The Cognitive Advantages of Counting Specifically: An Analysis of Polynesian Number Systems

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Abstract

Distinct number systems for certain objects within the same language seem to reveal a lack of abstract thinking and are therefore often taken as cognitively deficient. The case of the Polynesian languages will prove this assumption to be mistaken. In addition to a general, perfectly consistent, and decimal system with high numerals, some of these languages also traditionally applied apparently mixed-base systems for a range of frequently used objects. Māori, Hawai'ian, and Tongan in particular are conspicuous for such 'irregularities', and one broadly accepted conjecture is that their decimal systems were introduced by Western missionaries. In order to demonstrate that the opposite is in fact true, this article scrutinizes their main numeration principles. The results indicate that the original Polynesian system was always abstract and decimal and that the supplementary systems were developed as—cognitively efficient—tools for calculating without notation.

Keywords: Counting; Polynesia; cross-linguistic analysis.

Introduction

There is no doubt that a coherent number system, applicable in general to all objects worth counting, is cognitively efficient. The reverse conclusion—that everything else is less efficient and consequently must be a remnant of premathematical comprehension (e.g., Ifrah, 1985; Menninger, 1969)—is, at best, questionable. The case of Polynesian languages illuminates why.

According to contemporary dictionaries, general and perfectly consistent systems with base ten do prevail throughout Polynesia. But indicators for other systems prior to Western influence can be found as well: apparently irregular ways of counting certain objects in Tongan that emphasize pairs and scores (Churchward, 1953), special lexemes in Marquesan for 20 (Lynch, Ross & Crowley, 2002), an allegedly vigesimal system in traditional Māori (Best, 1906), and even a 'mixed-base' 4 and 10 system in Hawai'ian (Hughes, 1982). All these cases seem to provide evidence that the traditional Polynesian base was not decimal. But is this conjecture conclusive? Or is it also conceivable that decimal and other systems were used simultaneously?

Amazingly, we can—often together with these peculiarities and despite the fact that a notation was lacking—find numerals, up to which nobody would count, such as 100 000 or even 4 000 000. Why then, if Polynesians were so obviously interested in high numbers, did they stick to so unwieldy a system as the ones with 'mixed bases'? If they always had both types, why did they not give up the complicated systems in favor of the decimal system?

Before we can address this question we will have to prove that the decimal system was indeed no Western introduction but as indigenous as the 'mixed-base' systems. Although it cannot be resolved with complete certainty in retrospect which system is the older one, we will present some evidence for the assumption that the latter were not *preceding*, but *derived* from the general system.

We begin with a short analysis of the elements of number systems and some of their cognitive implications. After depicting general patterns of the Polynesian number systems, we will exemplify their numeration principles for the counting systems in Tongan. When putting the respective findings into context, it will become clear that Polynesian cultures did indeed have use for high numbers. By speculating on how they might have handled these without notation, we will find that the questions of base and extent are inextricably linked and that some of the Polynesian peculiarities are actually very sensible from a cognitive point of view.

Elements of Number Systems and Their Cognitive Implications

For the representation of natural numbers a one-dimensional system would, in principle, be sufficient, that is a system with a distinct lexeme for each number. However, since this is cognitively not efficient for large numbers, many languages apply a two-dimensional system of base and power (cf. Zhang & Norman, 1995). The composition of the power terms follows the multiplication principle, while the complete number word is generated by joining all power terms according to the addition principle. The number word for 652 in English, for instance, which has a decimal base, is accordingly composed as "six hundred" ($6 \cdot 10^2$), "fifty" ($5 \cdot 10^1$), and "two" ($2 \cdot 10^0$).

The same advantages of such a two-dimensional system apply to both oral numeration and notations: Cyclic patterns keep the number words compact while dramatically reducing the amount of lexemes needed (cf. Ascher, 1998; Zhang & Norman, 1995). A small set of basic number words (i.e., numerals) suffices, even if we consider that most natural languages also use specific lexemes for the powers of their base. A strict decimal system, for instance, requires nine numerals for the basic numbers 1 to 9 and one numeral each for the base and its higher powers (10, 100, 1 000, …). The higher the base, the more numerals are needed for basic numbers, but the less numerals are needed for the power dimension.

