Fostering the Acquisition of Transferable Problem-Solving Knowledge with an Interactive Comparison Tool and Dynamic Visualizations of Solution Procedures

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Abstract

Learning from worked-out examples is seen as a very efficient way to foster the acquisition of problem schemas. In this paper we demonstrate that computer-based instruction provides possibilities to further enhance example-based learning. In an experiment carried out with 59 pupils with an average age of 14.0 years from a German high school we first demonstrated that a tool which prompted learners to compare examples across different problem categories in the domain of algebra fostered performance on near transfer problems, which differed from the instructional examples with regard to their surface features. However, only the dynamic visualizations of the examples’ solution procedures additionally improved performance on far transfer problems, which differed from instructional examples with regard to their structural features. It is assumed that while the comparison tool supports the induction of an abstract problem schema, the visualizations help to understand relations below the category level, which is required to successfully adapt known solution procedures to changes in the problem structure.

Keywords: skill acquisition; problem solving; visualization; animation; worked-out examples; transfer

Acquiring Transferable Problem-Solving Knowledge from Worked-Out Examples

The problem-solving knowledge that is characteristic for expertise in well-structured domains like mathematics, science, or programming is usually assumed to consist of interrelated and hierarchically organized sets of problem schemas (Sweller, van Merriënboer, & Paas, 1998; VanLehn, 1996). Problem schemas are cognitive structures that represent problem categories together with category-specific solution procedures in an abstract way. Schemas can be acquired by either solving or studying concrete instances of problem categories (i.e., example problems). However, they go far beyond these concrete instances by highlighting structural problem features that are important for a problem’s category membership and by detaching these structural features from merely incidental surface features of the domain context or cover story that are irrelevant to the problem’s solution. Because of their abstract nature problem schemas allow to efficiently solve problems that belong to one of the represented problem categories. Once a to-be-solved problem has been identified as belonging to a known problem category the relevant schema can be retrieved from memory, can then be instantiated with the information that is specific to the problem, and finally the category-specific solution procedure attached to the schema can be executed in order to generate a solution. Studying worked examples (i.e., example problems together with a step-by-step solution) has been demonstrated to be an efficient instructional method to foster the acquisition of problem schemas (for an overview see Atkinson, Derry, Renkl, & Wortham, 2000).

With regard to problem-solving transfer, abstracting problem schemas from concrete examples is assumed to be a pivotal cognitive process to overcome the so-called near-transfer problem, which occurs when learners have to deploy knowledge that has been acquired in one concrete problem-solving situation to solve structurally equivalent problems that merely differ with regard to their superficial problem features. However, the availability of problem schemas per se does not seem to be sufficient to tackle the far-transfer problem that occurs when learners have to solve novel tasks that do not fall into known problem categories and that accordingly require an adaptation of a known solution procedure. To improve learners’ ability for far transfer it is necessary to help them understand relations below the category level, that is, relations holding irrespective of category membership such as relations between individual structural task features and individual solution steps (Gerjets, Scheiter, & Catrambone, 2004). Understanding the rationale behind the overall solution procedure might result in more meaningful knowledge on modular solution elements that enables learners to directly translate individual structural task features into characteristics of the problem solution. This knowledge might be much more helpful than conventional knowledge on problem categories and solution recipes for adapting solution procedures to novel problems beyond the known categories (cf. Catrambone, 1998).
As research on problem-solving transfer has shown, the mere availability of worked-out examples is not sufficient to guarantee an adequate schematic representation of problem categories and an understanding of solution procedures. Rather, a profitable processing of worked examples has to be ensured. Such processing is likely to include example comparisons and example elaborations as the most important activities. Therefore, we will investigate two instructional devices in this paper that can be expected to facilitate the acquisition of transferable problem-solving knowledge. First, we will explore whether a computer-based comparison tool that prompts learners to compare worked-out examples with regard to their similarities and differences might facilitate the induction of problem schemas and therefore improve near-transfer performance. Second, we will test whether the use of dynamic visualizations for the illustration of meaningful solution elements might be suitable to deepen learners’ example elaborations and their understanding of the rationale behind the overall solution procedure, thereby improving their ability to solve far-transfer problems.

