of proto-plans, dynamically change course to deal with an emergency.

We do not claim that this proto-plan is always consciously created or institutionally practiced, even though we believe it is observable. We transplanted the cottage cheese problem into a laboratory situation, where the same problem can be repeatedly presented to a number of subjects in different conditions to yield sufficient process data to clarify the nature of a proto-plan.

1.3. Nature of situated reflection

Humans use external resources actively, not passively. This active use leaves physical traces of the trials, the primary functions of which are to confirm their progress toward a solution. In this sense, they tend to perceive in the external traces what they expect to confirm subjectively. However, opposing views claim that because the external resources are external, they should be, and are, perceived objectively.

Reisberg (1987) describes one such case in a somewhat opportunistic view of this issue. According to Chambers and Reisberg (1985), when an ambiguous figure, such as a Duck–Rabbit, was shown for a brief period and the image was stored in memory, the subjects could form just one of two possible interpretations. The subjects were then asked to draw what they had in mind on a sheet of paper, and it is reported that they most often immediately noticed the other interpretation. Reisberg (1987) obtained similar results using auditory stimuli and suggested that externalized traces can lead to the restructuring of the original interpretation when they are observed from a specific, possibly goal-oriented, perspective (also see Suwa, Gero, & Purcell, 2000).

This initially appears to be a clear case indicating that individualistic externalization can be viewed and used objectively, but there was a strong demand for the subjects "to take a different perspective" in their drawings (they had been instructed in several ways to "try to see the mental image differently"). The re-interpretation was possible, but with a strong trigger like the experimental demand. This indicates that reflection, or the perspective shift, occurs only rarely and is constrained strongly by what is available or who is around in any particular situation.

Lave et al. (1984) report their cottage cheese case as "at no time did the [dieter] check his procedure against a paper and pencil algorithm . . . . Instead, the coincidence of problem, setting, and enactment was the means by which checking took place." This implies that re-interpretation of the problem was indeed difficult, but we should also remember that the dieter did not have much chance to re-interpret the traces. With the cottage cheese example, there is no way to know whether the cheese in the cup served as an objective trigger for any other solution than the one actually taken by the dieter himself.

We introduce in our experiments a sheet of origami paper instead of cottage cheese and assign a task of drawing oblique lines on it instead of isolating a specified amount. Subjects can compare the intermediate and resultant solution states with the original state using origami, considering the original state to be the whole sheet of origami. This was impossible for the dieter in de la Rocha's study, since the original state of the whole cup ceased to exist when the cheese was moved from the measuring cup onto the cutting board. It is at least possible for our subjects to grasp the resultant area as half of the whole. Consequently, this awareness could mediate a re-interpretation of the overall problem situation, which was not the case in de la Rocha's study. We use the origami task to seek the answers to two questions—whether our subjects would take an objective view to reflect upon the external traces on the origami paper, and whether external support would enhance that reflection. The process analyses would also reveal the nature of this reflection.

1.4. Nature of collaboration

Collaboration that is developed constructively can help participants deepen their conceptual understanding or develop abstract representation (Greeno, Sommerfeld, & Wiebe, 2000; Miyake, 1986; Schwartz, 1995). What is the mechanism underlying this conceptual or abstract understanding? One possibility is that a multi-agent situation has a better chance of including a highly abstract solution than does a single-agent situation and thus, this solution is adopted as the group solution (Okada & Simon, 1997), possibly in search of convergence among the members (Roschelle, 1992). The other possibility is to assume some built-in mechanism in the collaboration that triggers a gradual generation and integration of various solutions, from the externally bound and situated ones of each individual at the start to more abstract ones proposed in turn by the members, leading each member to a more abstract conceptual understanding independently from the other member. We will briefly review some studies of constructive interactions in collaboration to determine their positive features before providing insight into this mechanism.

Miyake (1986) asked her subjects to comprehend how a sewing machine made stitches and found that the process was iterative in the sense that some understanding of one stage prompted additional questions, which then led each member to a deeper understanding. Collaboration in this study was a trigger for individualistic efforts to restructure each member's knowledge independently. There was little chance that one member would completely agree with a possible explanation offered by the other member. This conflict serves as a trigger for reflection and impels each subject to further explore and reorganize his or her own explanations.

Schwartz (1995) demonstrated that dyads tend to develop external representations into abstract ones that can bridge the members' perspectives and provide a way to clarify a structure across multiple contexts. He observed a pair of middle school students who were required to construct visualizations on the topic of biological transmissions. One student drew a picture of a monkey and a tree, with the letter H ambiguously connected to the monkey by an arrow. The second student considered the first student's representation and drew a picture of a banana with an arrow from the monkey, which confused the first student, who had intended to use the arrow as a label (H = monkey), not to indicate transmission. They negotiated the conflict between their active views on the similar externalized objects and developed a more abstract, elaborate use of the notations. Conflict resulted in abstraction, but we could not tell how generalizable this result is from his report.

Another example can be found in Greeno et al. (2000), wherein three middle school students worked on a problem of successive multiplication. The question was how many mice there would be in the eighth season from 20 original adults with a birth rate of four per couple. They first arrived at the solution of $20/2 \times 4 \times 8 = 320$. One student noticed by plotting the number of mice on a graph that the number should increase exponentially, not linearly (also see Stenning & Sommerfeld, 2000). The graphing practices during this trial involved step-by-step plotting of points. Those externalized points were studied collaboratively as intermediate results, which helped them reconsider their calculations in the context of the step-by-step sequence of events in mice reproduction, leading them to reconstruct their knowledge. This is a compelling case, but again there remains a lot to be done to generalize the results.

There are two common features in these studies. First, collaboration tends to be more constructive when the pair's intermediate results can be shared, easily understood, and even controlled by both members of the pair (also see Hastie, 1986; Miyake, 1994, 1996). Second, these collaborative reflections yield rich solution variations and trials, which generate conflicts that iteratively trigger deeper reflection of each member's solution and ultimately lead to an integration of the variations. We wish to learn if there is some mechanism underlying such integration.

In a typical collaborative problem-solving situation, while one member acts out his/her solution, the other tries to understand it. Miyake (1986) called the former a task-doer and the latter a monitor. She observed that the monitors who tended to propose more topic-divergent motions or drastic changes in the problem-solving process, and attempted to propose an explanation for the mechanism. When the task-doer declared that s/he had reached a local solution, the monitor could not always share or identify the doer's individualistically active version of the solution. The monitor could only observe the doer's solution process externally, possibly from a broader perspective that included the solution within the overall problem situation. This meta-cognitive reviewing of the other's solutions from a broader perspective could have triggered them to start restructuring their own solutions at a different conceptual level.

The variations contributed by pairs in joint problem-solving situations thus represent different degrees of abstraction, rather than just a mix, if we generalize what Miyake observed. They may differ in their abstraction levels because a role change occurs between the task-doer and the monitor, whereby the monitor's view is always slightly less focused on the specific actions of the task-doer. We assume that what the participants would do here depends on each member's individual proto-plan, each highly situated in the external world. The monitors are also situation-bound and can only re-interpret the situation from a new, only slightly more abstract, standpoint. In other words, the monitors are bound to the external form the doer contributes to the situation, but they are sufficiently free to offer a different interpretation, which increases the solution variations in terms of the degree of abstraction. This implies that the shift from external to internal is highly interactive and gradual, if it happens at all.

We had pairs of subjects solve the same fraction problem to test this. We attempted to identify their gradual rise through the stages of different degrees of abstraction in their processes, from one externally bound to its re-interpretation, and further to the mental re-interpretation of the original problem itself as a mathematical one. We also tested whether this gradual rise corresponded with their role exchange, in order to identify some reason why the collaborative setting is more advantageous than solo situations.

In the following sections, we will describe the behavioral data of how our subjects solved the problem in different conditions, and the results of some detailed analyses of the processes. Section 2 describes the experimental results. In Section 3, we analyze our solo subjects' solution processes to examine the nature of their proto-plans. In Section 4, in order to understand the nature of collaboration, we analyze the pairs' processes to identify the types and scope of the variations of their solutions, and to know how such various solutions are integrated through the shift from external to internal solutions.

2. The performance data

The task we use throughout this paper is to obtain three-fourths of two-thirds, or its reverse, of a square material such as a sheet of origami. At first, we compare its mathematical solution to non-mathematical solutions. The mathematical solution is to solve it by calculation, as 2/3 multiplied by 3/4, while non-mathematical solutions do not involve any calculations. We show two sets of basic data in this section. First, we will reveal that this problem was mostly solved using non-mathematical measures, and second, that these non-mathematical solvers shifted their strategy to a mathematical one to solve a similar problem the second time, rather often in collaborative situations but only rarely in solo situations.

2.1. Experiment 1: Consistent use of external resources

The first question we want to answer is whether the subjects' use of external resources is *active* or *passive*. If the subjects actively searched for usable aspects of external resources, they would not be influenced by material appearances or by immediate usability of such materials.

The core task in Experiment 1 was to ask the subjects to shade 3/4 of 2/3 of the area of a sheet of origami to observe how they use external resources to solve this problem. It can be solved by mathematical knowledge, but it can also be solved by folding, cutting, or pencil marking the origami paper. These are the external features, or resources, that origami paper can offer the problem solver.

Other materials do not share the same features. A thick sheet of cardboard appears harder to fold than origami, while it can still be cut. An acrylic board does not appear foldable or even cut-able at all. The empirical question here is whether these apparent differences in possible actions will prohibit the use of external resources.

Paper-folding is a cultural practice for our subjects, where folding a square into fourths, not into thirds, is common.[1] Moreover, folding it into fourths is physically easier than folding it into thirds. Thus, comparing the solving patterns between 3/4 of 2/3 and its reverse, 2/3 of 3/4, would be a further test of the ease of folding as well as of the effects of cultural and everyday practices.