A word for zero, essential in strict place-value notations, is not required in oral number systems; most natural languages express the powers of their base explicitly and can therefore simply omit empty places (Greenberg, 1978).

As languages consist of a finite set of words, their number systems are also finite. The limiting number is defined as the next number beyond the highest possible composition (Greenberg, 1978), usually one power higher than the largest numeral. In a decimal system with a lexeme for "hundred" as the highest numeral, for instance, the limiting number is 999 + 1 = 1 000.

Several factors influence the ease, with which number words are learnt and operated (e.g., Wiese, 2003). Irregularities in number words and order of power terms, for instance, slow down the acquisition of the number system (Miller, Smith, Zhu & Zhang, 1995; see also Geary, Bow-Thomas, Liu & Siegler, 1996). The shorter the words, the greater is the memory span (Dehaene, 1997). With regard to bases, Zhang and Norman (1995) identified a cognitive trade-off in base size: While large bases are more efficient for encoding and memorizing big numbers, they also require the memorization of larger addition and multiplication tables when operating with them.

The General Polynesian Number Systems

The Polynesian languages belong to the Oceanic Subgroup of the Austronesian language family, which dates back at least some 6 000 years. Proto-Austronesian yielded a common set of numerals from 1 to 10, and Proto-Oceanic contained a numeral for 100 (Lynch, Ross & Crowley, 2002; Tryon, 1995). When we compare contemporary Austronesian languages, ranging from Madagascar through insular Southeast-Asia into the Pacific, we find that numerals do still show a considerable degree of convergence and that decimal systems are by far prevailing. These findings support the conjecture that the ancestors of the Polynesian voyagers brought a number system with base 10 and extending to (at least) a limiting number of 1 000 with them when they entered the Pacific (cf., Bender & Beller, 2005).

Among contemporary Polynesian languages, most numerals are still widely shared, although with a few exceptions (see Table 1). Particularly for the numerals from 1 to 9, lexical coincidence—within the range of typical sound shift—is striking, and even among the numerals denoting 10, for which variation is highest, we still can find similarities for the first three languages and traces of replaced numerals for the latter two. The numeral for the second power of the base (i.e., 100 or 400 in Hawai'ian respectively) is nearly the same in every language (*teau/lau/rau*); *mano*, on the other hand, denotes one further power, equalling 1 000 in Tahitian and Māori, and 10 000 in Tongan and Rapanui. In addition, it reappears in traditional Hawai'ian, where it refers to 4 000 (e.g., Hughes, 1982).

Even beyond this common stock, most Polynesian languages contained numerals for high numbers—up to allegedly 4 000 000 in Hawai'ian. But variation in extent is remarkable.

While, for the sake of simplicity, we have only depicted the regular aspects of Polynesian number systems in Table 1, some peculiarities need to be mentioned. In the Hawai'ian number system, for instance, the indigenous numerals did not apply to the pure powers of ten but to the powers of ten *times four*, such as 400, 4 000, and so on. Since Hawai'ian provides just one instance of an apparently widespread pattern of 'mixed bases' in Polynesian number systems, some scholars concluded that the original Polynesian system was non-decimal (e.g., Bauer, 1997; Best, 1906; Hughes, 1982; Large, 1902). Even if we refute this conclusion on the basis of the clearly decimal forms in Proto-Austronesian and Proto-Oceanic, it could still be that Polynesian number systems had shifted from an initial decimal to non-decimal systems at an early stage and that decimal systems were re-introduced under Western influence.

In order to scrutinize this assumption, an even more complex case with apparently 'mixed bases' and some additional irregularities will be detailed and analyzed in the following.

Table 1: Polynesian numerals (adapted from Bender & Beller, 2005).[a]

Nº	Tongan	Tahitian	Rapanui	Hawai'ian	Māori
1	taha	hōʼē, tahi	tahi	kahi	tahi
2	ua	piti	rua	lua	rua
3	tolu	toru	toru	kolu	toru
4	fā	maha	hā	hā	whā
5	nima	pae	rima	lima	rima
6	ono	ono	ono	ono	ono
7	fitu	hitu	hitu	hiku	whitu
8	valu	va'u	va'u	walu	waru
9	hiva	iva	iva	iwa	iwa
10	hongofulu	'ahuru	'angahuru	'umi	ngahuru
10^2	teau	rau	rau	*lau	rau
10^3	afe	mano	piere	*mano	mano
10^4	mano	manotini	mano	*kini	(tini)
10^5	kilu	rehu		*lehu	
10^6		iu		*nalowale	

[a] Prefixes are omitted for easier comparison. Non-decimal powers are indicated by *.