Comparing Instructional Examples

It has often been advocated to provide learners with multiple examples that allow them to compare examples within and among problem categories with regard to their differences and similarities. These comparisons might enable learners to identify the defining features of problem categories and to avoid confusions by examples’ surface features (Quilici & Mayer, 1996). For instance, learners can infer that shared properties of examples from the same problem category may potentially be the structural features that determine a problem’s membership to a specific problem category. These features cannot be altered without altering the solution procedure that applies to a problem. Without these comparison processes learners might tend to induce superficial problem schemas, to categorize test problems according to their surface features, and in turn to apply inappropriate solution procedures to them. In previous research (Scheiter, Gerjets, & Schuh, 2004) we could, however, demonstrate that comparing multiple examples within problem categories is not a necessary prerequisite for the acquisition of problem schemas. Rather, there are promising example-processing strategies that rely on single examples per problem category and that are similarly effective, namely comparing examples across problem categories. However, in order to have learners profit substantially from across-category comparisons it is necessary to carefully design example combinations so that they enable useful inferences with regard to structural and surface features of the examples presented. In particular, the surface features of examples should be kept constant across problem categories in order to allow learners to recognize that these surface features are not suitable to determine a problem’s membership to a specific problem category (cf., Quilici & Mayer, 1996). Accordingly, the comparison tool used in this study is designed to prompt learners to compare worked-out examples that share surface features, but differ with regard to the problem category they belong to according to their communalities and differences. These across-category comparison processes can be expected to facilitate the identification of structural and surface features of the examples and thus to stimulate the acquisition of correct problem schemas that enable learners to solve near-transfer problems.

Dynamic Visualizations of Solution Procedures

A common finding in learning from worked-out examples is that learners “tend to form solution procedures that consist of a long series of steps – which are frequently tied to incidental features of the problems – rather than more meaningful representations that would enable them to successfully tackle new problems” (Catrambone, 1998, p. 355). To overcome these shallow representations of solutions, learners have to draw inferences concerning the structure of example solutions, the rationale behind solution procedures, and the goals that are to be accomplished by individual solution steps. Without such an understanding it cannot be expected that learners are able to flexibly modify a known solution procedure in order to adapt it to novel problems beyond the known problem categories. In this paper we investigate whether embellishing worked-out examples with dynamic visualizations of solution procedures is a suitable method to stimulate learners’ inferences with regard to the relations between solution steps, goals, and abstract principles that provide the rationale behind a solution procedure.

Visualizations of solution procedures may act as external representations, which support reasoning in the domain by facilitating the interpretation of the problem situation and inferences based on the information given. Thus, they can be more computational effective than a mere text-based representation of the same information (Larkin & Simon, 1987). Moreover, as Stemming and Oberlander (1995) have noted, visualizations may reduce the ambiguity of a verbal problem statement, because they depict the intended interpretation rather than allowing for multiple interpretations like verbal representations often do. Thus, visualizations may be more specific than verbal representations and may guide learners in understanding what a problem is about and how it can be solved. According to Mayer’s multimedia principle (2001) embellishing textual learning materials by static pictures or dynamic visualizations (i.e., animations) helps promote learners’ understanding of instructions. With regard to the acquisition of problem-solving knowledge, visualizations of worked-out examples may first help learners to understand the situation described in the problem statement (i.e., the initial problem state) and thus to correctly represent its meaning in a situation model. Second, visualizations of the solution steps may promote the understanding of changes with regard to the initial problem state, which are achieved by applying a solution step to a problem (Scheiter, Gerjets, & Catrambone, in press).

Several findings suggest that animations can be used successfully for delivering abstract content such as mathematical rules, Newton’s laws, or computer algorithms (Byrne, Catrambone, & Stasko, 1999; Catrambone & Seay, 2002). With respect to conveying problem-solving knowledge, the visual-spatial properties of the visualization may be used to deliver
information on the current problem state and its relevant structural features. Moreover, the changes over time that can be depicted in an animation may be used to reflect the changes in problem states that result from applying a solution step to a specific problem state of the example (Scheiter et al., in press). Having this type of dynamic visualizations of solution procedures at their disposal might enable learners to better understand individual solution steps as a prerequisite for solving not only near-transfer problems but also far-transfer problems.

**Hypotheses**
As a first hypothesis we advocate that the provision of an interactive comparison tool as well as the provision of dynamic visualizations leads to a superior overall problem-solving performance.

Second, it is hypothesized that an increase in problem-solving performance when learning with the interactive comparison tool can be traced back to improvements for near transfer test problems.

Third, we assume that an increase in problem-solving performance when learning with dynamic visualizations can be traced back to improvements both for near and far transfer test problems.

**Experiment**

**Method**

**Participants.** Subjects were 59 pupils (9th grade) from a German high school who attended a project day on “learning with new media”. The average age was 14.0 years.