2.1.1. Experimental design

In Experiment 1, we asked subjects to shade 3/4 of 2/3 of the area of a sheet of origami (hereafter referred to as the "3/4-first" condition) or its reverse, 2/3 of 3/4 of the area ("2/3-first" condition).[2] When we refer to these two conditions together, we refer to them as the "origami" conditions.

We compared these to two other conditions to examine harder-to-fold materials, a thick cardboard ("cardboard" condition) and an acrylic board ("acrylic board" condition). The task with these materials was to indicate 3/4 of 2/3 of the area.

2.1.2. Method

2.1.2.1. Subjects. Forty college students served as subjects for course credit. All subjects were Japanese and knew the tradition of origami. They had enough mathematical knowledge to solve simple fraction problems.

2.1.2.2. Materials. Standard origami paper of a square shape (15 cm × 15 cm) with color on one side was used in the origami conditions (both the 3/4-first and the 2/3-first conditions). The sheets of thick cardboard and acrylic board were the same shape and size as the origami. A thick sheet of cardboard and a pencil with an eraser were given to the subjects in the cardboard condition; a sheet of acrylic board of 2 mm in thickness and a marker with a white board eraser were provided in the acrylic board condition.

2.1.2.3. Procedure. Ten subjects were randomly assigned to each of the four conditions. The subjects were tested individually. Each subject in the origami condition was given a sheet of origami first (coloredside up) and then a pencil. Next, the instructions were given orally; the instructions were to "designate 3/4 of 2/3 of this origami paper by shading with oblique lines" for the 3/4-first condition. The phrase "3/4 of 2/3" was converted to 2/3 of 3/4 for the 2/3-first condition. When the subjects finished their task, the experimenter asked, "How did the answer come out?" and accepted the finished product, correct or incorrect. A post-experimental inquiry was made to ascertain whether the subjects were aware of the one-half factor of the resultant area or the possibility of fractional calculation, and whether they executed any calculations.[3] The subjects' hand movements and speech were videotaped throughout the trial.

Each subject was seated at a table in the cardboard and acrylic board conditions, with the materials already in place. The instruction was to indicate 3/4 of 2/3 of the thick cardboard or the acrylic board by shading. The experimenter retrieved the products when the subjects finished. No feedback was provided. Other details of the procedure were the same as the origami conditions.

2.1.3. Results

2.1.3.1. Mathematical and non-mathematical strategies. All the subjects solved the problem successfully. The most abstracted representation of the task can be solved by calculation, proving the result to be one-half, and there should be no difference between the two origami problems, "3/4 of 2/3" or "2/3 of 3/4." Subjects who used internal calculations only had to first multiply 3/4 (or 2/3) by 2/3 (or 3/4) to get the answer 1/2, and then map it out on a sheet of origami in just one step, drawing a line in the middle of the paper and shading the half. When this occurred, we identified it as using the *mathematical strategy*. In contrast, there were different ways to solve the problem when subjects used external resources by folding or partitioning. Each part included in both the 2/3 and the 3/4 was marked by creases on a sheet of origami when folded, whereas each part was compartmentalized by pencil (or marker) marks, pencil (or marker) lines, or subjects' finger positions when partitioned. We call these *non-mathematical strategies*.[4]

2.1.3.2. Strategy coding. We categorized the subjects' videotaped solution processes into strategies to examine their pattern of using external resources. We describe the details of the coding procedure in Section 3.3.1. This procedure enabled us to identify the mathematical and non-mathematical strategies in all but one of the 40 trials. In the exception, a subject's answer to the post-experimental inquiry served as the criteria to identify it as using the mathematical strategy.[5] The inter-coder reliability was 100%.

2.1.3.3. Strategy preference. As Table 1 shows, the non-mathematical strategy was consistently preferred to the mathematical strategy in all four conditions. Thirty-seven of 40 subjects solved the task by using external resources. The order of the problems and the material types had no

Table 1
Number of subjects using mathematical and non-mathematical strategies

	Origami		Cardboard	Acrylic board
	2/3-first	3/4-first		
Non-mathematical	9	10	9	9
Mathematical	1	0	1	1

Note: Subjects solved the 3/4-first problem in the cardboard and the acrylic board conditions.

influence on the performance. In addition, the subjects apparently had no difficulty finding and pursuing their non-mathematical strategies. The average solution time of the non-mathematical strategy in the origami conditions was 53.1 s ($SD = 42.8$ s), which is shorter than that (63 s) of one mathematical strategy user.

We could assume that the subjects actively perceived and picked out usable functions of the materials and solved the task regardless of the material's appearances, like the cardboard or the acrylic board, not by reacting to the physical features of the materials. The cultural practice did not influence them either. One interpretation of this pattern could be that the subjects tended to use an external-oriented strategy rather than a mathematical one because the task required a physical outcome. Even so, the results indicate that upon such interpretation the subjects easily succeeded in doing so, regardless of the condition differences. When presented with things they could either fold or cut, or just partition, and asked to answer the problem using such things, they somehow found a way to do that externally and did not resort to mathematics, their inner repertoire of solving the same problem.

2.2. Experiment 2: Solution shift for a subsequent task in collaboration

The second question we want to answer is whether the subjects could reflect their external traces objectively, and if they could, when and in what conditions. Such objective reflection on the correct answer would make the subjects realize the mathematical solution, which, in this case, is quicker and requires less effort. The occurrence of this objective reflection can be identified by the shift of strategies from externally oriented ones to the mathematical one.

2.2.1. Experimental design

We introduce two new factors in this experiment, other people as a different kind of external resource and the follow-up task. The former is implemented by having paired subjects work together to solve problems, and the latter, by adding a second task. Both the paired subjects (the Pairs) and the individual subjects (the Solos) were tested twice, first with the 2/3-first problem and then with the 3/4-first problem, or vice versa. We will refer to the task sequence of first asking 2/3 of 3/4 followed by 3/4 of 2/3 as the 2/3 of 3/4-first condition, whereas its reverse, the 3/4 of 2/3-first condition, will indicate that the first task given was 3/4 of 2/3, followed by 2/3 of 3/4.

We examined how often they used the mathematical strategy again to verify the effects of these new factors, in particular whether they shifted from the non-mathematic to the mathematic strategy as they moved to the second trial. We will interpret such a shift as an indication of the objective reflection on the outcomes of the first trials.

2.2.2. Subjects

Ninety university students were randomly assigned to either paired conditions (30 pairs of subjects) or solo conditions (30 solo subjects). The two conditions in both the Pairs and the Solos differed in their task sequence, either 2/3 of 3/4-first or 3/4 of 2/3-first. All members of the paired subjects knew each other and were of the same sex. None had participated in Experiment 1.

2.2.3. Procedure

The procedure for both the Pairs and the Solos was the same as the origami conditions of Experiment 1, with the following exceptions. All the subjects were given a new sheet of origami after finishing the first trial and were presented with the second problem, with 2/3 of 3/4 and 3/4 of 2/3 switched. The subjects were again asked, "How did the answer come out?" after the second trial.

The paired subjects were given just one sheet of origami and explicitly directed to collaborate, while the Solo subjects were asked to think aloud. The Solo subjects had practiced thinking aloud prior to the trial with the game of *Master Mind*.

2.2.4. Results of a strategy shift

All the subjects solved the problem successfully in this experiment, too. The strategy shift was coded in the same manner as explained in Section 2.1.3. As Table 2(a) shows, the subjects in both conditions tended to use the non-mathematical strategy in the first trial. Combining the two task sequence conditions, only 5 of the 30 pairs and 1 of the 30 solo subjects used the mathematical strategy. This proportion did not significantly differ between the conditions ($p > .10$; Fisher's exact test, two-tailed).

Twenty-six solos also used the non-mathematical strategy in the second trial (Table 2 a). Of the remaining four solos, one solo used the mathematical strategy for both trials and three shifted, which represents only 10% of the whole. However, a larger proportion of the Pairs shifted the strategy in the second trial (Table 2 a). Specifically, 19 pairs used the mathematical strategy while only 4 solos did so. The proportion of the Pairs (63.3%) was significantly larger than that of the Solos (13.3%; $p < .01$; Fisher's exact test, two-tailed). The proportion was also larger than that of the nominal Pairs (24.9%, calculated by a truth–wins combination of individual performances, see Lorge & Solomon, 1955). Five of the 19 pairs used the mathematical strategy on both trials. Fourteen pairs, or 46.7% of the whole, shifted to a mathematical strategy from a non-mathematical one in the first trial.

Table 2 (b) shows the numbers sorted into the different orders of the problems posed. The performance level of the first tasks remained relatively constant across all conditions.

Table 2

	Pair	Solo	Nominal Pair
(a) Number of Pairs vs. Solos using mathematical strategy ($N = 60$)			
First trial	5/30	1/30	1.97/30
Second trial	19/30	4/30	7.46/30
(b) Number of Pairs vs. Solos using mathematical strategy by problem orders ($N = 30$ per each)			
3/4 of 2/3-first			
First trial	3/15	0/15	0/15
Second trial	8/15	0/15	0/15
2/3 of 3/4-first			
First trial	2/15	1/15	1.93/15
Second trial	11/15	4/15	6.93/15

The tendency that the Pairs shifted while the Solos did not is clearer in the 2/3 or 3/4-first sequence than in the other sequence.

As far as the shift rate is concerned, the Solo subjects do not seem to have reflected their outcome of their first trials objectively, while some objective reflection was possible for the Pairs.

2.3. Summary of performance data

The basic data pattern is clear. (1) Our subjects, whether they were individual or paired, consistently used external resources on their first encounter with the problem. (2) Individual subjects consistently used external resources, whether a sheet of origami or an acrylic board, whether the task was easily doable or not. They were persistent enough to continue using the same strategy even for the second trial, a similar task with only the order of fractions changed. (3) The only conspicuously dissimilar pattern we have uncovered so far is the pairs' tendency to shift their strategy to a mathematical one in their second trials, particularly after solving the 2/3-first problem. We will explore the details of the processes of these patterns to further demonstrate the Solo subjects' consistency (Section 3) and how this consistency interacts with the others' perspectives in collaborative situations (Section 4).