The Tongan Number Systems

The Tongan language, spoken by about 100 000 people in the Kingdom of Tonga, traditionally contained different number systems: One general, perfectly consistent system, and four diverging ways of counting some of the most common objects.

The general number system is a decimal system with numerals from 1 to 9 and for the powers of the base up to 100 000 (see Table 2). Nowadays, this extent is increased through the use of English loan words for higher numerals such as *miliona* ("million"). In addition, the contemporary system contains a lexeme for "zero", *noa*, but this term was most likely not used in the sense of numerical zero until Western arithmetic was introduced.

The composite number words in between are generated regularly by two or more lexemes according to the multiplication and addition principle. The multiplier directly pre-

Table 2: Traditional Tongan number words (general system).

Nº	Word	Nº	Word	Nº	Word
0	(noa)				
1	taha	10	hongofulu	100	teau
2	ua	20	uofulu	200	uangeau
3	tolu	30	tolungofulu	300	tolungeau
4	fā	40	fāngofulu	…	
5	nima	50	nimangofulu	1 000	afe
6	ono	60	onongofulu	2 000	ua afe
7	fitu	70	fitungofulu	…	
8	valu	80	valungofulu	10 000	mano
9	hiva	90	hivangofulu	100 000	kilu

cedes the multiplicand, and the larger summand precedes the smaller with a link in front of the last term.

Several objects, however, were counted not by using the general system, but rather by diverging systems with at least partly specific terms for certain numbers (see Table 3). Without exception, these objects are natural products used for subsistence: pieces of sugar-cane thatch, pieces of yam for planting, whole yam, fish, coconuts, and one type of pandanus leaves. The counting of these objects followed specific patterns that all have one feature in common: The smallest unit is the pair (*nga'ahoa* for sugar-cane thatch, pandanus, yam, and fish, and *taua'i* for coconuts).

While the counting of sugar-cane then proceeds in tens of pairs (*tetula*), hundreds and thousands of pairs (using the regular numerals, yet omitting the lexeme for "pair"), coconuts, yam, and fish are, from 20 onwards, counted in scores. The term for "one score" is even glossed differently depending on the counted object. For the counting of coconuts and yam, a further term refers to "tens of scores" (*tefua* for coconuts and *tefuhi* for yam). The scores (*kau*) of fish, however, are regularly counted in number words from one to hundreds.

The counting for each of these objects thus differs from the others, either with regard to the gradation—that is, whether it proceeds in pairs only, scores, or 10-scores—or with regard to terms, or both.

Examining Table 3, two further peculiarities catch the eye: First, while some terms refer to a particular object and its number, as *tefua* (one 10-score of coconuts), other terms can change their absolute value, depending on the object counted. The most variable term in this regard is *teau*, which refers to 100 ordinary things, 100 pairs of sugar-cane thatch (i.e., 200 pieces), or 100 scores of coconuts, yam, or fish (i.e., 2 000 pieces). We may therefore conclude that the counting unit for ordinary things was 1, for sugar-cane it was 2, and for coconuts, yam, and fish it was 20.

Second, the number 20 seems to play an essential role. But do these specific ways of counting follow a vigesimal system, as was argued for related Polynesian languages by Best (1906) or Large (1902)? In a strict vigesimal system, we would expect numerals for the basic numbers 1 to 19 as well as for the base and all higher powers (i.e., $20^1 = 20$, $20^2 = 400$, $20^3 = 8\,000$, and so on). This is not the case here (and not in any other Polynesian language, either). Instead, all the specific systems combine the fundamental base 10 with 2, and should therefore, from a mathematical perspec-

Table 3: Tongan specific counting systems (adapted from Bender & Beller, 2004).

