

# A Computational Model of Human Planning in the Traveling Salesman Problem

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## Abstract

Planning is a fundamental cognitive function frequently employed in usual daily activities. The Traveling Salesman Problem (TSP), in which the participant decides what order between a number of locations optimizes total travel distance, is a paradigm that allows the study of planning and strategy choice. We present a computational model that simulates the human performance observed during the execution of a variant of the TSP task, including bottom-up and top-down influences. The performance of the model simulated the continuous monitoring observed in human participants; moreover, after a lesion to the network's units corresponding to the prefrontal cortex, we found a decrease of strategy changes like those observed in frontal patients.

**Keywords:** TSP, planning, computational modelling, frontal damage.

## Introduction

Planning is a mental simulation which envisages the circumstances and runs possible actions, evaluating their consequences and selecting the optimal order for executing them (Cohen, 1989). It depicts the skill of producing mental representations of the future behavior prior to action and sequential reasoning about the consequences of acting, in order to properly choose among the possible courses of action. TSP is a useful tool to investigate planning, because, in this task, participants are required to generate a strategy in order to optimize the pathway.

TSP, as a paradigmatic example of non-polynomial optimization problem, has been extensively studied by mathematicians and computer scientists (see Lawyer et al.,

1985, for a review) and much less by psychologists. Nevertheless, there has been a growing interest in the analysis of human performance in TSP-like problems (Cadwallader, 1975; Gärling, 1989, 1994). In the TSP, given a set of  $n$  interconnected towns, represented by nodes on a graph, the task consists in finding an itinerary that visits each town exactly once, returning to the starting town, ensuring that the total travelled distance is as short as possible.

TSP involves essentially visuo-spatial planning, a particular kind of problem solving in which participants try to optimize a performance measured in spatially constrained environments. In comparison with more typical planning tasks, spatial planning requires a stronger interaction between central and peripheral processes: visual, attentive and motor factors play a fundamental role, in addition to reasoning, in determining the final behavior.

Our research is inspired by a series of studies conducted with an open version of the TSP task (Basso et al., 2001), consisting in choosing the order of locations with the start and end point fixed (fig. 1b). The behavioral data showed the presence of three distinct spatially-based heuristics supporting the production of successful strategies. The first heuristic, widely described in previous studies (Barr & Feigenbaum, 1981; Hirtle & Gärling, 1992) is the Nearest Neighbour (NN): it states that each location is chosen on the basis of the local minimum distance from the actual location. The other two Direction Heuristics (DH) described in the studies conducted with the TSP variant take place when subjects start from a location placed on a border and reach the next locations following one of the main spatial axes (horizontal or vertical) (Basso et al., 2001) and a direction (up or down for vertical axis, left or right for

horizontal axis). In the studies we refer to, the starting point was always in the upper left corner; accordingly, the heuristics have been renamed with Direction Right (for horizontal heuristic) and Direction Down (for vertical heuristic).

Another interesting finding concerns the interaction between heuristics: participants often show on-line changes of heuristic during the execution of the task (Basso et al., 2001). These results confirm that subjects operate a continuous monitoring and flexibly adapt their behavior to the requirements of the task (or environment), as pointed out in previous studies (Hayes-Roth & Hayes-Roth, 1979).

From a neuroanatomical point of view, the substrates of planning skills are located in frontal areas: planning deficits have long been associated with frontal lobe lesions (Cooper & Shallice, 2000; Duncan, 1986; Goel & Grafman, 1995, 2000; Shallice, 1982, 1988). More specifically, the crucial role of the frontal lobe in visuo-spatial planning has been demonstrated using the TSP variant described above. Patients with frontal lesions and normal subjects under repetitive transcranial magnetic stimulation (rTMS) over the frontal lobes showed a significant reduction in the number of heuristic changes which are usually performed during the execution of the task (fig.1a) (Basso et al., submitted).