3. Persistent individuals: Situated reflection

We must gain a better understanding of the nature of individual subjects' consistent use of external resources, or of their proto-plans, and how they repeatedly interact with the external traces of their own solutions before we can understand why the Solo subjects did not shift their strategies while the Pairs did. Therefore, we examine three topics in this section—the distribution of their strategy choices (Section 3.2), the steps involved in their solutions (Section 3.3), and their perception of the resultant shaded areas (Section 3.4). Sections 3.2 and 3.3 both aim to see how externally oriented the proto-plans are, and Section 3.4 examines how persistent the orientation is. The target data of analyses came from the Solo subjects of Experiments 1 and 2 using the origami paper. Before the analyses, we explain the various solution strategies with explications of their two salient features (Section 3.1).

3.1. Solution variations

This seemingly simple task, obtaining two-thirds of three-fourths, is often solved in a number of intricate ways, reflecting the complexity of the internal–external interaction of our cognitive processes. To use one of the non-mathematical strategies, the task is literally a two-step problem. The first part of solving 2/3 of 3/4 is to obtain the 3/4. There are several ways to do this on a sheet of square paper, and what one does next depends on what was done first. Thus, non-mathematical strategies come in physically different forms and develop into different courses, some of which are more straightforwardly two-step than others. Some end up with a clearly obvious one-half as the answer, while others do not. Fig. 1 schematically shows possible solutions on a square of paper or other materials.

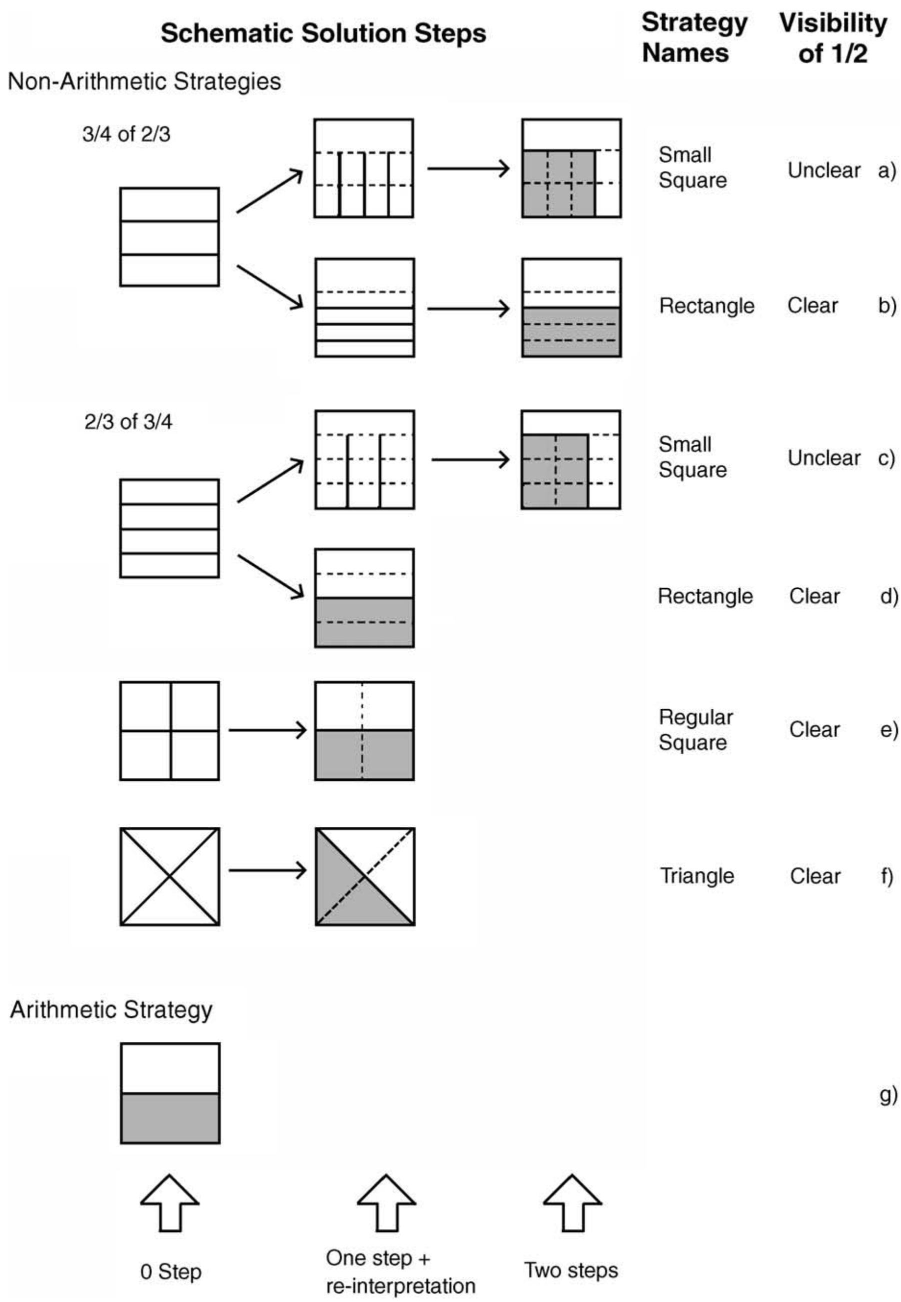

Fig. 1. Strategies.

In Fig. 1, the left column shows how the original square can be worked on at the beginning of each problem type. The rows correspond to different strategies; the top two are for the 3/4-first problem, the middle four are for the 2/3-first problems, and the bottom row is the end result arrived at using a mathematical solution. Subsequent squares on each row demonstrate how the solving process proceeds from left to right, with schematic depictions of each step involved in the solution. The bold lines indicate where actions like folding and partitioning take place at each step for each square.[6] These actions leave traces in forms of creases or lines, some of which may only be drawn mentally. The dotted lines represent these traces from their previous step. The strategies are categorized as (a) and (c) *the small square strategy*, (b) and (d) *the rectangle strategy*, (e) *the regular square strategy*, and (f) *the triangle strategy*.

The images in Fig. 1 are sufficiently self-explanatory, except for two features. One is the number of steps in each solution strategy. Strategies (a)–(c) require two steps, corresponding to the two fractions in the problem. These steps must be physically taken to obtain the answer; i.e., all of them require an additional folding or partitioning to advance from the middle column to the right. A two-step solution can be solved step-by-step, or by subdividing the original problem into two sub-parts and by solving them one at a time. This is what the instructions literally asked for.

The other three, (d)–(f), all stop at the end of the first step, seemingly requiring only one step. However, all of these one-step solutions require a hidden procedure necessary to solve the problem. Let us explain this with the 2/3-first problem. When the subjects are faced with the original square with three straight creases or lines, they can create two-thirds, either vertically or horizontally, the difference of which separates the course into strategy (c) or (d) shown in Fig. 1. However, in (d), the task of creating thirds horizontally has already been achieved as a by-product of the first step of obtaining 3/4 (Fig. 1(d)). Subjects who understand this can skip the second physical step, which instead requires them to preform a mental re-interpretation, or new reading, of the intermediate traces. A similar re-interpretation is also required in the other apparently one-step strategies of (e) and (f).

In this sense, all non-mathematical strategies involve two steps, either physically achievable without much re-interpretation, as in strategies (a)–(c), or one physical step plus a mental one, requiring a perspective shift to perceive the end result of the first step differently, as in strategies (d)–(f). The former strategies are more dependent on external resources, while the latter ones are integrated methods of internal and external resources. We can see to what degree the subjects' proto-plans are externally oriented by examining their preference of type of strategies in Section 3.2.

Fig. 1 represents some possible courses of solving, only schematically. It clearly delineates the left traces on the whole square of paper at each solving step. However, some subjects did not have opportunities to see the traces as they are in Fig. 1. For example, subjects who first folded the paper into fourths and then into thirds successively without opening it did not see the three creases at the intermediate step as in Fig. 1(c) or (d). However, if they unfolded the paper to its original form at the midpoint, they had an opportunity to see the traces. It could have enabled them to confirm the expected three-fourths as the intermediate result, which also marked the completion of their first solution step more clearly than in the case without opening the paper. Re-reading the two-thirds area in the intermediate result as in strategy (d) was also only possible with this opening operation. The operation is embedded in the instructions and can be easily implemented in the external material, and has its own merits, though it requires an additional manipulation. We analyze the solution steps involved in the subjects' actual solving processes to see whether they actually preferred this intermediate opening operation in Section 3.3.

The answer to the question asked at the end of the trials, "How did the answer come out?" is another test of the persistency of the external orientation of the subjects' proto-plans. The end products, the right squares on each row, do not end up in the same shading. In particular, some are visibly half, such as (b), (d), (e), (f), and (g), while (a) and (c) are not. We refer to strategies (a) and (c) as *unclear appearance strategies*; unclear in terms of the answer not being an obvious one-half. The other strategies, (b) and (d)–(f), are called *clear appearance strategies*. The subjects could be active enough to interpret the final solution as the outcome of their own plan. For example, they might have perceived their final products as the three-fourths of the taken two-thirds area, not one-half of the entire origami, even when facing the clear appearance answers. We will examine the relationship between the subjects' answers to the question and their strategies in Section 3.4.

3.2. Strategy choice

First we examine whether the subjects preferred two- or a one-step strategy, as shown in Fig. 1. The non-mathematical strategies explained in Section 3.1 (Fig. 1), can be implemented both by folding or partitioning methods. Thus, they were combined with the methods used to yield eight strategies, such as the "*folding and small square strategy*" for the 2/3-first problem and four strategies for the 3/4-first problem.