General numerals		Category	Sugar cane	Category	Coconuts	Pieces of yam[a]	Fish
1	taha						
2	ua	1 pair	taha [nga'ahoa]	1 pair	[taua'i…'e] taha	taha [nga'ahoa]	taha [nga'ahoa]
4	fā	2 pairs	ua [nga'ahoa]	2 pairs	[taua'i…'e] ua	ua [nga'ahoa]	ua [nga'ahoa]
	…	…	…	…	…	…	…
10	hongofulu						
20	uofulu	10 pairs	tetula	1 score	tekau	tekau	kau … 'e taha
40	fāngofulu	20 pairs	uangotula	2 scores	uangakau	uangakau	kau … 'e ua
	…	…	…	…	…	…	…
100	teau						
200	uangeau	100 pairs	teau	1 10-scores	tefua	tefuhi	kau … 'e hongofulu
400	fāngeau	200 pairs	uangeau	2 10-scores	uofua	uangofuhi	kau … 'e uofulu
	…	…	…	…	…	…	…
1 000	taha afe						
2 000	ua afe	1 000 pairs	taha afe	100 [scores]	teau	teau	kau … 'e teau
4 000	fā afe	2 000 pairs	ua afe	200 [scores]	uangeau	uangeau	kau … 'e uangeau
	…	…	…	…	…	…	…

[a] Whole yam was also counted specifically, however not with a distinct system, but partly with the system for pieces of yam and partly with the one for fish. The *kie* pandanus leaves were also counted like pieces of yam.

tive, be rather accounted for as decimal systems that do not operate on single objects but on pairs of objects. This view is supported by the emphasis laid on pairs in other linguistic domains and even in spatial conceptions (Bennardo, 2002).

Was the general system introduced in colonial times to substitute for the specific systems? European missionaries, arriving in Tonga in the early 19th century (cf. Campbell, 2001; Rutherford, 1977), did not have great influence on these counting systems, except for the fact that with compulsory school attendance young Tongans do no longer learn the traditional systems in the traditional way. A linguistic analysis revealed beyond doubt that all numerals of the Tongan system are indeed indigenous (Bender & Beller, 2004). We can therefore entirely dismiss the hypothesis that the general number system in Tongan, with its strict decimal base and high numerals, was a Western introduction. Instead, we propose that this general system was always a decimal system of considerable extent, which, in certain cases, operated with pairs or scores instead of single units.

Counting in Context

This leads us to the question of why certain objects were counted specifically, while others were not. As this is rather a question of origin than of practice, it cannot be satisfactorily answered in retrospect. However, a thorough look at the cultural context of these objects and of counting them may help to shed some light on this question and on the controversy over whether the supplementary systems have to be regarded as 'non-efficient primitives'.

The Objects of Specific Concern

As there is nothing peculiar about a score itself—in Tonga, it is rather 10 that appears to have been of cultural significance (Gifford, 1929), in other Polynesian cultures it was 8 (Biggs, 1990)—it cannot be "the score of coconuts" that was significant. Therefore, it must be something more general about these objects that meant they deserved special treatment in counting. But what is so particular about them to justify specific counting systems? What do they have in common?

To begin with the food, fish, yam, and coconut are among the most important foods, providing protein, starch, and fat as well as water respectively. In addition to whole yam, pieces for planting are considered—and counted—separately. The sugar-cane leaves were used to thatch traditional houses, and the pandanus-leaves (*kie*) to weave fine, white mats.

These products were not only important for subsistence, but also of high cultural significance. Yam, for instance, is the most prestigious vegetable food and preferred gift in social obligations. The ripening of the yam was the time for the first fruit presentations and the beginning of a new year, as was the case in other Pacific societies and most likely even in ancient Polynesia (Gifford, 1929; Kirch & Green, 2001). Yam and coconut are prototypical for cultivated food that differentiates between civilized and non-civilized people (e.g., Gifford, 1924). Several *tapu* applied to certain species of fish or are connected to successful fishing in general (Collocott, 1921), and coconuts were used for divination (Gifford, 1929). Still, there were other products, either essential or significant, that were not counted specifically.