**Materials and Procedure.** The pupils used a computer-based learning and problem-solving environment, which conveyed knowledge on how to solve algebra word problems. The experiment was divided into two phases, a learning and a test phase. During the learning phase pupils were asked to read three textbook chapters on different school subjects, namely biology, chemistry, and politics, which were presented on the computer screen. They started with a chapter taken from a biology textbook that dealt with forest dieback, which was followed by a chapter from a chemistry schoolbook that explained alcoholic fermentation. Finally, pupils worked on a chapter from a politics book, which pointed out some general rules about the election process of the German parliament.

Embedded in each of the three chapters were three algebraic worked-out examples that illustrated how to solve specific problems in these domains. The following biology example is related to forest dieback. It can be solved by using the algebraic formula: \( A = a_1/b_1 * c_1 + a_2/b_2 * c_2 \).

In an experimental area for the observation of forest dieback, one part of the trees are deciduous trees and the other part are coniferous trees, the latter including spruces and pines. 1/10 of the 489 pines and 1/4 of the 1793 spruces are damaged. How many coniferous trees have been damaged?

For each of the nine worked-out examples, the solution formula as well as each step of the solution procedure were presented. Depending on the experimental condition the worked-out examples were either presented as text only or they were accompanied by an interactive comparison tool, a dynamic visualization of their solution procedure, or both.

Within each chapter, the three worked-out examples differed with regard to the complexity of their underlying solution procedure, that is, each worked-out example required a different algebraic formula. These three formulas were identical across the three school subjects. That is, participants read nine worked-out examples embedded into three different domain contexts (variation of superficial example features) and they were confronted with three different algebraic formulas (variation of structural example features).

Learners were asked to intensively study the three chapters and especially to pay attention to the worked-out examples before entering the test phase of the experiment. In the test phase pupils were instructed to solve 21 algebraic word problems. The worked-out examples of the learning phase were not available during the test phase. Learners were asked to write down their solutions on a sheet of paper. For solving the problems, they were allowed to use a calculator. The overall time for learning and problem solving was restricted to 120 minutes.

**Design and Dependent Measures.** As a first independent variable the provision of a comparison tool for the worked-out examples during learning was varied. As a second independent variable we implemented dynamic visualizations of solution procedures that learners either could retrieve or not. Thus, the following four experimental conditions of the resulting 2x2-design can be differentiated:

In an experimental condition without comparison tool and without dynamic visualizations (baseline condition) learners were presented with text-based worked-out examples only.

The experimental condition with comparison tool but without dynamic visualization (comparison condition) contained the same three textbook chapters and the same worked-out examples as the baseline condition. But after the learners had read the three chapters they were instructed to use an interactive comparison tool before switching to the test phase (cf. Figure 1). This interactive comparison tool was based on hyperlinks and multiple pop-up windows that allowed learners to quickly compare different worked-out examples from the biology content domain with regard to their similarities and differences. Based on our prior research (Scheiter et al., 2004), the comparisons across problem categories (i.e., examples with different solution formulas) that were highlighted by this tool were considered to be particularly helpful for learning. Learners were instructed to use the comparison tool as often as they wanted, but in minimum three times before entering the test phase.
### How to use the comparison tool

In total you have seen nine worked-out examples: three biology examples, three chemistry examples, and three politics examples. Now we want you to compare these examples with regard to their commonalities and differences by using the interactive comparison tool. Please proceed in the following way:

- Compare example pairs by clicking the comparison button.
- A new window will pop up, where the two examples are presented next to each other. Compare these examples with regard to their commonalities and differences. Please try to figure out why these examples have to be solved by using different solution procedures.
- When you think you have recognized the commonalities and differences of an example pair, please choose another pair of examples.
- Use the comparison tool as often as you want, but select at least each example pair once!

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**Example 1 versus Example 2**

**Example 1 versus Example 3**

**Example 2 versus Example 3**

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### Biology: Example 2

In an experimental area for forest dieback, 3912 trees have been examined. One part of these trees is coniferous wood, including spruces and pines. 1/8 of the examined trees are pines and 1/3 are spruces.