Table 3 shows the strategy distribution of the Solo subjects in Experiments 1 and 2. We analyzed the results of 48 non-mathematical solvers' first tasks (two solos in the 2/3-first condition calculated the answer). There we can see a bias toward the folding and small square strategy. The small square strategy totals 25, or 52.1%, when we collapse the problem types and the methods. This strategy requires two physical steps and thus is not an "economic" strategy. However, a two-step strategy like this one is a more straightforward interpretation of the instructions, which required each fraction to be individually made on the paper in steps. As a consequence, the subjects were farther from realizing the abstract strategy. The less preferred strategies are one-step, seemingly economic strategies, but they in fact required additional mental re-interpretation. Overall, the results illustrate the subjects' inclination to external resources, a basic form of their externally oriented proto-plan.

3.3. Intermediate check

The subjects that were active in selecting and pursuing the solution paths could also be active in confirming the progress of their solution on the externalized intermediate traces. It is

Table 3
Strategy distribution of Solos

	3/4-first problem		2/3-first problem			
	Small square	Rectangle	Small square	Rectangle	Regular square	Triangle
Folding	11	2	3	2	4	1
Partitioning	8	4	3	3	4	3
Sum	19	6	6	5	8	4

Note: Two solo subjects in the 2/3-first condition calculated the answer.

advantageous to unfold and open the origami at the middle point to confirm the progress, as we explained in Section 3.1. We analyzed all the videotapes in regard to their solution paths and report here the results of the 23 solo trials where the folding strategies were used in Experiments 1 and 2.

3.3.1. Solution path coding

Fig. 2 shows some of the possible solution paths of the folding and small square strategy in the 3/4-first problem. The figure consists of origami shapes that illustrate the states of the products and arrows that follow the behavioral processes, starting from the top, the initial state, and progressing through each possible state on different paths (for more detail, see Shirouzu & Miyake, 1999a).

Similar flowcharts were created for the other non-mathematical strategies, on which the first author (H.S.) identified the paths for each subject in the entire dataset. An independent coder coded the 20 trials of the 10 subjects (five in the 2/3-first and five in the 3/4-first conditions) and arrived at 100% inter-coder reliability. This reliability also guarantees the procedure of the strategy coding in Sections 2.1 and 3.2.

3.3.2. Results

The typical paths and the subjects' preferences in this strategy are schematically represented in Fig. 2 by lines, the thickness of which roughly corresponds to the relative number of subjects who took that path. They preferred paths (a), (b), and (c) over the other paths, as seen in the figure. The difference between the preferred paths and the other paths is that the preferred paths have the unfolded original whole square in their middle point, while the others do not, even though the former (i.e., paths (a), (b), and (c)) require more steps. The midpoint corresponds to the end of making the first fraction, two-thirds. Nine of the eleven subjects that utilized this strategy followed paths that returned them to the original stage at the end of making the first fraction, while only two followed those paths that did not. We call this route the going-back-to-the-original route. One subject laboriously went backwards to the initial state along the path he had followed before entering into path (d) (this is represented by wavy lines in Fig. 2). The subjects evidently preferred paths on which they could view the outcome in relation to the original whole at the point where the first fraction was achieved.

Table 4 summarizes the number of observed trials taken from all the folding strategies, dichotomized into those that followed the going-back-to-the-original path and others.[7]

It is clear that the subjects tended to go back to the original unfolded stage (total 15 out of 23), as if they subdivided the overall task at this point to confirm their intermediate results and to plan the next step of their solution. They seemed willing to devote extra effort to this. The results in this section imply that the subjects' proto-plans involved some decomposition of the original task so that they could take advantage of their external traces.

3.4. Verbal references of the outcome

Our analyses here focus on whether the subjects subjectively perceived their answers as literally "3/4 of 2/3" and referred to them as such, rather than objectively perceiving them as one-half of the original whole and explicitly stating this. We now consider the 47 solo subjects

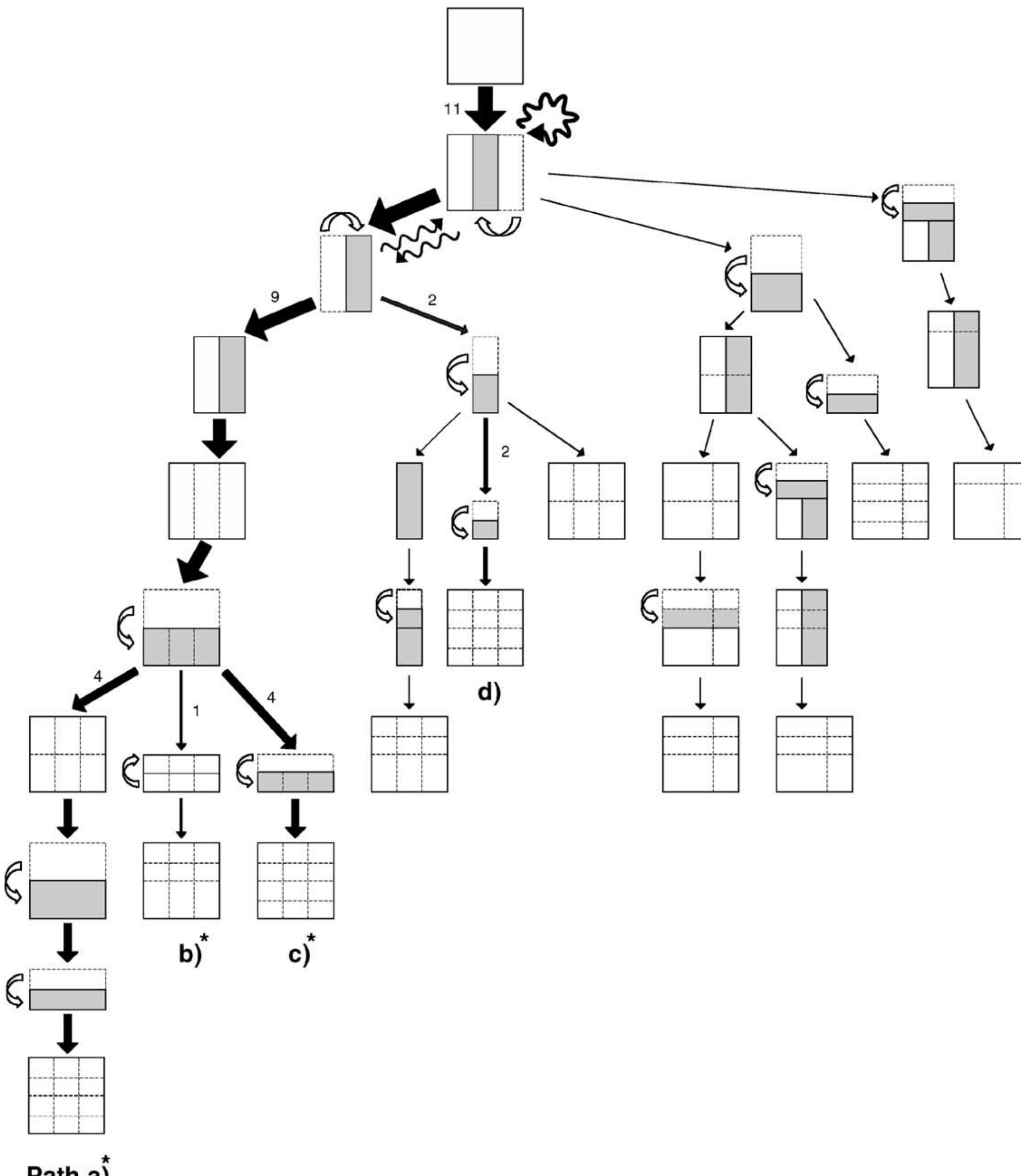

Fig. 2. Solution paths taken in the folding and small square strategy for the 3/4-first problem. The shaded areas aid distinction between origami sides. The curving arrow represents folding, and the part surrounded by dotted lines corresponds to the folded part. The broken line is a crease. The straight arrow between the figures represents the transitions between states. The paths with asterisks are defined as the going-back-to-the-original routes. The wavy lines indicate that one subject went back to the original size by unfolding the thrice-folded paper before entering into path (d).

Table 4
Number of trials followed the going-back-to-the-original route and others in folding strategies

	Strategies						
	3/4-first problem		2/3-first problem				Total
	Small square	Rectangle	Small square	Rectangle	Regular square	Triangle	
Going-back-to-the-original	10	2	3	–	–	–	15 (65.2%)
Not-going back	1	0	0	–	–	–	1 (4.3%)
Unidentifiable	–	–	–	*2*	*4*	*1*	*7 (30.4%)*

Note: It is difficult to identify whether the purpose of opening the folded paper was to go back to the original or to perceive the goal area in the rectangle, the regular square, and the triangle strategies for the 2/3-first problem. Thus, they are classified as "Unidentifiable."

who used the non-mathematical strategy for the first trial in Experiments 1 and 2. They all ultimately shaded the answer area correctly. Thirty-two (68.1%) of those forty-seven subjects[8] did not refer to one-half in their replies to the question, revealing additional evidence that the subjects were pursuing their own solution processes and possibly not objectively seeing what was physically there.

Fourteen subjects out of the thirty-two who did not verbalize the one-half reported the solution procedure literally; e.g., "I isolated two-thirds of the origami, then proceeded to make three-fourths by folding." This indicates that they did actively subdivide the task and went through the solution process step-by-step. Ten subjects replied with demonstrative words like "Here" to indicate the products before them, four reported that their outcome was the result of their trials ("it turned out like this"), three said nothing, and one smiled. Overall, the subjects appeared to perceive in their products what they had planned to achieve and not the objective one-half. They actively (or subjectively) viewed the final solution as the outcome of their own trial, or of their proto-plan, and were not always able to take a more objective view. These results imply that the individual subjects did not take a second look at the end results, and probably did not re-interpret the traces at the midpoint.

