The Role of Strategies and Instructions in Relational Deductive Reasoning

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Abstract

Deductive reasoning is often seen as being composed of an immutable mechanism, universal to all reasoning situations and consisting of either mental models (e.g., Johnson-Laird, 1983) or formal-rules (e.g., Rips, 1994). Many researchers have questioned whether these positions are truly mutually exclusive (e.g., Roberts, 1993, 2000). Most deductive reasoning research has largely ignored the influence of instructions and strategies on the reasoning process. The present experiment was conducted to investigate reasoning strategies along with metacognitive measures of those strategies. Instructions were given to use a particular strategy (e.g., spatial, verbal). Items were separated into two levels: simple and complex, based on the amount of premises. Premise times, accuracy, and strategy reports were collected. Instructions had an effect on performance, as seen in premise times and accuracy. Also, strategy reports indicated a distribution of strategies utilized by participants. Strategy reports proved vital in corroborating differential patterns of performance indicative of varied approaches to solving this task.

Introduction

We are constantly faced with the need to derive new information from prior information. Reasoning is a vital part of that derivation process, whether it is solving a problem, invoking the proper strategies in memory, or drawing inferences to test hypotheses.

Individuals can solve a reasoning task in many different ways. In recent years, there have been two main theories of representation in reasoning—mental models (e.g., Johnson-Laird, 1983) and formal-rules (e.g., Rips, 1994). Proponents of the mental models view claim individuals construct analog representations of the information presented to them and attempt to find counterexamples to the purported conclusions, often in the form of a spatial model. In contrast, rule-based reasoning advocates claim individuals invoke relatively content-free algorithms to reason, usually in the form of verbal rules. These general definitions have quite different implications for the reasoning process. The mental models definition would imply the use of a spatial strategy to construct a representation that is not unlike the information encountered. In contrast, the formal rules definition implies a verbal approach to analyzing information, based on syntactic rules and abstract content. Each theory claims superiority in most tasks of reasoning, implying an all-or-none situation for reasoning, ignoring the possibility of individual differences in strategy use or task-related performance differences.

Formal rules (e.g., Rips, 1994) characterize reasoning as the application of logical inference rules. Individuals have a set of syntactic, content-free rules that can be used to draw inferences (Polk & Newell, 1995). These rules are seen to be similar to such classical logical structures as modus tollens and modus ponens (Kahane, 1990; Rips, 1994). Reasoning by rules is highly discrete and syntactic, with only one right answer (if the problem is determinate) in an all-or-none manifestation. Difficulty is a direct result of an increase in the amount of rules (or repeated steps in using those rules) needed to solve a set of premises in a reasoning problem. Also, there is no need to translate information, because the rules and premises are presumed to already be in a verbal form. These are points of divergence between mental models and rule-based reasoning (cf. Polk & Newell, 1995; Sloman, 1996).

Mental models theorists take a different approach to the reasoning process. Rather than using strict, syntactic, abstract rules of reasoning, they propose analog mental models invoked in reasoning situations (Johnson-Laird & Byrne, 1993). People are believed to form mental models, which are representations of the situation, as a starting point for the reasoning process (Johnson-Laird & Byrne, 1993). These models are sketches of reality used for deriving conclusions from the information given (Johnson-Laird, 1999). First, one constructs a model of the premises, then formulates a putative conclusion, and finally, searches for counterexamples to refute the initial conclusion (Johnson-Laird & Byrne, 1993). Procedural rules are used to translate verbal propositions into a spatial or symbolic array (Kurtz, Gentner, & Gunn, 1999).