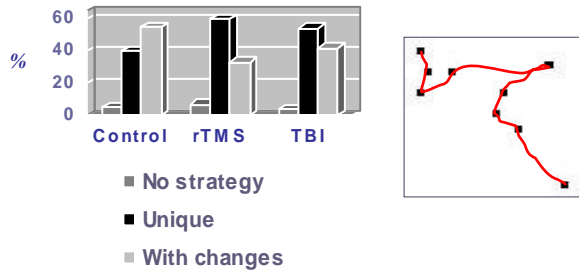


Fig.1. a) Percentage of strategies from the experiment with the TSP variant. From left to right: normal subjects, subjects under rTMS, and traumatic frontal lobe brain injured patients (TBI). b) An example of an open version of TSP.

The aim of this article is to develop a computational model simulating the perceptual and cognitive processing involved in the human solution of the TSP. The model shows a good match to human performance when tested on the same patterns administered to the healthy participants. Moreover, after an artificial lesion to its “frontal lobe” component, the model accounts for the behavior exhibited by traumatic brain injured patients (TBI) with damage to the frontal lobe (Basso et al., 2001).

## The model

The computational model is composed by three interconnected modules (fig. 2), with a broad hierarchical organization and feedback connections, that loosely simulate the occipito-parieto-frontal circuit involved in the TSP task. These components comprise: 1) a visual module,

in which the input pattern is processed by Gabor filters to simulate the processes responsible of perceptual grouping; 2) a competitive selection module that simulates the internal dynamics for the choice of the heuristic; 3) a motor map, based on population codes, that executes the pathway at the spatial level. Moreover, the presence of saliency maps, recurrent connections and inhibitory mechanisms allow us to simulate the incremental aspect of visuo-spatial planning and the interaction of bottom-up and top-down processes.

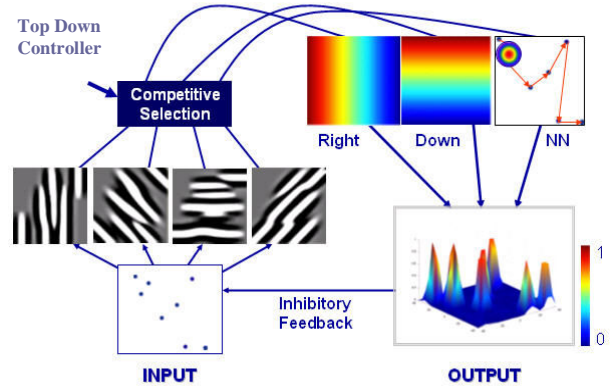


Fig. 2. The architecture of the model.

## The visual module

This module is designed to enhance the directional features intrinsically present in the input pattern. The input consists of a two-dimensional image of 161x161 pixels representing the pattern. Each city is represented by a black region of 4x4 pixels coded with ones, whereas empty space is coded with zeros.

The visual image is processed by a family of eight Gabor filters. Gabor filters are band-pass filters with tuneable center frequency, orientation and bandwidth. Gabor filters provide an excellent approximation of the response of simple cells in the primary visual cortex and have been widely used in computer vision. In particular, we used the family derived by Lee (1996):

$$\psi(x, y, \omega_o, \theta) = \frac{\omega_o}{\sqrt{2\pi\kappa}} e^{-\frac{\omega_o^2}{8\kappa^2}(4(x \cos \theta + y \sin \theta)^2 + (-x \sin \theta + y \cos \theta)^2)} \cdot \left[ e^{i(\omega_o x \cos \theta + \omega_o y \sin \theta)} - e^{-\frac{\kappa^2}{2}} \right] \quad (1)$$

where  $x$  and  $y$  represent the center of the wavelet,  $\omega_o$  is the radial frequency in radians per unit length and  $\theta$  is the wavelet orientation in radians. Moreover, a Gabor filter is defined as a complex function, and its real and imaginary part are used as two real filters (odd and even). In the present work, we varied only the wavelet orientation ( $0, \frac{1}{4}\pi, \frac{1}{2}\pi, \frac{3}{4}\pi$ ) for a total of eight filters (four even and

four odd). A set of eight maps was then obtained by convolving the input image with the eight Gabor filters.