We further examined whether this subjectivity of the subjects was sufficiently strong to be solely independent of the external shapes. Some shapes were visibly one-half, others less so, as shown in Fig. 1. Table 5 shows the number of subjects who verbalized the answer clearly as one-half according to the appearance conditions. None in the unclear appearance category

Table 5
Number of subjects who referred to one-half in relation to the visibility of the resultant area in the first trial ($N = 47$)

	Verbalized	Not verbalized	Sum
Clear appearance	15	14	29
Unclear appearance	0	18	18
Sum	15	32	

Note: Two solo subjects calculated and one subject's answer was not recorded.

verbalized that the result was one-half. All of the 15 subjects who replied so (even if they had not used a mathematical strategy) were in the clear appearance category. However, not all members of that category could verbalize the one-half result. The subjects apparently could not verbalize it as one-half unless the outcome was readily seen as one-half. The external shape did matter, but it was not sufficient.

Remember that in Experiment 2, there were three solo subjects who shifted their strategies in the subsequent task. They all had clear appearances at the end of their first trial, and they all explicitly stated the outcome as one-half. Though the number is small, this result indicates that it was possible for some subjects to reflect upon the external shape and change the interpretation, as a future source for a gradual but flexible solution shift.

3.5. Summary: Proto-plans and their situated reflection

To summarize, this section made clear three aspects of the suggested proto-plans for our subjects. They were externally bound, schemed to take advantage of externally left traces, and persistent enough to lead the subjective perspective toward confirmation of their progress as well as the resultant shadings.

We assume, from the behavioral data in Section 2, that each individual more or less held a certain proto-plan upon hearing the instructions, and proceeded to pursue it. The subjects preferred the two-step, externally oriented strategy (Section 3.2) as well as the paths involving an extra step of external backtracking (Section 3.3). These results converge to suggest that the proto-plans were indeed quite different from the mathematical strategy. The gap between them appears to require successive re-interpretations of the intermediate outcomes, hinting the transition, if it occurs, would be gradual.

We are going to analyze the collaborative processes based on this assumption that the individuals are basically individualistic and their proto-plans are externally oriented. These characteristics could be a cause for the collaborators to develop solution variations, a candidate condition for the gradual shift to take place.

4. Flexible pairs: Interactive and gradual shifts

In order to propose some explanation on how collaboration works toward more flexible problem solving, we analyze the pairs' shifts of strategies in this section. We have thus far demonstrated that the individuals are subjective, active external problem solvers who leave traces while solving problems and interpret such traces according to their own solution plans. We also demonstrated that the pairs shifted their strategies on their second trials, which was not always easy for individuals to accomplish, precisely because they were active in the sense defined in this paper. The analyses of individual solving processes in Section 3 suggested that the shift would be gradual from an externally bound individualistic one to a more abstract, flexible one. We will first propose a model of steps involved in the shifts and then examine the process data (protocols) of collaborative settings to see how the collaboration triggered them.

We selected the nine pairs of the 2/3 of 3/4-first condition that shifted to the mathematical strategy in the second trial to analyze their verbal protocols (see Table 2 b). We focused on the

2/3-first problem since the solution process for this can branch out into two different strategies, providing opportunities to identify any re-interpretation of the intermediate outcome. All pairs shifted at the onset of the second trial, indicating that whatever contributed to the shift had occurred during the first trial. Therefore, we analyzed their processes in the first trials.

We developed two sets of coding schemes, one to capture the subjects' mental steps reflected in their verbalization and the other to define the role exchanges. The first coding scheme uses cues in the conversation to identify the steps, which follow the four solution steps briefly introduced in Section 3.1 and to be fully explicated in Section 4.1.1. It specifically focuses on how the traces are perceived, on the viewpoints of the solver, and on the focus of his attention. It is used to examine the stepwise shift from a non-mathematical, external strategy to a mathematical one. The second scheme involves the role exchange that often occurs during joint problem solving, the task-doer–monitor exchange described in our previous work (Miyake, 1986). We will combine the results of these two sets of coding to examine whether any consistent relation is evidenced between the role exchanges and the shifts through the steps.

4.1. Gradual shift: From external to internal

The Pairs initiated their solution using external resources in their first trials. Table 6 shows the strategy distribution of the Pairs in Experiment 2. They often used the folding strategy (23 out of 25 pairs) and unfolded the origami paper after obtaining the first fraction (20 out of 23 pairs) as most of the Solos did. However, more Pairs applied the rectangle strategy, not the small square strategy, to the 2/3-first problem than Solos (8 out of 11, or 72.7% vs. 5 out of 11, or 45.5%). As described in Section 3.1, solutions (c) and (d) in Fig. 1 separate from each other after completion of the first step; (d) requires some re-interpretation of the intermediate outcome and (c) requires pursuit of a physical second step. The Pairs' bias toward the rectangle strategy, (d), implies that there was more re-interpretation in the Pairs, while the Solos tended to stay on their externally oriented two-step proto-plans, which let the Pairs shift more than the Solos.

4.1.1. Framework for levels analysis

We assume that there are four successive *levels* from the most external to the most abstract strategies, each corresponding to one of the four *steps* to be explained below. They are

Table 6
Strategy distribution of Pairs

	3/4-first problem		2/3-first problem				Sum
	Small square	Rectangle	Small square	Rectangle	Regular square	Triangle	
Folding	10	1	3	8	1	0	23
Partitioning	0	1	0	0	1	0	2
Sum	10	2	3	8	2	0	25

Note: Five pairs calculated the answer.

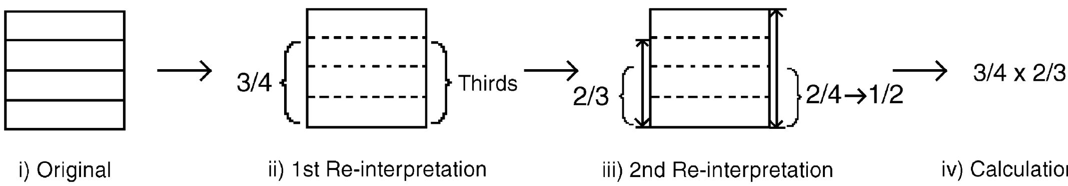

Fig. 3. Schematic steps involved in the interpretation shift.

schematically depicted in Fig. 3. If the shift is gradual, we assume it would take all of these steps in a stepwise fashion. To illustrate this, we will briefly describe the subjects' perspectives at each step.

4.1.1.1. Level 1. The view of Level 1 concerns to paper with creases of four equal rectangles as shown in Fig. 3(i). The subjects may have created this state in an attempt to get three-fourths and complete the first subtask in the original task, depending on their externally situated proto-plan. They focus on the three-fourths area on this externalized trace and plan to divide this area into three as their second physical step (so that they can isolate two parts out of the three).

4.1.1.2. From Level 1 to Level 2. The three-fourths area with the two lines in it makes the desired three by itself, i.e., the denominator of the second fraction, as shown in Fig. 3(ii). Thus, the subject can perceive that there are three parts already demarcated by creases and therefore that the second folding has already been done if he focuses on the same three-fourths part as in Level 1 but with a slightly more objective view. It further allows him to take two parts from the three to obtain the answer without any physical action.

4.1.1.3. From Level 2 to Level 3. This level change involves realizing that the two-thirds area within the three-fourths equals one-half of the original frame of the origami paper as shown in Fig. 3(iii). The subject can perceive the appearance as 2/4, or 1/2 of the original square, if he adopts a view sufficiently broad to put the two rectangles back into the four.

4.1.1.4. From Level 3 to Level 4. Once one perceives that 2/4 is in fact 1/2, it can be understood that it equals the result of 3/4 multiplied by 2/3 as indicated in Fig. 3(iv). The abstract numbers, not the shapes or creases, are the target of reflection at this level.

4.1.2. Perspective level coding

Our purpose here is to determine whether the pairs shifted their views from Level 1 to Level 4 step-by-step, as in Fig. 4(a), or jumped directly from Level 1 to 4, as in Fig. 4(b).

We identified the levels by detecting the key expressions associated with each level. The operational definitions and the key expressions used for the coding of the levels are shown in Table 7. The coding is based on the contents of the expressions, not their forms. We used the behavioral cue to distinguish Level 1 from Level 2. When a subject referred to the 2/3, such as "the 2/3 is here," he could either be continuing to fold the paper or expressing his

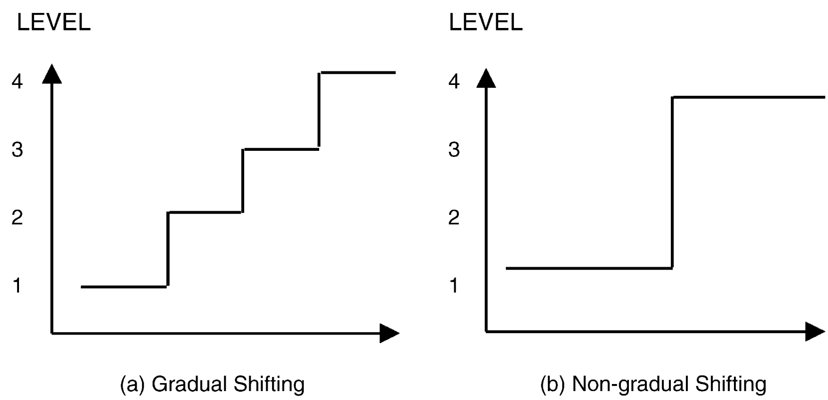

Fig. 4. Schematic diagrams of gradual (a) and non-gradual (b) perspective shifting for the 2/3-first problem. The vertical line represents the levels of perspective; the horizontal line represents time.

re-interpretation without attempting to fold it further. We took the former behavior as a sign of Level 1, and the latter as an indication of Level 2.

A set of transcripts was prepared from the videotapes. They were first divided into utterance units carrying minimum meaningful contents. The units were assigned to a level when any of the pre-defined key expressions for the levels was detected. The current level was maintained until a new explicit key expression appeared.

We use a typical pair, Pair A, as an illustrative case to demonstrate how the described coding scheme was applied to generate the coded transcript. An excerpt is shown in Table 8. The subject ID and his utterances are shown in lines of its left column. Some lines were omitted or integrated to save space. The adjacent right column shows the number of the identified level. Key expressions used for the coding were italicized in the table. The levels were assigned to pairs, not to each individual, to capture the upward shift by pairs, not the minor ups and downs that occur in each individual's solution protocols.