**How many coniferous trees have been examined?**

| Number of examined coniferous trees (A): | > |
| Number of examined trees (n): | 3912 |
| Examination rate spruces (a/b): | 1/8 |
| Examination rate pines (a/b): | 1/3 |
| Solution formula: A = (a/b)_c * n + (a/b)_p * n |
| A = (1/8) * 3912 + (1/3) * 3912 |
| A = 489 + 1304 |
| A = 1793 |

### Biology: Example 3

In an experimental area for forest dieback, one part of the trees are deciduous trees and the other part are coniferous trees, the latter including spruces and pines. 1/10 of the 489 pines and 1/4 of the 1793 spruces are damaged.

**How many coniferous trees have been damaged?**

| Number of damaged coniferous trees (A): | > |
| Number of pines (c): | 489 |
| Number of spruces (c): | 1793 |
| Rate of damaged pines (a/b): | 1/10 |
| Rate of damaged spruces (a/b): | 1/4 |
| Solution formula: A = (a/b)_c * c + (a/b)_p * c |
| A = (1/10) * 489 + (1/4) * 1793 |
| A = 48.9 + 448.25 |
| A = 497.15 |

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**Figure 1:** The comparison tool

(Note: The upper part depicts the start screen of the comparison tool. The lower part illustrates how to-be-compared examples were presented, once a learner had selected a comparison on the start screen).

In the experimental condition without comparison tool but with dynamic visualizations (visualization condition), we provided learners with dynamic visualizations of worked-out examples and their solution procedure (cf. Figure 2). For each worked-out example there was one dynamic visualization of the problem statement and one visualization for each of the individual steps of the solution procedure. The visualizations of a problem statement illustrated the transition between a concrete problem representation and a more abstract representation of its structural features. The problem statement was first depicted by using the concrete objects mentioned in the example, which then changed into more abstract objects so that only the structural problem features were represented in the visualization, while superficial features were excluded. For instance, if an example dealt with calculating a certain number of damaged coniferous trees out of two different forest species with different damage rates, the concrete visualization depicted the two forest species. This visualization was transformed into an abstract visualization, in which the trees were represented by small squares (in the abovementioned example 489 small squares represented pines and 1793 small squares represented spruces). The small squares were merged into two bigger squares, which each represented the numbers of objects mentioned in the example problem (i.e., 489 and 1793). The rates mentioned in the word problem were represented by dividing these bigger squares into parts of equal size (in the abovementioned example 1/10 and 1/4). The final addition of the two rates was illustrated by merging the two square parts that represented the numbers of damaged pines and spruces. This type of abstract visualization using squares was used for all examples so that the visualizations of all examples shared a common representation of objects and their relevant relations. Learners could replay each visualization as often as they wanted, but were asked to view it at least once for each of the example’s components.

**Figure 2:** Screenshot of a worked-out example with dynamic visualizations of the solution procedure

We assumed that the common abstract visual representation across examples would help learners to focus on the structural similarities and differences between the examples while being able to ignore their surface features. Furthermore, breaking down a solution procedure into a sequence of meaningful steps and visualizing the rationale behind each step should...
enhance learners’ understanding of the solution procedure and thus also foster far transfer.

The experimental condition with comparison tool and dynamic visualization (combined condition), provided learners with a combination of both instructional methods. Thus, we offered to them the dynamic visualizations as described before while they studied worked-out examples. Additionally, learners were asked to use the interactive comparison tool to compare different worked-out examples at the end of the learning phase. However, the worked-out examples that could be retrieved by using the comparison tool were purely text-based and did not comprise any pictorial information.

As a dependent measure, we registered learners’ performance for solving 21 problems in the test phase of the experiment. These problems differed in their transfer distance with regard to the examples presented in the learning phase. We distinguished between two levels of transfer, namely, near transfer and far transfer. For both, near and far transfer problems, new cover stories were used that learners had not encountered in the learning phase. Some of these cover stories were related to domain contexts already known from the learning phase (e.g., biology), while others were related to new domain contexts (e.g., physics). Whereas near transfer problems could be solved by using one of the three solution formulas presented during the learning phase (e.g., $A = a_1/b_1 + c_1 + a_2/b_2 + c_2$), far transfer problems could only be solved by constructing a new solution formula, for instance by modifying a known formula (e.g., $A = a_1/b_1 + c_1 + a_2/b_2 + c_2 + k$). Each of the 21 problems resulted in a maximum of five points (i.e., two points for setting up the right formula, two points for inserting the right values into the formula, and one point for solving the formula correctly). For each individual the percentage of correct responses in relation to the maximum score was calculated for easier interpretation.

**Results**

Due to the time limitations in the experiment all conditions where comparable with regard to learning times. In a first step we compared participants’ overall problem-solving performance across the four conditions (Figure 3) by means of an ANOVA (comparison tool x dynamic visualization).