An important difference between these competing claims is that the translation procedure would likely be more intensive and time consuming than dealing with the verbal propositions in their verbal form, which is an implication of rule-based theories, yet the product of the translation would
Johnson-Laird and colleagues have used a spatial relations task in several studies (e.g., Byrne & Johnson-Laird, 1989; Ehrlich & Johnson-Laird, 1982), to argue for a mental models representation. In this task, participants are presented serially with propositions, such as, “A is on the right of B; C is on the left of B; D is in front of C; E is in front of B; What is the relation between D and E” (Byrne & Johnson-Laird, 1989, p. 568)? Participants are required to use the information given to deduce the answer—D is on the left of E. In this case, there is one underlying model:

\[
\begin{array}{c}
\text{C} \\
\text{D} \\
\text{E}
\end{array}
\]

Alternatively, changing the above example slightly would yield a multiple-model problem with an ambiguous structure. For example, A is on the right of B; C is on the right of B; D is in front of A; E is in front of B (and asking for the relation between D and E), would result in the possibility of two valid models (both yielding the correct answer that D is on the right of E):

\[
\begin{array}{c}
\text{B} \\
\text{C} \\
\text{D} \\
\text{E}
\end{array} \quad \text{or} \quad \begin{array}{c}
\text{B} \\
\text{A} \\
\text{C} \\
\text{D} \\
\text{E}
\end{array}
\]

By manipulating the validity of the propositions (valid conclusion is possible or not) and how many alternative representations could be formed, Byrne and Johnson-Laird (1989) claimed convincing support for mental models. This support was based on the assumption that rule-based theories should not differentiate between propositions that evoke single- and multiple-models (because they capture the relationship between key propositions where the number of rules are the same), where mental models theories would predict differences based on the number of models necessary to deduce the conclusion. The data seem to support of a mental models representation. Overall, participants had more errors in trying to solve multiple-model than single-model problems, which is predicted by the mental models theory.

There are two things to note about the task—1) participants were instructed to “imagine” a spatial array (images and mental models are very similar, see Johnson-Laird, 1983), and 2) each item consisted of four premises, making the item rather difficult to solve. Although there are studies looking at the role of presentation on strategy preferences (e.g., Roberts, 2000b), where the argument has been made that serial presentation forces a spatial strategy, few studies have investigated the role of instructions and item complexity (see, Sternberg & Weil, 1980). These two facets of this task were the impetus behind the present study. What effects do item complexity and instructions have on participants’ strategies in solving spatial deductive reasoning problems? The following experiment attempted to answer this question.

An underlying argument of both theories is that all individuals fundamentally reason either using a mental model or formal rule at all times. This view would assume there are no individual differences in the fundamental mechanisms of reasoning. “The problem of individual differences affects any research intended to identify the fundamental reasoning mechanism” (Roberts, 2000a, p. 33). Furthermore, related to the particular task just mentioned, which was arguably designed to have a spatial strategy predominate, there is a small percentage of individuals that still use a verbal strategy (Roberts, 2000b). How can a “spatial” reasoning task, designed to evoke a dependence on spatial strategies, fail to completely support the notion of deductive reasoning by spatial mental models? There is one “simple” answer—individual differences in strategy choice.

This paper will address inter-strategic individual differences (different strategies used by different individuals) as opposed to intra-strategic individual differences (different levels of the same strategy in different individuals) (Roberts, 2000a). Also, strategies will be defined as, “a set of cognitive processes which have been shown to be used for solving certain types of deductive reasoning tasks” (Roberts, 1993, p. 576). Roberts (2000a) proposes three groupings of strategies—a) spatial strategies, where information is represented spatially and is analogous to the state of affairs in the world and is exemplified by the mental models theory of Johnson-Laird (1983), b) verbal strategies, where information is verbally or abstractly represented and various content/context free rules of syntax enable one to draw new conclusions from given information, with rule-based theories (e.g., Rips, 1994), and c) task-specific shortcut strategies, which result in massive gains of performance, rely on various representations, and are extremely task specific.

A powerful set of studies using the sentence-picture verification task highlighted the potent influence individual differences can have on task performance. As Roberts (2000a) noted, this literature is often neglected by researchers in deductive reasoning, although it has produced important findings that are useful for the reasoning strategies literature. Namely, individuals can willingly use alternative strategies to arrive at the correct problem solution in the sentence-picture verification task.