Abundance cannot have been the single criteria either, as many objects that are plentiful in the islands were not counted specifically. However, if we combine importance, or rather cultural significance with abundance, we obtain an intersection that precisely maps onto the group of specifically counted objects. While things like kava, lobster, or pig are culturally salient, they are not plentiful; and breadfruit, taro, or mango, on the other hand, are abundant, but not appreciated as much as comparable products. Only coconut, yam, fish, and the material for thatching houses and weaving mats are both important *and* abundant in Tonga.

A common pattern can be identified in other Polynesian languages, for instance in Samoan, New Zealand Māori, and Hawai'ian. In all four languages, fish belonged to the category of specifically counted objects, and in some of them, coconuts, the most prestigious tubers, and material for fabrics were also included (cf. Bender & Beller, 2005).

But still the main question remains unanswered: What could be the reason for the parallel use of these different counting systems?

The Objective of Specific Counting

Taking for granted that the Polynesian languages inherited a general system with base 10 and extending well beyond 1 000—why then did they develop further systems with 'mixed bases' that were restricted to certain objects?

Again, Tongan provides a particularly interesting case. Its general way of counting follows a perfectly consistent, decimal system with a limiting number of 1 000 000. With the exception of slight irregularities for terms in the tens and hundreds, this system was easy to learn and memorize as the lexemes for basic numbers were comparatively short; the order of the power terms was fixed, ranging from the higher to the lower ones; and the base was of medium size. In order to answer the question as to why people would—for a small number of frequently used objects—give up such an efficient system in favor of apparently more complicated ones, we need to turn to the controversy over the limiting number.

In most cases, and particularly in Tongan and Hawai'ian, these 'mixed-base' systems go together with numerals for high numbers. One of the reasons why some scholars doubt that the high numerals were used in a numerical sense (e.g., Elbert & Pukui, 1979) is that they were far beyond countable amounts. However, high number words are not required in counting—they are required in calculating.

There is no doubt that at least some Polynesian cultures had a great interest in genuine high numbers and number words. Even in daily life, when preparing and weaving mats, for instance, large amounts of certain objects were required. In addition, large numbers of people had to be provided with food at special occasions like ceremonial feasts or during war. Particularly in the hierarchical societies of Hawai'i, Tahiti, and Tonga, goods were regularly centralized and redistributed by chiefs and kings (e.g., Goldman, 1970; Kirch, 1984).

As this was done by way of gift or tributes and allocation, not only addition (as in counting) was required, but also multiplication (as in calculating): Several families contributed to a village's share, and several villages to an island or district share, thus eventually producing considerably large amounts.

Keeping an account of these goods and coordinating their redistribution was an important task. As no notation system was available, dealing with large numbers was rendered also a difficult task. And it is exactly in this context of accounting where the specific number systems make sense. In extracting a certain factor (such as 2, 4, or even 20) from the absolute amount, numbers can be abbreviated and the cognitive effort required to operate with them facilitated. As Zhang and Norman (1995) stated, a cognitive trade-off is associated with base size: Despite being efficient for encoding and memorizing big numbers, large bases also require the memorization of larger addition and multiplication tables for calculations. The specific, 'mixed-base' systems, however, combine the advantages of both the medium-sized decimal base as well as the larger 'semi-base' 20. While still sticking to the restricted amount of lexemes necessary for basic numbers and to the respective addition and multiplication tables, as in the former case, encoding and internal representation of large numbers (in terms of absolute amounts) as well as operating with them was facilitated, as in the latter case.

There is some evidence to support this abbreviation hypothesis.

(1) One indication is that the general numerals were sometimes used even for the special objects as long as the total numbers were small. Or, the other way around: It was particularly for large numbers that these specific systems were used.

(2) A second indication is provided by the group of objects that were counted specifically. What all these objects have in common is the fact that they were important enough to be counted with more than sporadic frequency, and at the same time they were sufficiently abundant to make an abbreviation desirable. Accordingly, it is precisely this combination of features that not only characterizes but even *legitimizes* the supplementary use of counting systems with their 'mixed bases' and high numerals.