To calculate the strength of the directional features extracted by the different filters, all local values evaluated from the patterns (composing the training set) were scaled to the [0 1] range and the highest value was collected for each map. This procedure allows to establish whether the maximum value of a filter is above the average global response, indicating a strong presence of the directional features enhanced by the filter. The structure of this module is consistent with the hypothesis of Field (1994) that oriented edge features constitute a sparse representation of the images. This means that for any image, only a few of the features are needed to represent that particular image, and that over an ensemble of images a particular feature will seldom be significantly active.

### The competitive selection module

The simulation of the selection process was implemented with a self-organizing, competitive learning network consisting of feedforward excitatory connections (between input and output units) and lateral inhibitory connections (only between the output units). The training set consisted of 100 randomly created patterns, composed of six to ten cities.

This network was able to discover three main categories of input images on the basis of the features detected by Gabor filter processing. In the model, the competitive learning network has one layer of eight input neurons corresponding to the normalized highest value of the eight Gabor filter responses, and a layer of output neurons. Each input unit indicates the strength of a particular orientation axis, whereas each of the three output nodes represents an image class that calls for a specific heuristic (see below). Inserting more than three units in the competitive layer did not produce substantial changes in the results, as most of the patterns (95%) were still classified by three units.

In the learning phase the weights  $w$  are updated according to the following equation:

$$\Delta w_i = \eta(x_i - w_i)y \quad (2)$$

where  $\eta$  is the learning rate,  $x$  is the activation of the input vector and  $y$  is the activation of the winning output unit. Note that in a winner-takes-all circuit, the output unit receiving the largest input is assigned the full value of 1, whereas all other units are suppressed to a 0 value. The competitive selection takes place thanks to the lateral inhibition in the output layer.

The result of the competitive selection is produced by an entirely bottom-up process (spatial analysis). The presence of directional features in the input pattern determines the choice of a heuristic, which in turn biases the execution of the pathway by means of a saliency map.

### The saliency maps

The saliency maps have been implemented to simulate the influence of the chosen heuristic through the sequential selection of stimuli. The saliency maps have the same size of the motor output map (see below): each unit in the motor map is pre-activated by the corresponding unit of the saliency map. The Nearest Neighbor (NN) is implemented as a Gaussian-shaped hill of activity centered on the last visited city, whereas Direction Right (DR) and Direction Down (DD) saliency maps consist of linear gradients that cover the entire visual space.

The effect of a saliency map is to relatively enhance the activation of one city by reducing (in accordance with the specific gradient) that of all other cities on the map. The competition among neurons in the motor output map produces a single winning location that corresponds to the most salient city, which constitutes the next target. Each unit of the competitive selection module activates the heuristic (that is, a saliency map) that corresponds most closely to its coded directional feature.

We have observed that most of the subjects tend to select the DR heuristic when the cities are principally distributed along the diagonal axis from upper-left to lower-right, while the DD is often chosen when the cities are principally distributed along the opposite diagonal axis. In intermediate or ambiguous situations, subjects tend to use the NN heuristic. Similarly, after training on the visual patterns, two output units of the competitive network are mainly driven by the Gabor filters oriented at 45° and 135°, respectively. Accordingly, they have been connected to the DR and DD saliency maps. The last unit, being equally driven by many Gabor filter outputs, has been connected to the NN saliency map.

### The Motor Output

The output map is composed of 21 x 21 units. The location of the activations correspond to the positions of the cities in the pattern. Each neuron  $O_i$  receives inputs from the corresponding units of the visual input, and from all the other output units.

These different inputs are combined in the following way:

$$O_i = \frac{2}{1 + e^{-NetInput_i}} - 1 \quad (3)$$

(if  $NetInput_i \leq 0$   $O_i = 0$ )

$$NetInput_i = V_i(1 + Saliency_i) + \sum_j W_{ij}O_j \quad (4)$$

where  $NetInput_i$  is the input to the unit and  $Saliency_i$  is the activation value of the  $i$ -th unit in the Saliency map.