Hunt and colleagues (Hunt & Macleod, 1978; Macleod, Hunt, & Mathews, 1978; Mathews, Hunt, & Macleod, 1980) performed an elegant set of experiments using the sentence-picture verification task. In this task, participants first are presented with a simple sentence (e.g., plus is above star) and then a picture (e.g., */+). The goal was to verify whether the picture is true or false of the sentence. Response times (comprehension and verification) were used to infer the type of strategy used.

Individual differences in strategy use during task performance have been identified across a variety of domains such as syllogisms (Johnson-Laird, Savary, & Bucciarelli, 2000; Galotti, Baron, & Sabini, 1986), categorical syllogisms (Ford, 1995), three-term series (Egan...
& Grimes-Farrow, 1982), and relations (Morra, 1989). Even in a task like Byrne and Johnson-Laird’s (1989) spatial relations task mentioned earlier, where a spatial strategy is presumed a necessity, Roberts (2000b) found 10% of participants matched the verbal strategy, which is an example of the prevalence of individual differences in reasoning.

How do we determine what type of strategy a participant uses in reasoning? One method is the analysis of response times and inferring from them what strategy was used (e.g., Macleod et al., 1978; Mathews et al., 1980; Roberts, 2000b). A spatial strategy should entail translation and integration at the premise level, which would result in longer comprehension times and shorter solution latencies. Another approach is to ask individuals to report their strategies as they solve problems. Several of the studies in deduction that found individual differences used some type of retrospective report (e.g., Egan & Grimes-Farrow, 1982) or verbal protocol (e.g., Ford, 1995) to assess strategy choice and usage.

In a different domain where strategy use can be important and variable—namely, paired-associate learning, item-level encoding strategies also have been assessed using self-reports (Dunlosky & Hertzog, 2001). Dunlosky and Hertzog (1998, 2001) found that self-reported strategies highly predicted associative recall performance. From the previous review of the literature, there are two aspects of performance on the spatial relation task that deserve greater attention. The first is the role of instructions on a participant’s method of reasoning in the task. The second is the role of strategy choice.

Overview of Experiment

The experiment was based on the spatial relations task developed by Byrne and Johnson-Laird (1989) previously discussed in the introduction. Individuals were instructed to use a certain strategy (e.g., spatial, verbal). The present experiment manipulated instructions and complexity (premise and model number), analyzing their effects on performance and strategy choice.

We predicted instructions and item complexity would affect strategy choice. Specifically, the spatial group would take the longest on premise times, and the verbal group would be the quickest. Also, the spatial group would perform the best overall, especially on complex items, and would show performance differences between one- and two-model problems.

Compliance with strategy instructions was expected to be imperfect, but it would be highest for the spatial group. The preferred strategy of the naïve group would be spatial, but verbal strategies would be selected by a proportion of participants. Also, there will be a shift to spatial strategies for the complex items.

Method

Participants. One hundred and thirty undergraduates from the Georgia Institute of Technology participated for course credit. Participants were between 18 and 32 years older (M = 20.04, SD = 1.79)

Materials and procedure. Items were constructed to contain either two premises (“simple”) or four premises (“complex”). Complexity is defined by the number of propositions contained in an item (e.g., two propositions which vary relation in one-dimension or four propositions varying relation in two-dimensions). An item with four propositions, containing related and concrete objects, would be “A pen is to the left of a pencil. A paperclip is to the right of a pencil. A ruler is in front of a pen. A notebook is in front of a pencil.” Furthermore, the complex problems were broken into one- and two-model items (explained in the introduction).

There were 16 simple items (two-premise) and 16 complex items (four-premise). The complex items were either one-model (8) or two-model (8). All problems contained equivalent numbers (16 each) of true and false verification statements, concrete objects, and were serially presented on a computer.