In Tonga, the application of these specific ways of counting can still be observed in at least two contexts. One is the presentation of food to the sovereign (cf. Bender & Beller, 2004; Bott, 1982; Evans, 2001), in which the ceremonial character demands the observance of traditional counting. The other context is part of women's work in daily life and occurs when pandanus leaves (*kie*) are prepared for weaving. When tied together to be taken to the sea for bleaching, people continue to make bundles in the literal sense at the score. And as the mats made from *kie* are among the most valuable goods of a Tongan family and do, at the same time, require huge amounts of leaves, the *kie* may serve as perfect example for the category of objects to be counted specifically.

(3) A third argument in favor of our abbreviation hypothesis has to do with the 'mixed bases' themselves. There is no indication for a genuine vigesimal system in any of the Polynesian languages as was assumed, for instance, by Best (1906) or Bauer (1997). Instead, all powers denoted by numerals in all Polynesian systems result from multiplication of lower powers by 10. What we do find—and in many cases *in addition to* a regular decimal system—are systems that seem to mix a second base, most often 2 or, as in Hawai'ian, 4, with the fundamental base 10.

With regard to the 'supplementary base', we can find evidence for the fact that exactly this number is reflected in the customary way of counting. Within the Tongan lexicon, for instance, "counting in general" (*lau*) is distinguished from "counting one by one, not in pairs" (*lau fakamatelau*). Even in practise, many Tongans still prefer to count objects by taking two items at a time (cf. Bender & Beller, 2004). The same is reported for Māori (Best, 1906). And for Hawai'ian with its 'supplementary base' four, Alexander (1864) refers to the custom of counting particular objects in pairs of pairs.

We therefore consider it justified to conclude that, what might appear as a second base at first glance, should be rather interpreted as a counting unit, on which an actually decimal system operates. That means, not single objects were counted, but pairs of objects (or pairs of pairs of objects as presumably in Hawai'ian) and even scores of objects as in certain cases in Tongan or Māori.

It is also no coincidence that all the 'secondary bases' are *even* numbers, and predominantly 2, as counting in pairs is fairly fast and comfortable, both from a cognitive and a practical point of view. It then even makes sense that partly different lexemes are chosen for similar reference numbers as these make it easier to differentiate between the number systems and to identify their respective value.

This assumption also explains why the apparently 'mixed-base' systems were not used exclusively but did supplement more general decimal systems: They were not entirely distinct but derived from the general system. In fact, they 'transposed' the original system into a higher order system.

Conclusion: Expanding the Limiting Number

Distinct systems for specific objects can be found in several Polynesian languages (Bender & Beller, 2005). Since they were used besides a general decimal system, they may be, from an evolutionist perspective, regarded as primitive remnants. However, as we have shown, these specific systems did *not* precede the general one, but were derived from it. As a derivative they were even quite sophisticated and can therefore not be regarded as indicating a lack of mathematical comprehension, either.

We argue, to the contrary, that they were invented for rational purposes. One was to enhance counting of frequent objects by using pairs or tens or scores as the counting unit. At the same time, this extraction of a certain factor abbreviated higher numbers and consequently facilitated mental arithmetic.

The other purpose might have been to expand the limiting number of the respective system, up to a factor 20 in the case of the Tongan specific systems.

The distribution of 'mixed base' systems and limiting numbers among Polynesian languages supports this assumption. Despite the overwhelming convergence in numerals, Table 1 reveals some variation with regard to the limiting number in contemporary Polynesian number systems. This variety may result basically from expanding or contracting the original system according to local requirements. One of these requirements could have been the size of the population and the degree of stratification. In islands with powerful chiefs or kings and their strong concern with collecting and redistributing resources (such as Tonga, Tahiti, and Hawai'i), higher numbers might have been required, while societies with less centralized political forces or small communities (such as Rapanui or Māori) might not have felt a need for the very large numbers. It is therefore no coincidence that particularly in languages with high numerals—and we may add: with a concern for high numbers—supplementary systems were in use as well.

The existence of specific number systems does not contradict an abstract interest in numbers because coconuts and fish cannot reasonably be added anyway, and when people wished to operate generally they still could work with the general number system. The evolutionist claim that specific number systems are premathematical remnants can therefore be rejected. Under the unfavorable conditions of mental arithmetics without external representation, specific number systems do serve remarkably rational purposes—and they do so in a cognitively very efficient way.

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