All the saliency maps consist in a gradient that produces a pre-activation of 0.2 at motor level in the position of maximum enhancement. In particular, the DR saliency map produces a preactivation of 0.2 at the extreme left side that decreases linearly to zero at the extreme right side. The DD

saliency map employs an equivalent gradient from the upper to the lower side of the output map. The NN saliency map is represented by a broad Gaussian-shaped hill of activity centered on the last visited city, with a peak of 0.2. The left-hand side of equation (4) computes the visual input  $V_i$  modulated by the corresponding Saliency unit, while the right-hand side computes the internal input, resulting from the connections  $W$  with the other output units. Indeed, the output map contains symmetric lateral connections with fixed-value inhibitory weights that depend on the distance between neurons. When more than one stimulus is presented to the network, the competition between neurons belonging to different populations resolves over time in favor of one of the stimuli.

Every time a city is visited, the corresponding population code is subsequently suppressed in the motor map and inhibition spreads to the input module via a feedback connection. This ensures that the city with the highest activation value will be visited and that the same population will not be selected again during the sequential selection process.

### The Top-Down Controller

After each step, The Top-Down Controller resets the activation of the heuristic units in the competitive selection module. This endorses the required flexibility of the model in order to make it capable of a change of heuristic during the execution of the task.

### Simulation

The model is able to perform planning as an incremental process. For each pattern, the visual input is analyzed with Gabor filters to enhance the most influential spatial features, then the competitive selection module chooses the most appropriate heuristic for the pattern. The winning heuristic is implemented in terms of a saliency map, whose activation influences the motor map in determining the city to be visited.

At the end of each step, the units in the motor map corresponding to the visited city are inhibited and the activation of the same city in the input pattern is reduced via the inhibitory feedback loop (fig. 2). This reduction allows a possible change of heuristic: indeed, the visual input is processed again in the visual module and, if a different heuristic emerges from the competitive selection, the Top Down Controller inhibits the previously selected heuristic to foster the new heuristic. This process takes place at every single step; therefore, heuristics can be changed more times during the execution of a single path.

## Results

The performance of the computational model has been compared with the behavioral data of the experiments based on the TSP variant (Basso et al., submitted). The percentage of heuristics chosen by the model in performing the TSP on

the set of input images, using the selection mechanism described above, was compared with the data collected on both healthy and impaired participants.

We evaluated the heuristics considering the relationships among the cities: the heuristic used to move from a city to another was judged considering the distance from that city and the other cities not yet visited. At each step, we determined the horizontally closest city, the vertically closest city and the absolute closest city, with respect to the last visited city. Following this method, we obtained a precise description of the heuristics used by the model at every single step.

### Skilled performance

The resulting percentage of heuristics used by the model was very close to that observed in healthy adults (fig. 3a). We also analyzed the overall pathways chosen by the model (fig. 4). The test consisted in comparing each pathway executed by the model with the frequencies of solutions produced by the human participants for the corresponding pattern. For this test, we used the patterns which have been administered to the largest number of subjects: indeed, for these patterns the frequencies of the pathways chosen by the participants were clearly more reliable.

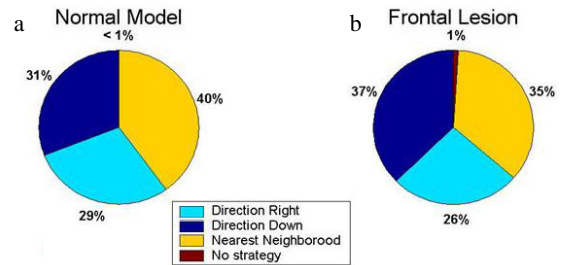


Fig. 3. Percentage of heuristics chosen by a) the unimpaired model and b) the lesioned model.

The results demonstrate that the pathways chosen by the model are often among the most frequently produced by healthy adults. In particular, in 50% of patterns, the pathway chosen by the model is the most frequently executed by the human participants.

