The Truth about Possibilities-based and Truth-based Truth Table Tasks

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Abstract

Truth table tasks are a popular way to investigate reasoning performance. In this paper, we make a direct comparison between the answer patterns yielded by two task formats used to investigate the mental representation of conditional sentences: the possibilities-based truth table task and the truth-based truth table task. In a truth table task, participants are asked to evaluate four logically possible cases produced by permuting the truth values of the antecedent and the consequent: true-true (TT), true-false (TF), false-true (FT) and false-false (FF). In the possibilities-based truth table task, they have to indicate whether a case is either possible or impossible according to the rule and in the truth-based truth table task, participants have to evaluate whether the case makes the given rule either true, false or is irrelevant with respect to the truth of the rule. Lately, a discussion of the status of "irrelevance" has emerged. The experiment shows that when participants have the opportunity to do so, they are inclined to judge the false-antecedent cases to be irrelevant rather than true with respect to the truth of a given rule.

Keywords: Conditional Reasoning; Truth Table Tasks; Irrelevance

Introduction

The interest in the linguistic, psychological and logical meaning of "if" has provided us with a long history of research on thinking and reasoning about conditionals. It is not surprising that the word "if" is one of the most important in our language: Conditional statements occur extremely frequent in everyday discourse and in all thinkable contexts. We use them explicitly in promises or threats, but conditionals are also omnipresent in an implicit way: we know that if we are thirsty we have to drink and that if we cross the street without paying attention, we might get run over. Conditional reasoning is a central part of our thinking, allowing us to reflect on the implications of what may come, could have been or will be. And it goes without saying that this is a key factor in decision making.

Many studies have been designed in order to externalize people's understanding and mental representation of conditionals. By conditionals, we mean conditional sentences of the form *"If A then C"*, with *A* being the antecedent and *C* the consequent of the declarative clause. It's important to know how we represent and use the information we acquire. For example: How do we get to a conclusion on the basis of that knowledge? And what information serves as a confirmation or a falsifying counterexample for that conclusion?

In an implicit way, classical logic is omnipresent in the interpretation of the external world in our daily lives. For example, either it is snowing or it is not, either Belgium is my current physical location or it is not, either an angle is orthogonal or it is not. On a more abstract level, classical logic provides the foundation for all program control expressions in computer source code: Boolean expressions evaluate to zero if false and to one if true.
Consider the following example:

> *Either my present physical location is Belgium or it is Brazil, but not both.*

The truth of this assertion is a function of the truth of its constituent atomic propositions. A proposition is a statement that can be either true or false; it must be one or the other, and it cannot be both. An atomic proposition is a proposition that cannot be divided into smaller propositions. So "my physical location is Belgium" and "my physical location is Brazil" are atomic propositions. Logicians present those propositions in a truth table. Each row in a truth table represents a different possibility. Connectives, i.e., words connecting statements into more complicated compound statements, that can be defined in a truth table are known as *truth* functional: their meanings are functions that take the truth values of propositions as inputs and deliver a truth value as an output (Jeffrey, 1981).

Table 1: Truth Tables for Conditional Statements.

A	C	Material implication	Material equivalence	Defective implication	Defective equivalence
T	*T*	T	T	T	T
T	*F*	F	F	F	F
F	*T*	T	F	I	F
F	*F*	T	T	I	I

Since classical logic has been the starting point and framework for research in conditional reasoning for many decades, one frequently used methodology investigating the mental representation of conditionals sentences is the truth table task, introduced by Johnson-Laird and Tagart (1969). In this method, participants are asked to evaluate, with respect to a given rule, the truth or the possibility of four logically possible cases produced by permuting the truth values of the antecedent and the consequent. For instance, given the rule *"If the letter is a B, then the number is an 8"*, four possible combinations or cases arise:

- a B and an 8 signify a true antecedent and a true consequent (TT)
- a B and not an 8 signify a true antecedent and a false consequent (TF)
- not a B and an 8 signify a false antecedent and a true consequent (FT)
- not a B and not an 8 signify a false antecedent and a false consequent (FF)

Truth tables are a type of mathematical table used in logic to determine whether an expression is true or valid. According to classical logic, the connective *if* follows the truth table for the *material implication* (see Table 1), meaning that only the TF case is false and that the three other cases are true. Therefore, the rule *"If B then 8"* is falsified only when there is a B, but not an 8 (TF). This material implication truth table has the form TFTT.

An alternative logical possibility for the meaning of the connective *if*, is that of the *material equivalence*: *"C if and only if A"*, the situation in which A implies C and C also implies A. According to this TFFT truth table, not only TF, but also the case FT falsifies the rule. So the conditional *"If B then 8"* is falsified both when there is a B but not an 8 (TF), and also when there is not a B but there is an 8 (FT). Material implication and material equivalence are the only two truth tables for conditionals recognized under classical logic.