The level of Pair A at a particular line in Table 8 was determined by verbal expressions uttered by either of the member at around that line. Thus, they were at Level 1 at line 2 because

Table 7
Categorization cues for perspective level categories

Level	Operational definition	Key expressions
1	Reference to dividing the end result of the first step, three-fourths, into three[a]	"You can get the 3/4 this way, and the 2/3 that way." "I am folding it into thirds this way."
2	Reference to the existence of three parts as the basis for obtaining the second two-thirds	"The thirds is already here."
3	Reference to the one-half answer	"This is one-half."
4	Reference to multiplication	"You can calculate the answer."

[a] Special cases include questions such as "Should I fold this again?" requests such as "Would you make 2/3?" and evaluations "Making 2/3 looks difficult, doesn't it?" all of which explicitly refer to dividing the original paper into three.

Table 8
Example of level coding: Pair A's protocol in their intermediate step

	ID	Utterance [behavior]	Identified level
1.	A1:	This line is the three-fourths [opening the paper and tracing the line]	
2.	A1:	so *folding this as futon-like, we can get a one-third*, you see? [*starting to fold it into three-fourths again*]	Level 1
3.	A2:	Of three-fourths . . .	
4.	A2:	aha	
5.	A2:	two-thirds of three-fourths is [opening the paper folded by A1]	
6.	A2:	so, of three-fourths . . . [pressing each part of the three-fourths]	
7.	A2:	*the two-thirds is here* [*pointing at the two-thirds area*]	Level 2
8.	A1:	Oh, you, silly	
9.	A1:	this is *the half* [pressing the area pointed to by A2]	Level 3
10.	A2:	Oh, my [with the facial expression of amazement]	
11.	A1:	Why, you should use your brain more	
12.	A2:	Two-thirds of three-fourths is going to be . . .	
13.	A2:	first, where is the 3/4? [sliding the unfolded paper in front of A1]	
14.	A1:	Yes, here it is, this is the 3/4 [surrounding the 3/4 area]	
15.	A2:	Then, the 2/3. Where? [sliding the 3/4 folded paper]	
16.	A1:	The 2/3 of this is . . . ahhh, here [leaning over the paper]	
17.	A1:	oh, I've got it. We could have solved it *with multiplication*	Level 4
18.	A2:	If We could have	

Note: Some lines are omitted or integrated due to the space constraint. Key expressions are italicized. See text for details.

A1 said, "Folding this, we can get a one-third," and at Level 2 at line 7 because A2 mentioned "The two-thirds is here." In the same fashion, they were at Level 3 at line 9 (A1: "This is the half"), and Level 4 at line 17 (A1: "We could have solved it with multiplication"). Even though A1 individually regressed from Level 3 to Level 2 at lines 10–16, apparently to confirm the location of the emergent answer, we coded the Pair at Level 3 throughout these lines because they had already been exposed to that verbalized level at a previous line, line 9 (for more details, see Shirouzu & Miyake, 1999b).

The protocols for the nine pairs were analyzed from the beginning of the first trial to the beginning of the second trial (including the answers to the question "How did the answer come out?"). The inter-coder reliability was calculated on all the protocols between the first and second authors, yielding 96.4% reliability. The first author's coding will be used for the analyses throughout this paper because of this high reliability.

4.1.3. Results of perspective shifts

All nine shifting pairs started from Level 1. Seven of those reached Level 4, while the remaining two stopped at Level 3. The basic conclusion is that the shifted pairs gained perspectives at least higher than Level 3 while they sought solutions in their first trial.

Seven out of the nine pairs ascended stepwise through more than three levels from the lowest to the highest, while two pairs jumped directly from Level 1 to Level 4. Three among the seven pairs progressed through all four levels stepwise (as in Fig. 4a), and two ascended three levels stepwise. The other two pairs ascended through four levels but skipped the explicit reference

Table 9
Number of shifts following expected or unexpected moves within the framework of gradual perspective shifting

	Number of shifts
Expected moves	
Level 1–2	7
Level 2–3	5 (7)
Level 3–4	3 (5)
Sum	15 (19)
Unexpected moves	
Level 2–4	2 (0)
Level 1–3	0
Level 1–4	2
Sum	4 (2)

Note: Numbers in parentheses are those when an implicit shift to Level 3 is included as Level 3.

for Level 3. Nonetheless, the members of these pairs who jumped to Level 4 from Level 2 did search for or confirm the location of the answer by pointing to the center of the paper, without code-able verbalization. They could have worked on the answer area within the frame of the paper, thus, implicitly passing through Level 3 between Levels 2 and 4.

Table 9 dichotomizes the above shifts according to whether the shift followed the expected step-by-step fashion or not. Out of all 19, 15 shifts, or 78.9%, were step-by-step, as expected. This percentage is higher if we include Level 3 as not explicit but weakly incremental as just described above (numbers in parentheses in Table 9). These results indicate that the majority of pairs gradually accomplished the calculation through repeated re-interpretations of the externalized traces.

4.2. Role exchange and shift patterns

The basic mechanism of role exchange in joint actions may be responsible for the gradual shift we have demonstrated thus far. One member in a joint work situation performs a task and the other monitors it. We can expect some direct relationship between the role exchange and the level shift if this simple mechanism puts the monitor in a position to gain a broader view of the solution processes, thus, compelling the monitoring member to lead a shift to a higher level. We will describe how we define the roles of the monitor and the task-doer in Section 4.2.1 to clarify which member assumed the monitoring role at specified times. The question is whether it is the monitor that led the shift to a higher level.

4.2.1. Role coding

The roles were assigned on the basis of two factors: what is talked about, and who has the origami. The member who talked about task-relevant contents or was engaged in task-relevant actions while manipulating the origami was defined first as the task-doer, and then the other member was designated the monitor.

We categorized all the utterances into 18 categories according to the actions taken or to be taken, such as whether they were explicitly planned, reported, or identified, to define the task-relevant verbalizations. All the categories are listed in Appendix A. Among them, the categories "plan," "execution report," "identification," "reference to one-half," "reflection" and "execution guide" were considered task-relevant without exceptions. Examples of irrelevant utterances are "consent," "non-understanding," "criticism," "evaluation," and so on.

The second criterion was who controlled the origami paper. Mostly one member who held the paper was coded as the owner, a condition for a doer. However, when one member reached for and touched the paper in front of the other, it was categorized as both of them controlling the paper (this occurred primarily at "identification" points). Calculation and clear reference to one-half were exceptions and independently assigned to task doing, even though the actor was not manipulating the paper.

When a member satisfied both criteria, we defined him at that point as the task-doer and the other as the monitor. One exception to this was when a member repeated the instructions ("instruction" in Appendix A) while manipulating the origami. Reciting instructions suggests a monitoring action rather than active efforts of the solving. These cases were thus conditionally coded as task doing only when the member assumed the task-doing role in the subsequent utterances. The role assignments were mutually exclusive.

This analysis was conducted on all the utterances in the seven pairs who shifted gradually from Level 1 to Level 4 (five pairs) and Level 1 to Level 3 (two pairs). In total, 128 lines were clearly coded by this procedure out of all 269 lines of the seven pairs' protocols. We coded the other lines as remaining in the same role as in the previous line. The inter-coder reliability about the coding of a Pair (all 73 lines of Pair A's protocol) between the first and second authors was 76.9%. When the coders disagreed, the discrepancy was resolved by discussion. This coding identified 49 turns in the seven pairs in total. Thirty-six turns, or 73.5%, had expressions that corresponded to one of the levels coded by the scheme described in Section 4.1.2.

4.2.2. *Results of role exchange and level shifts*

Table 10 shows the correspondence between the role exchange data and the level shifts of Pair A. We added the role exchange data in this table to the right columns of their level shifts (see Table 8). We divided the eighteen lines for Pair A into seven turns, lines 1–2 (A1), 3–8 (A2), 9–11 (A1), 12–13 (A2), 14 (A1), 15 (A2), and 16–18 (A1), where the member in parentheses was the task-doer. One case may require some explanation. A2 started to repeat the instruction ("instruction") while reaching for the origami on line 3. He was coded as a task-doer because he assumed that role on line 7 ("identification" of the answer on the paper).

The shift from Level 1 to Level 2 was made by A2 at line 7 in Table 10, after he changed his role from the monitor in the previous turn, which is indicated in the table by an oblique arrow. During this turn, A1 was the monitor, who became the doer and shifted the level up to Level 3 in the next turn. The shift to Level 4 was again made by A1 at line 17, after he resumed the monitor's role in line 15. He said, "this is the half," which was not directly followed by his next statement, "we could have multiplied." Every shift in this dataset is associated with turn-taking, where the monitor in the previous turn became the doer and shifted up the level. We call this the monitor-initiated shift. Every shift in Table 10 was initiated by the monitor, whether it was A1 or A2. A contrasting case would be a single member stays in the role of task-doing and shifts up the levels by himself. We call this the doer-initiated shift.