Two example problems were given at the beginning of the simple block and two were given at the beginning of the complex block. The simple block was presented first, then the complex block. Each example gave the correct solution with feedback to the participant, with the proper method for solving the item (according to what was instructed). In the case of the naïve group, the problem was restated and no method of solution was presented. Before the sample items, individuals in the verbal and spatial groups were given specific instructions as to the strategy to use, whereas the control (naïve strategy) group was not given any instructions regarding the type of strategy to use.

Individuals in the spatial group were told to use an image and add objects to that image every time they were presented with a premise. In contrast, the verbal group was told to use a set of verbal rules that were provided. Participants in the naïve group were told to solve the problems in any manner preferred.

Once all of the items were presented and solved in the simple block, participants were presented with the items again and asked to report the strategy used for each item, using a forced-choice format. The options were spatial, verbal, other, both and no strategy. These retrospective reports prevented problems of contamination for the naïve group, since the strategy report scale will have to list the possible strategies, and this could contaminate subsequent strategy selection if other items followed (as in concurrent strategy reports for the complex block of items).

After the simple block, participants were then instructed that the next set of items contained two more premises (for a total of four), and they were reminded of
their particular instructed strategy, if applicable. Strategy reports were collected after every item.

Results

The significance level was \( p < .05 \) for all analyses. ANOVAs consisted of a between-subjects Instructions factor (spatial vs. verbal vs. naïve), and either a Complexity factor (simple vs. complex) or a Model factor (one vs. two), which were both within-subjects.

Solution Accuracy. Table 1 presents the means and standard errors of the proportion correct and mean premise times for correct items in both simple and complex blocks, with the complex block broken into one- and two-model problems.

Table 1: Proportion correct and mean premise times for correct items

<table>
<thead>
<tr>
<th></th>
<th>Spatial</th>
<th>Verbal</th>
<th>Naïve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple</td>
<td>.96 (.01)</td>
<td>.89 (.02)</td>
<td>.91 (.02)</td>
</tr>
<tr>
<td>Complex</td>
<td>.78 (.02)</td>
<td>.66 (.02)</td>
<td>.74 (.02)</td>
</tr>
<tr>
<td>1 Model</td>
<td>.80 (.02)</td>
<td>.65 (.03)</td>
<td>.78 (.02)</td>
</tr>
<tr>
<td>2 Model</td>
<td>.75 (.02)</td>
<td>.67 (.03)</td>
<td>.70 (.02)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Spatial</th>
<th>Verbal</th>
<th>Naïve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple</td>
<td>4.59 (.29)</td>
<td>3.06 (.26)</td>
<td>4.18 (.22)</td>
</tr>
<tr>
<td>Complex</td>
<td>4.75 (.21)</td>
<td>3.61 (.26)</td>
<td>4.11 (.19)</td>
</tr>
<tr>
<td>1 Model</td>
<td>4.37 (.20)</td>
<td>3.44 (.26)</td>
<td>3.93 (.19)</td>
</tr>
<tr>
<td>2 Model</td>
<td>5.18 (.23)</td>
<td>3.75 (.28)</td>
<td>4.32 (.21)</td>
</tr>
</tbody>
</table>

There was a difference between the three instructional groups: marginal \( M = .87 \) for spatial, marginal \( M = .78 \) for verbal, and marginal \( M = .82 \) for naïve. As predicted, there was a significant effect of instructions on proportion correct, \( F(2, 130) = 9.38 \), with the spatial group performing better than the verbal and naïve groups.

Complexity had a robust main effect on proportion correct, \( F(1, 130) = 285.29 \). Individuals performed much worse on the difficult items (marginal \( M = .73 \)) than on the simple items (marginal \( M = .92 \)). The number of models had a significant effect on proportion correct, \( F(1, 130) = 9.38 \), with the spatial group performing better than the verbal and naïve groups.