More recent research however has questioned whether classical logic is to be considered as an adequate normative system for ordinary reasoning: Psychologically, there is evidence that, next to true or false, people make use of a third truth value absent in classical logic: irrelevant or indeterminate (e.g., Evans & Over, 1996, 2004). Wason (1966) was the first to introduce the "defective truth table", in which false antecedent cases (FT and FF) are considered irrelevant rather than true. The *defective implication* has a truth table of the form TFII and the *defective equivalence* of the form TFFI. In the defective truth table view, people think that false-antecedent cases have nothing to do with the truth or falsity of a conditional and are therefore labeled as "irrelevant". This is referred to by Evans et al. (2004) as a "truth value gap" for the false-antecedent cases. For example the FF case "not a B and not an 8" would be interpreted as irrelevant with respect to the truth or falsity of the conditional rule *"If B then 8"*. This defective truth table representation of

conditionals has been replicated on a number of occasions (e.g., Newstead, Ellis, Evans & Dennis, 1997).

Throughout the psychological reasoning literature, one can broadly discern two types of truth-table tasks: a possibilities-based and a truth-based variant. In the possibilities-based truth table task, people have to indicate for each of the four possible antecedent-consequent cases whether that specific combination is either possible or impossible with respect to the given rule (e.g., Barrouillet & Lecas, 1998). In the truth-based truth table task, participants are asked to evaluate for each of the four cases whether the combination makes the given rule true, false or is irrelevant with respect to the truth of the rule (for a review, see Evans, Newstead & Byrne, 1993).

In this context, Evans and Over (2004) state: "We are a little suspicious of the 'possibilities' form of the truth table task that is exclusively used by followers of the mental-models theory and seems to us to obscure a distinction that the standard task reveals." They refer to the truth-based truth table task as the "standard task" and argue that with the dichotomy possible/impossible, used in the possibilities-based task, one can not point out the difference between true and irrelevant responses: "Possible" just means "not impossible" while "true" cannot be presupposed to mean "not false" when "irrelevant" is included as a third manner to classify cases.

We realize that, next to their high degree of resemblance, the two tasks discussed above differ concerning their directionality: The truth-based truth table task focuses on the evaluation of the given rule on the basis of the cases, and the possibilities-based truth table task on the assessment of the cases with respect to the given rule. Nevertheless, this is the format in which the tasks have been administered in the conditional reasoning research tradition for many years. So changing the directionality of one of the tasks is impracticable since it would make an explicit comparison between the possibilities-based and the truth-based truth table task in their traditional format impossible. And this is exactly what constitutes the problem statement of the present exploratory study: Since a direct comparison of the possibilities-based truth table task and the truth-based truth table task has never been subject to an experimental study, it is our aim to bridge this gap, shedding light on the truth table judgments and the shift in the response patterns caused by the task differences.

Experimental Study

Method

Participants Two hundred first-year Psychology undergraduate students at the University of Leuven (17-23 years of age, Me = 18) took part in this experiment as a partial fulfillment of a course requirement. None of the participants had ever taken a logic-course. One hundred participants received the possibilities-based truth table task and 100 participants the truth-based truth table task.

Materials and Procedure A truth-table task can be administered either in a construction or an evaluation format. In an evaluation truth table task, the participant is asked to evaluate each of the four presented cases with respect to the

given rule. In the construction variant, participants have to construct exhaustively all possibilities that verify or falsify the rule. We chose for the evaluation format in order to keep the possibilities-based truth table task as similar as possible to the truth-based task, that is usually administered in an evaluation format.

Participants were run in two groups, with one group performing the possibilities-based truth table task and the other group the truth-based variant. There was a random assignment of the participants to the task-type.

Both tasks were constructed with 'E-prime' software and presented to the participants on individual standard PCs in a self-paced manner. Responses were given with the arrow-keys on an AZERTY keyboard. The experiment lasted between 5 and 15 minutes. All parts of the experiment were presented in Dutch.

The instructions appeared on the computer screen and explained that the purpose of the experiment was to examine how people reason with conditionals. Participants were provided with a concrete example of a conditional statement, as well as with an example of the questions in the actual task (no correct answers were provided). The instructions were followed by one practice-trial.

In the two task-types, all participants were successively presented with both the conditional rule and one of the four combinations of the occurrence and non-occurrence of A and C. This means that each participant was presented with the four following cases in a random order:

- *Rule: if the letter is a G, then the number is a 7*
  Case: "a G" in combination with "a 7"

- *Rule: if the letter is a G, then the number is a 7*
  Case: "a G" in combination with "not a 7"

- *Rule: if the letter is a G, then the number is a 7*
  Case: "not a G" in combination with "a 7"

- *Rule: if the letter is a G, then the number is a 7*
  Case: "not a G" in combination with "not a 7"

In the possibilities-based truth table task, participants had to evaluate for each of the four cases whether that combination was either possible or impossible according to the rule. In the truth-based truth table task, participants had to evaluate for each of the four possible antecedent-consequent combinations whether the combination made the given rule either true, false or was irrelevant with respect to the truth of the rule. No feedback was provided. The rule followed by one of the four cases appeared each time on a different screen and presentation order of the four cases was completely randomized.