Table 10
Example of role coding: Pair A's protocol in their intermediate step

ID	Utterance [Utterance Category]	Identified Level	Task doer	Monitor
1.	A1: This line is the three-fourths [**Identification**]		A1	**A2**
2.	A1: so folding this as *futon*-like, we can get a one-third, you see? [**Plan**/ Question]	**LEVEL 1**	---------A1	**A2**
3.	A2: Of three-fourths… [Instruction]		A2	**A1**
4.	A2: aha, [Surprise]		A2	**A1**
5.	A2: two-thirds of three-fourths is, [Instruction]		A2	**A1**
6.	A2: so, of three-fourths… [**Identification**]		A2	**A1**
7.	A2: the two-thirds is here [**Identification**]	**LEVEL 2**	**--------A2**	**A1**
8.	A1: Oh, you, silly [Criticism]		A2	**A1**
9.	A1: this is the half [**Identification**]	**LEVEL 3**	**-------- A1**	A2
10.	A2: Oh, my [Surprise]		A1	A2
11.	A1: Why, you should use your brain more [Criticism]		A1	A2
12.	A2: Two-thirds of three-fourths is going to be… [**Plan**]		A2	A1
13.	A2: first, where is the 3/4? [**Execution guide**]		A2	A1
14.	A1: Yes, here is, this is the 3/4 [**Identification**]		A1	A2
15.	A2: Then, the 2/3. Where? [**Identification/ Execution guide**]		A2	**A1**
16.	A1: The 2/3 of this is… ahhh, here [**Identification**/ Surprise]		A1	A2
17.	A1: Oh, I've got it. We could have solved it with multiplication. [**Reflection**]	**LEVEL 4**	**-----A1**	A2
18.	A2: If we could have. [Consent]		A1	A2

Note: The utterance categories in bold represent task-relevant verbalizations. The member who verbalized them was coded as the task-doer when he also controlled the origami paper. The horizontal lines correspond to turn takings between A1 and A2. The left-downward arrows represent the monitor-initiated shift.

Focusing only on the first appearance of each level, we examine whether the member that assumed the monitor role initiated these level shifts. Table 11 summarizes the total number of shifts between levels and the number of monitor- and doer-initiated shifts.

The monitors tended to initiate the rise through the levels, particularly at lower ones, where the physical visibility was more responsible for the re-interpretation. The shift from Level 3 to Level 4 was less associated with turn taking, especially when we include implicit Level 3 as Level 3 (number in parentheses in Table 11). The shifts between these levels involved reference to internal knowledge and not a re-interpretation of externally emergent forms.

4.3. *Summary: interactive and gradual shifts*

The detailed analyses of the shifting pairs revealed the interactive and gradual integration processes of their solution variations of different abstraction levels. In a two-person solving

Table 11
Number of monitor- and doer-initiated shifts

	Number of shifts		
	Total	Monitor-initiated	Doer-initiated
Level 1–2	7	7	0
Level 2–3	5 (7)	3 (4)	2 (3)
Level 3–4	3 (5)	1 (1)	2 (4)
Level 2–4	2 (0)	1 (0)	1 (0)
Sum	17 (19)	12 (12)	5 (7)

Note: Numbers in parentheses are those when implicit Level 3 is included as Level 3.

situation, one's solution process and his externalized traces can be observed by another, who has an externally bound yet slightly broader, more abstract view of such traces. This leads a Pair to an iterative chain of re-interpretations. While the monitors are not free from the external form contributed by the doer, they are free to offer a different interpretation of the emergent externalization of the doer's solution process. We take the position that the incremental shift we observed resulted from an accumulation of such effects.

The physical states of the origami paper could also be a factor for the incremental shifts. Fifteen out of the seventeen shifts were made when the origami paper was dealt with, as opening it to its original form. The external traces were the resource for active interpretation and re-interpretation for each member, contributing to the collaborative reflection.

5. General discussion: interaction of situated reflection and role taking

A simple problem like getting two-thirds of three-fourths on a square sheet of origami can be solved in many different ways. Human beings are active users of external resources when solving such problems, and they leave physical traces of their solutions on the materials they use, which provides others with opportunities to observe the process. Verbalizations by the observer (called a monitor in our paper) regarding the process often refer to broader conditions than the focus of the solver (a task-doer), since the monitor does not necessarily share the proto-plan of the doer. This contributes to increases in the variety of solutions to be reflected, which vary not just in type but also in the degree of abstraction. Such reflection often led our paired subjects to adopt the abstract mathematical solution strategy in their second trials.

The solution strategies used by the Solo subjects varied to cover a broad range. When alone, half the subjects used a two-step solution of obtaining the three-fourths first and then delineating the two-thirds in the obtained three-fourths. Some applied the one-step solution of perceiving the three parts in the already-obtained three-fourths and directly taking two out of the three. Others, though fewer in number, saw the end result as being equal to a one-half, shifting the frame-of-reference back to the entire area of the paper. Finally, some could adopt an entirely origami-free solution by multiplying the 2/3 with 3/4 and obtaining 1/2 as the answer. In our analyses of the pairs' processes, though, we found that these variations of steps often occurred successively from the most externally oriented two-step to the most abstract one, and the shift between such steps coincided with role shifts between task-doing and monitoring. We propose that this indicates that having the monitor's perspective is separate from the task-doer's proto-plan is advantageous, and contributes to increasing the abstraction levels of the solutions. We also propose that it was this variation of solutions in the degree of abstraction that let the pairs explicitly re-interpret the problem situation as a mathematical one.

We summarized current controversies in research on cognitive interaction in Section 1. Based on the findings in this report, we take the position that (1) the external resource use of human beings is active, (2) their reflection on such external use is positively situated, and (3) during collaboration, participants have the advantage of integrating various solutions differing in the degree of abstraction, which is a natural outcome of the interaction between the task-doer's proto-plans and the monitor's re-interpretation.

We started this research with the goal of identifying the theory or underlying principles of collaborative learning, so that we could support it. We have demonstrated here that it is possible for people in collaborative settings to interact constructively so that the solution variations as the outcome of their interaction can lead them to more flexible problem solving. We consider that such resourcefulness in a collaborative problem-solving situation should entail more than just being able to solve the problem effectively. The ability to access various solution strategies is only the beginning. Selecting one strategy and modifying it is a necessary second step. It is also important to assimilate a new solution into our overall knowledge so that it can be portable and used in future new situations. We aimed our research at clarifying the collaborative cognitive processes that lead to this resourcefulness and at finding ways to support them.

Some results warrant future development, though they are still currently incomplete. The most prominent is the question of how the different solutions can be integrated once they are available. Integration does not occur easily. Professors at a university do not expect their students to thoroughly integrate the lectures given in a semester course; it often takes years for slightly different results under the same research topic to be formed into a widely acceptable unanimous scheme. Understanding this integration process would be a significant contribution from cognitive science to neighboring sciences such as learning sciences.

We have demonstrated that collaborative settings offer a systemic advantage in yielding abstraction variations that work favorably for integration. However, in the origami task, the abstract strategy was known to the subjects and explicitly cited by most of the pairs that shifted their strategies in the subsequent task. The integrated, abstract form was readily available to them. This differs significantly from cases where only varieties of lower level solutions are available and no sure method of abstracting them has yet been demonstrated. Two features of the processes we analyzed are relevant to contemplation regarding this integration process in general. They are the effects of verbalization in collaboration and the use of external traces. We will briefly review these two topics before concluding this paper.

If reflection is situated and externally bound as shown in this paper, it requires some additional support to lead itself into abstraction. Because verbalization is one of the most natural courses of abstraction, guiding verbalization could provide potential support for integration in learning situations. The literature indicates that a teacher's appropriation can lead to co-construction of the proximal development zone in a class (Newman, Griffin, & Cole, 1989) and that reciprocal verbalization of meta-cognitive instructions has sustainable effects on reading comprehension (Palinscar & Brown, 1984). However, these pioneering studies did not necessarily attempt to guide the verbalization to gradually increase its abstraction. We observed in one of our pilot studies that a teacher (the first author, H.S.) in a small, collaborative classroom gradually guided the talks of the entire class and led the children to understand the mathematical strategy using the same set of origami tasks utilized in this paper. The effect of this understanding endured more than 6 months for the students who could articulately verbalize the solution in its abstract forms (Shirouzu, 2001; Shirouzu & Miyake, 2002). More careful analyses are required with more variety of tasks and settings to verify the principles for such verbalization guidance.

Another feature, the external trace, is the inevitable residual of the active use of external resources. We analyzed in detail how the physical features of such traces play essential roles in soliciting and enabling re-interpretations in this paper. The effect of externalized outcomes is not a feature unknown to cognitive science (e.g., Chambers & Reisberg, 1985; Kirsh, 1995; Kirsh & Maglio, 1994; Larkin & Simon, 1987; Norman, 1988; Reisberg, 1987; Schwartz, 1995; Zhang, 1997; Zhang & Norman, 1994). The effect of external *traces* of one's cognitive processes and how they would enhance our cognition is still a new research topic. While traces are powerful, we have demonstrated that each individual perceives them in situated ways that are neither common nor objective. Additional studies are necessary to understand the nature of this situated reflection, to begin to know how we could free ourselves from our own confirmation bias.

Traces left behind lose their connection to their original situation as time passes. Proto-plans may be lost, leaving only the traces available for further objective re-interpretation. The ability to backtrack over external traces may have value not just for collaborating pairs but for independent individuals at different times and in different places. These prolonged effects of traces also generate new sets of research questions.

Collaboration also needs to be studied further. Once established, it works advantageously for solving problems, deepening each member's understanding, and creating new ideas. However, establishing collaboration itself can be an issue, particularly in learning situations. Some promising attempts are emerging (e.g., Holbrook & Kolodner, 2000), to which we wish to contribute more of the basic understandings of the type we gained in this research.

The externalized forms of individual problem-solving processes are not always easily understood by others since they depend on the task-doer's knowledge and experience. This enables re-interpretation to be a valuable contribution. We also demonstrated that it does not have to come solely from the doer but could be naturally provided by others. Thus, social diversity can contribute to each individual member's restructuring of inner knowledge, which can then be applied to different situations. Current technology provides us with more and better tools to keep records, to edit them and to share and examine them collaboratively. As we have seen in this paper, what we need appears to be a set of various solutions, differing from each other in appropriate abstraction levels, not just a thorough record of every deed we do. This raises questions like how detailed the records should be, how much editing should be performed before distribution, how to support integration of such accumulated records, and so on. These questions can and should be answered within the context of real-world knowledge acquisition. We plan to expand our work on the theorization of collaborative knowledge construction to support such tasks in authentic cognitive environments.