Mean Premise Times for Correct Items. There was a significant main effect of instructions, \( F(2, 130) = 7.81 \). Planned comparisons indicate the spatial group was slower at each premise than the verbal group. If participants are forming images at each premise, this extra time is likely a cost of translating the verbal statements into images. Mean premise times were significantly affected by whether an item was a one- or two-model problem, \( F(1, 130) = 40.55 \). One-model problems were performed faster, per premise, than two-model problems. One-model problems being performed faster would only be expected for the spatial group, and by extension, the naïve group (assuming the majority of people will choose a spatial strategy in the more difficult complex block). There was a significant Model X Instruction interaction, \( F(1, 130) = 4.32 \). There was a larger difference between premise times for one- and two-model problems for the spatial group (and not in the other groups), which would be expected if participants are trying to incorporate images into a preexisting model, and then trying to flesh those images out.

Proportions of strategies reported. Strategy reports consisted of selections from one of five options: spatial, verbal, both, other, and none. The “none” and “other” strategy options were extremely infrequent (less than 1.2% for simple items and less than 2% for complex items), and removed from subsequent analyses. Given the ambiguity of the meaning of “both” strategies, these will be excluded from the subsequent analyses.

Table 2 includes the proportions of strategies reported for all items. As expected for the proportion of spatial strategies reported, there was a main effect of instructions, \( F(2, 130) = 19.43 \). Also, there was a significant main effect of complexity, \( F(1, 130) = 4.75 \). Participants shifted slightly toward a spatial strategy on the more complex items. Complex items are used more frequently in this type of task, and those items may elicit spatial strategies because of difficulty.

Similar to the spatial strategies just reported, there was a main effect of instructions on the proportion of verbal strategies reported, \( F(2, 130) = 65.08 \). Complexity did have an effect on the proportion of verbal strategies reported, \( F(1, 130) = 18.89 \). Participants shifted slightly away from a verbal strategy on the more complex items.

Table 2: Proportion of Strategies Reported

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Spatial</th>
<th>Verbal</th>
<th>Naïve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple</td>
<td>.51 (.06)</td>
<td>.20 (.05)</td>
<td>.48 (.06)</td>
</tr>
<tr>
<td>Verbal</td>
<td>.17 (.04)</td>
<td>.69 (.05)</td>
<td>.29 (.06)</td>
</tr>
</tbody>
</table>

Complex

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Spatial</th>
<th>Verbal</th>
<th>Naïve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spatial</td>
<td>.64 (.07)</td>
<td>.17 (.04)</td>
<td>.57 (.06)</td>
</tr>
<tr>
<td>Verbal</td>
<td>.02 (.01)</td>
<td>.54 (.06)</td>
<td>.09 (.03)</td>
</tr>
</tbody>
</table>
Effects of Strategy Choice on Accuracy. Table 3 presents the conditional probabilities of being correct given a particular strategy was reported.

Table 3: Conditional Probabilities

<table>
<thead>
<tr>
<th>P(C)</th>
<th>Strategy</th>
<th>Spatial</th>
<th>Verbal</th>
<th>Naïve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple</td>
<td>Spatial</td>
<td>.97 (.01)</td>
<td>.89 (.04)</td>
<td>.91 (.03)</td>
</tr>
<tr>
<td></td>
<td>Verbal</td>
<td>.97 (.02)</td>
<td>.89 (.03)</td>
<td>.92 (.03)</td>
</tr>
<tr>
<td>Complex</td>
<td>Spatial</td>
<td>.76 (.02)</td>
<td>.67 (.07)</td>
<td>.71 (.03)</td>
</tr>
<tr>
<td></td>
<td>Verbal</td>
<td>.83 (.13)</td>
<td>.69 (.03)</td>
<td>.68 (.08)</td>
</tr>
</tbody>
</table>

Complexity had a significant main effect on the probability of being correct given a spatial strategy was reported, $F(1, 73) = 32.65$. Even though a spatial strategy may be less working-memory intensive than a verbal strategy, participants were having difficulties when premise number was increased.

One of the strong predictions of a mental models approach is that there should be better performance on one-model problems than two-model problems. Limiting the analysis to just those participants in the spatial and naïve groups produced the expected difference in performance as a result of model number, $F(1, 67) = 6.58$. Participants in these two groups performed significantly better on one-model problems than two-model problems when they reported using a spatial strategy.