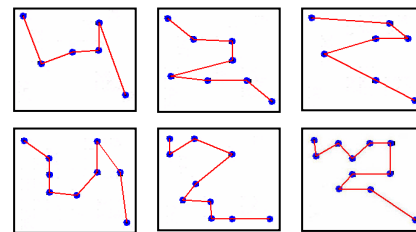


Fig. 4. Examples of pathways performed by the skilled model. These pathways correspond to those most frequently observed in human subjects.

Moreover, in order to distinguish the type of strategy used by subjects to solve the TSP task, the number of heuristics used in every single trial has been examined: if, during the pathway, participants operated a switch between two or more heuristics, the resulting strategy was considered with changes; otherwise, the strategy was classified as unique. The ratio between unique strategies and strategies with changes showed by the model is very close to the performance showed by healthy adults (compare fig. 1a with fig. 5a).

### Lesioned model

We simulated a lesion to the Top Down Controller by decreasing its capacity to reset the competitive selection module. This damage produced a conspicuous decrease of strategies with changes (fig. 5b). This matches the decline of changes observed by Basso et al. (2001) in frontal patients and in normal subjects under rTMS over the frontal lobe (fig. 1a). As for the behavior of frontal patients, the performance of the model was still adequate because of the preserved bottom-up mechanism (fig. 3b), but the damage of the Top Down Controller caused a loss of flexibility and adaptivity in the behavior, resulting in a greater difficulty in switching between heuristics.

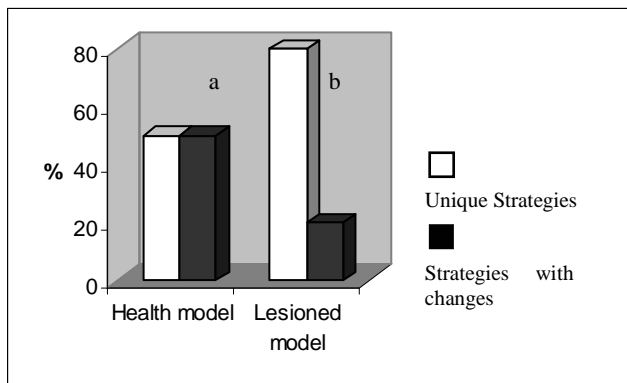


Fig. 5. Performance of a) the unimpaired model and b) the lesioned model.

### Discussion

Overall, the simulations closely mirrored human performance in both normal and pathological states. This suggests that the model captures the basic cognitive processes involved in the human solution of the TSP. There are several points that deserve consideration. First of all, the spatial analysis performed in the visual module through the Gabor filters appears to successfully account for bottom-up influences in the TSP. That is, the directional features of the input pattern, enhanced by perceptual grouping, are the main determinants of the TSP solution. Note that Gabor filters have been previously used to successfully simulate perceptual grouping (Carreira et al., 1998).

Another remarkable finding concerns the selection among the possible heuristics. Even inserting more than three units in the competitive layer, most of the patterns (95%) were still classified by three units. This means that the images can be divided into three broad categories with respect to their directional features.

However, the most intriguing characteristic of the model regards its capacity to change heuristics during the execution of a single pathway. The flexibility of the model is based on the online updating of the problem representations. Healthy adults appear to execute the TSP in an iterative manner; the incremental process requires a minimum cognitive load because participants do not need to generate a comprehensive plan resolving the entire situation but only the next move. The model executes the pathway operating the same incremental process observed in human participants, by selecting and having the chance to choose among different heuristics at every point of the pathway. The interaction of bottom-up (Gabor filters and competitive selection module) and top-down (Top Down Controller) influences have been successfully implemented in our model.

Finally, there are strong similarities between our computational model and the ATA model (Norman & Shallice, 1986). In our model, the Top Down Controller allowed the inhibition of the previously chosen heuristic, when at a given point the use of another heuristic was more appropriate. The rare use of strategies with changes showed by the damaged model confirmed the key role of the Top Down Controller in this task, in agreement with Norman and Shallice's (1986) proposal regarding the Supervisor Attentional System.

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