This analysis yielded a significant main effect for the comparison tool ($F(1,55) = 6.42; MS_E = 199.27; p < .02$), showing that this tool led to an increase in problem-solving performance as expected. Additionally, dynamic visualizations also fostered learning outcomes ($F(1,55) = 18.72; MS_E = 199.27; p < .001$). The interaction between these two factors was not significant ($F < 1$).

To test whether improvements in performance in the comparison conditions are due to a performance increase for near transfer problems only, we conducted two separate ANOVAs (comparison tool x dynamic visualization) for performance on near and far transfer problems (Figure 4 and 5). As expected, this analysis yielded a significant main effect of the comparison tool on near transfer problems ($F(1,55) = 6.66; MS_E = 268.48; p < .02$), but not on far transfer problems ($F(1,55) = 2.58; MS_E = 250.71; p > .10$).

Finally, we were able to demonstrate that dynamic visualizations improved performance for both, near transfer problems ($F(1,55) = 6.38; MS_E = 268.48; p < .02$) and far transfer problems ($F(1,55) = 35.15; MS_E = 250.71; p < .001$). There was no interaction between the two factors ($F < 1$).

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**Figure 3:** Overall problem-solving performance (in %) as a function of the provision of the comparison tool and dynamic visualizations.

**Figure 4:** Problem-solving performance (in %) for near and far transfer problems as a function of the provision of the comparison tool.

**Figure 5:** Problem-solving performance (in %) for near and far transfer problems as a function of the provision of dynamic visualizations.
Summary and Conclusions

In this paper we were able to demonstrate that a tool that helps learners abstract from the irrelevant surface features of examples by having them compare these examples facilitates performance for solving near transfer problems. While near transfer problems require learners to apply their knowledge to problems that are unfamiliar with regard to the cover story they are embedded in, far transfer problems additionally make it necessary to adapt known solution procedures to differences in structural features. Providing learners with dynamic visualizations that depict the initial problem state as well as changes to this problem state resulting from applying a solution step to the problem did not only foster learners’ performance when solving near transfer problems (like the comparison tool), but also improved far transfer.

This experiment confirms prior findings (Quilici & Mayer, 1996; Scheiter et al., 2004) that comparing examples with the same surface features across categories supports the acquisition of abstract problem schemas. Unfortunately, this schematic knowledge is tied to the boundaries of problem categories, thus only fostering performance on near transfer problems, which originate from the same categories as the examples previously studied. However, in order to foster knowledge on relations below the category level as required for solving far transfer problems, an instructional approach is needed that guides learners’ attention to the individual solution steps carried out to solve a problem. While the dynamic visualizations used in this experiment proved very suited to provide this guidance, there is evidence that not every visualization is similar effective. In the study by Scheiter et al. (in press) visualizations that used the examples’ concrete objects throughout the whole solution procedure actually led to deteriorations in problem-solving performance. Thus, the effectiveness of the visualizations of the current study seem to reside in the fact that they help learners to translate a concrete example into an abstracted representation, based on which mathematical operations can be carried out more easily (e.g., determining ratios for different objects). This conclusion is line with the work by Larkin and Simon (1987) hinting to the fact that the effectiveness of external representations is based on the way they support specific cognitive processes that have been deemed important in the domain in question. Accordingly, the statement that text and pictures aid learning more than text does alone – as expressed in Mayer’s multimedia principle (2001) – is an oversimplification. Rather, the usefulness of a visualization very much depends on the learning domain itself, the kind of cognitive processes required for achieving deeper understanding in this domain, and the way the visualization is designed to support these processes. Thus, the current results do not necessarily tell us anything regarding the effectiveness of visualizations in general; rather, they show that abstraction as a cognitive process relevant to the acquisition of transferable problem-solving knowledge can be facilitated if the dynamic visualization is designed to reflect this abstraction by showing the transition from a concrete problem statement to an abstract mathematical representation of the problem and its solution. Further research, however, needs to be conducted to investigate whether the benefits of visualizing worked-out examples can also be demonstrated for more difficult problems that require more extensive adaptations of known solution procedures than the far transfer problems used in the current study.

It is important to note that both the comparison tool and the dynamic visualizations were more effective than the baseline, while not requiring more time for learning (thus they were also more efficient). From an instructor’s perspective the efficiency of an instructional method is an important argument that needs to be considered in particular when designing instructional materials for classroom settings.

References