As would be expected, there was also a significant effect of complexity on the probability of being correct given a verbal strategy, $F(1, 51) = 12.03$. In the simple block, there was a marginal $M = .91$, while in the complex block there was a marginal $M = .76$. These differences indicate the cost of using a verbal strategy on the type of complex items typically used in this task.

Effects of Strategy Choice on Mean Premise Time. One would expect that using a spatial strategy would show differences in premise times for two-model problems. There was a significant effect of model number, $F(1, 76) = 14.21$, with one-model problems taking less time per premise than two-model problems. This is intriguing, because it lends support to the notion of valid self-reports. Individuals using a spatial strategy would be subject to delays in processing based on model number as they try to construct a model that is composed of ambiguous parts (i.e., multiple possible configurations that are all true).

The patterns of mean premise times when verbal strategies were reported seem quite different. There were no significant main effects or interactions, $F$s $\leq 2.27$. These lack of differences lend credence to the validity of the strategy self-reports, especially when compared to the spatial strategy reports. If participants are truly using a verbal strategy, there should not be any differences between one- and two-model problems, because a verbal strategy would not be sensitive to the number of underlying spatial models. Indeed, we find no effects of model number, $F$s $\leq 1$. Because verbal strategies based on formal rules ignore multiple model configurations, this finding is yet more support for individuals accurately reporting the use of a verbal strategy.

Discussion

Do instructions affect how individuals approach the task the spatial relational deduction task? From the data just presented, the overwhelming response is—yes. Instructional effects were seen throughout the analyses. When collapsing across strategies, the proportion an individual answered correctly was directly affected by the instructions given. One claim was that individuals will naturally prefer to use a spatial, and presumably optimal, strategy on this type of task, which is why the spatial group performed the best. If this is true, then the verbal group should have great difficulty, due to being told to approach the task in a non-optimal way. This argument seems invalid for several reasons. First, even the verbal group performed well above chance, showing it is possible to perform the task relatively well under these instructions, granted it may not be the “best” strategy. Second, if the argument was valid, then performance in the naïve group should have exactly mirrored performance in the spatial group, which was not the case. Individuals in the naïve group did not perform identically to the spatial group, which could indicate not all participants in the naïve group were using a spatial strategy. Finally, given a valid argument, 100% of the strategies reported in the spatial and naïve groups should have been spatial—this did not occur. If the spatial strategy was the optimal strategy, then individuals should have been eager to use it every trial. Instead, many individuals either never used it or did not use the spatial strategy on all trials, even under instructions to do so.

The effects of instructions on latency were also robust and provide convergent evidence for differential approaches. As predicted, the spatial group took the longest for each premise and the verbal group had the fastest latencies. Similar to the argument made by Hunt and colleagues (Hunt & Macleod, 1978; Macleod, Hunt, & Mathews, 1978; Mathews, Hunt, & Macleod, 1980), individuals using a spatial strategy would take longer to translate the words into a pictorial format. We see support for this hypothesis.

Do people use various strategies in this task and is it important to attend to individuals differences in strategy use? From a purely information-processing view, it is clear that individuals were not performing the task identically, with robust differences emerging and resulting from instructions and the strategy reported. Furthermore, when we analyzed strategy self-reports, we found a variety of strategy preferences.
Given that compliance was not perfect, it was necessary to look at the actual strategy reported in relation to other variables. The patterns of accuracy and premise times both lend credence to the validity of strategy self-reports. Namely, spatial strategy reports corresponded to differences in accuracy and premise times when model number is considered. Rarely are strategy self-reports collected in this type of task, yet they can be another valuable and valid tool in assessing approaches to task performance by participants.

In sum, even though the spatial relations task has been cited as a “spatial” reasoning task, strategy self-reports and instructions indicate this is not the full story.

Acknowledgments

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