Notes

1. Japan has a long tradition of folding origami into many shapes; the most famous of which is a "crane." Most Japanese can fold at least one or two shapes. Instructions for how to fold a shape are often given in a series of folding diagrams with a pre-creased sheet. Thus, they are accustomed to folding paper and to viewing creases as "records of folding."
2. The designation of 2/3-first (or 3/4-first) represents the nominal order of the fractions in the problem. Procedurally, a subject must deal with 3/4, rather than 2/3, to solve the 2/3-first problem externally. This complication does not arise in Japanese, the language we used throughout our experiments, because the fractions are mentioned in the procedural order ("3/4 of 2/3" is "*3 bun no 2 no 4 bun no 3*").
3. The subjects had a second trial of a similar fraction problem, and the post-experimental inquiry was posed at the end of the entire experiment, not immediately subsequent to the task described here. See Section 2.2.3.
4. We do not maintain that external resources without internal cognitive workings are preferred or that such preferences are the essential human cognitive nature. The external resource of the origami paper could not be used without the internal cognitive workings of dividing the original sheet and examining the appropriate parts to identify where to fold next. Instead, our interest is ultimately on the interaction between the two resources, and a clear understanding of human fundamental use of the external resources is crucial for that purpose (also see Schwartz & Black, 1996).
5. The subject changed the 3/4-first problem into the 2/3-first problem voluntarily by altering the order of fractions and used the folding and regular square strategy in the first trial. For the second trial (the 2/3-first problem), he directly folded the paper into the half from the start, about which he clearly mentioned his awareness of the availability of the calculation in his post-experimental interview, and thus, his strategy was identified as a mathematical one.
6. The lines are drawn only to the point where they are relevant to the solution. Actual folding and pencil marking can leave more traces than depicted here.
7. A small piece of evidence comes from the paths of the partitioning and the small square strategy. All subjects extended the multiplier line only to the point of intersection with the multiplicand line. They did not draw the line to the edge of the square, indicating that they selectively worked on the area demarcated with the line drawn first, not the original area of the origami.
8. Out of 50 in all, two solved the problem mathematically. One subject's answer was not recorded and thus is not analyzed here.

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Appendix A. Categories for role coding

Categories	Example
Task-doing (relevant)	
1. Plan	"We must fold it into fourths first of all." (before folding)
2. Execution report	"I am folding it into thirds in this way." (with folding), "Two-thirds multiplied by three-fourths is . . . " (with calculating)
3. Identification	"The 2/3 of 3/4 is here," "This is one-half." (with pointing at particular areas or lines)
4. Reference to one-half	"It is one-half." (without any pointing)
5. Reflection	"We could have multiplied."
6. Execution guide	"First, where is the 3/4?" (guiding the other member for "identification")
Task-doing or monitoring (conditional)	
7. Question	"The answer is here, isn't it?" "Should I fold this again?" (to the other), "Is the answer here?" (to oneself)
8. Answer	"(What should we do?) Drawing oblique lines." (replying to the other's question)
9. Repeat	"(This is thirds) Thirds . . . " (repeating the other's words)
10. Consent	"Oh, yes," "I see."
11. Denial	"No, I don't think so," "Oh, you, silly."
12. Non-understanding	"It does not make sense," "I can't see," "Mmm."
13. Criticism	"Why, you should use your brain more," "You have a different image than I."
14. Evaluation	"Making 2/3 looks difficult," "This is the trap."
15. Surprise	"Ahhh, (here)," "Oh, you are right," "Aha," "Oh, is it this?"
16. Request	"Would you make 2/3?" "You should take the initiative next."
17. Instruction	"The 2/3 of 3/4," "3/4 . . . " (without meaningful actions on origami)
18. Interaction with experimenter	(questions or confirmation)

References

Brown, J. S., Collins, A., & Duguid, P. (1989a). Situated cognition and the culture of learning. *Educational Researcher*, *18*, 32–42.

Brown, J. S., Collins, A., & Duguid, P. (1989b). Debating situation: A rejoinder to Palinscar and Wineburg. *Educational Researcher*, *18*, 10–12.

Chambers, D., & Reisberg, D. (1985). Can mental images be ambiguous? *Journal of Experimental Psychology: Human Perception* & *Performance*, *11*, 317–328.

de la Rocha, O. (1986). *Problems of sense and problems of scale: An ethnographic study of arithmetic in everyday life*. Unpublished Doctoral Dissertation, University of California, Irvine.

Greeno, J. G., Sommerfeld, M. C., & Wiebe, M. (2000). Practices of questioning and explaining in learning to model. In L. R. Gleitman & A. K. Joshi (Eds.), *Proceedings of the 22nd annual conference of the cognitive science society* (pp. 669–674). Mahwah, NJ: Erlbaum.

Hastie, R. (1986). Review essay: Experimental evidence on group accuracy. In B. Grofman & G. Owen (Eds.), *Decision research, Vol. 1, Information pooling and group decision making: Proceedings of the second conference on political economy* (pp. 129–157). University of California, Irvine.

Holbrook, J., & Kolodner, J. L. (2000). Scaffolding the development of an inquiry-based (Science) classroom. In B. Fishman & S. F. O'Connor-Divelbiss (Eds.), *International conference of the learning sciences* (pp. 221–227). Mahwah, NJ: Erlbaum.

Hutchins, E. (1995). *Cognition in the wild*. Cambridge, MA: MIT Press.

Kirsh, D. (1995). The intelligent use of space. *Artificial Intelligence*, *4*, 415–452.

Kirsh, D. (1996). Adapting the environment instead of oneself. *Adaptive Behavior*, *4*, 415–452.

Kirsh, D., & Maglio, P. (1994). On distinguishing epistemic from pragmatic action. *Cognitive Science*, *18*, 513–549.

Larkin, J., & Simon, H. (1987). Why a diagram is (sometimes) worth ten thousand words. *Cognitive Science*, *11*, 65–99.

Lave, J., Murtaugh, M., & de la Rocha, O. (1984). The dialectic of arithmetic in grocery shopping. In B. Rogoff & J. Lave (Eds.), *Everyday cognition: Its development in social context* (pp. 67–94). Cambridge, MA: The Harvard University Press.

Lorge, I., & Solomon, H. (1955). Two models of group behavior in the solution of eureka-type problems. *Psychometrica*, *20*, 139–148.

Miyake, N. (1986). Constructive interaction and the iterative process of understanding. *Cognitive Science*, *10*, 151–177.

Miyake N. in collaboration with the P.I.A. Group (1994). Personal knowledge: Its externalization, manipulation and sharing for scientific creative activities. In *Proceedings of the international symposium on intellectual facilitation of creative activities* (pp. 14–19). Special Coordination Funds for Promoting Science and Technology.

Miyake, N. (1996). Conditions for constructive interaction. *Cognitive studies: Bulletin of the Japanese Cognitive Science Society*, *3*, 26–27.

Newman, D., Griffin, P., & Cole, M. (1989). *The construction zone: Working for cognitive change in school*. Cambridge University Press.

Norman, D. A. (1988). *The psychology of everyday things*. New York: Basic Books.

Okada, T., & Simon, H. A. (1997). Collaborative discovery in a scientific domain. *Cognitive Science*, *21*, 109–146.

Palinscar, A. S. (1989). Less charted waters. *Educational Researcher*, *18*, 5–7.

Palinscar, A. S., & Brown, A. L. (1984). Reciprocal teaching of comprehension monitoring activities. *Cognition and Instruction*, *1*, 117–175.

Pea, R. D. (1993). Practices of distributed intelligence and designs for education. In G. Salomon (Ed.), *Distributed cognitions: Psychological and educational considerations* (pp. 47–87). Cambridge, MA: Cambridge University Press.

Reisberg, D. (1987). External representations and the advantages of externalizing one's thoughts. In *Proceedings of the 9th annual conference of the cognitive science society* (pp. 281–293).

Roschelle, J. (1992). Learning by collaborating: Convergent conceptual change. *The Journal of the Learning Sciences*, *2*, 235–276.

Salomon, G. (1990). If intelligence is distributed: What about the cultivation of individuals' abilities? *Paper presented at a symposium in American Educational Research Association*. Boston.

Schwartz, D. L. (1995). The emergence of abstract representations in dyad problem solving. *The Journal of the Learning Sciences*, *4*, 321–354.

Schwartz, D. L., & Black, J. B. (1996). Shuttling between depictive models and abstract rules: Induction and fallback. *Cognitive Science*, *20*, 457–497.

Shirouzu, H. (2001). Children's algorithmic sense-making through verbalization. *Paper presented at the 23rd annual conference of the cognitive science society*. Edinburgh, UK.

Shirouzu, H., & Miyake, N. (1999a). *Cognitively active use of external resources: Further analyses of the interactive problem solving processes*. Tech. Rep. No. 98-1-03. Nagoya: School of Computer and Cognitive Sciences, Chukyo University.

Shirouzu, H. & Miyake, N. (1999b). Roles of cognitive externalization for joint problem solving. In *Proceedings of the second international conference on cognitive science and the 16th annual meeting of the Japanese Cognitive Science Society Joint Conference* (pp. 337–342).

Shirouzu, H., & Miyake, N. (2002). Guided verbalization for conceptual understanding: A scaffold for making sense of multiple traces of cognition. *Paper presented at American Educational Research Association*. New Orleans, LA.

Stenning, K., & Sommerfeld, M. (2000). Heterogeneous reasoning in learning to model. In L. R. Gleitman & A. K. Joshi (Eds.), *Proceedings of the 22nd annual conference of the cognitive science society* (pp. 493–498). Mahwah, NJ: Erlbaum.

Suwa, M., Gero, J., & Purcell, T. (2000). Unexpected discoveries and S-invention of design requirements: Important vehicles for a design process. *Design Studies*, *21*, 539–567.

Wineburg, S. S. (1989). Remembrance of theories past. *Educational Researcher*, *18*, 7–10.

Zhang, J. (1997). The nature of external representations in problem solving. *Cognitive Science*, *21*, 179–217.

Zhang, J., & Norman, D. A. (1994). Representations in distributed cognitive tasks. *Cognitive Science*, *18*, 87–122.