Results

Table 2 summarizes the responses ("possible", "impossible") on the four cases presented in the possibilities-based truth table task. The TT case is evaluated to be possible with

respect to the rule by 91% of the participants, the FF case by 87% and the FT combination is judged to be possible by 52% of the participants. The TF case, the only impossible combination according to the material implication truth table, is evaluated to be possible by only 19% of the participants.

Table 2: Truth table judgments (%) for the possibilities-based task.

	TT	TF	FT	FF
Possible	91	19	52	87
Impossible	9	81	48	13

Table 3: Truth table judgments (%) for the truth-based task.

	TT	TF	FT	FF
True	84	15	8	38
False	5	76	63	8
Irrelevant	11	9	29	54

Table 4: Response patterns (%) for the possibilities-based and the truth-based task.

	PTT	TTT
Material implication	36	4
Material equivalence	37	20
Defective implication	/	17
Defective equivalence	/	25

Table 3 summarizes the responses ("true", "false" or "irrelevant") of the 100 participants that were presented the truth-based truth table task. The TT case yields 84% of true and 11% of irrelevant responses, the FF case 38% of true and 54% of irrelevant responses, and the FT combination 8% of

true and 29% of irrelevant responses. The TF case is judged to make the given rule true by 15% of the participants and considered irrelevant with respect to the truth of the rule by 9 % of the participants.

Focusing on both tasks regarding the amount of logically correct judgments according to the material implication truth table, we performed a Chi-square test for two independent samples. In Table 2 and 3, the data were arranged into a frequency table (n = % since both tasks were performed by 100 subjects). The expected frequency in each cell was computed by multiplying the two marginal totals common to a particular cell and then dividing this product by the total number of cases.

We compared the proportion of "possible" vs. "impossible" judgments with the proportion of "true" vs. "false" plus "irrelevant" judgments for TT, FT and FF. For the TF case, we made a comparison of the proportion of "impossible" vs. "possible" judgments with the proportion of "false" versus "true" plus "irrelevant" answers. There is no significant difference between the number of possible and true judgments on the TT case (91 - 9 vs. 84 - 16), neither is there a significant difference between the number of impossible and false judgments on the TF case (81 - 19 vs. 76 - 24). On the false-antecedent combinations (FT and FF), we do observe a significant difference between the amount of possible and the amount of true judgments, both on the FT case (52 - 48 vs. 8 - 92, $\chi^2(1) = 46.1$, $p < .001$) and on the FF case (87 - 13 vs. 38 - 62, $\chi^2(1) = 51.22$, $p < .001$).

In the possibilities-based truth table task, more than 70% of all responses reflect either material implication or material equivalence truth table responses, both patterns occurring equally frequent (see Table 4). In the truth-based truth table task, permitting the participants to answer "irrelevant" next to "true" or "false", only 4% of the participants respond according to a material implication pattern and 20% according to a material equivalence truth table. In addition, 17% of defective implication patterns and 25% of defective equivalence patterns are observed. All other patterns are given by less than 5% of the participants and therefore considered negligible.

Discussion

With the present study, we aimed to provide a first explicit comparison between two formats of the truth table task: the possibilities-based and the truth-based truth table variant. The most striking result occurs with the negative-antecedent cases: there is a major shift from "possible" to "irrelevant". This means that people judge false-antecedent cases to be irrelevant with respect to the truth of the conditional rule, when they have the opportunity to give that answer. This finding is incompatible with the material implication truth table and in favor of a defective truth table representation: The truth of the conditional is decided by true-antecedent instances, and false-antecedent-instances are classified as irrelevant.

The large proportion of irrelevant answers on the false-antecedent cases in the truth-based truth table task is

challenging for the mental-models theory (Johnson-Laird & Byrne, 1991, 2002; Johnson-Laird, Byrne & Schaeken, 1992). According to the "truth principle" of the mental-models theory, people construct mental models of the possibilities compatible with the premises, but they initially and by default do not represent what is false. Therefore, their conclusion is based on the initial model[1]:

[A] C
…

So mental models represent only one possibility, covering what is common to all the different ways in which the possibility may occur. Moreover, mental models represent explicitly what is true, but not what is false. At least, individuals do not by default represent what is false, but under certain circumstances they make 'mental footnotes' about the falsity of clauses (represented by the ellipsis or three dots). If they are able to retain these footnotes, reasoners can flesh out the implicitly represented information into fully explicit models, which represent clauses even when they have false antecedents[2]:

  A   C
¬A   C
¬A   ¬C

These possibilities correspond to the three rows of the truth table in which the material implication is true, including the false-antecedent cases FT and FF. So the mental-models theory has to provide an answer to the observation that the TF case is judged irrelevant by 30% of the participants and the FF case by more than 50% of the participants. This means that the possibilities "¬A C" and "¬A ¬C" would not occur in the list of the explicit models representing the given rule.

The mental-models theory is able to explain the TFII pattern, but only provided that they assume that people base their answer solely on their initial model: "[A] C". This initial model undoubtedly supports the "true" judgment of the TT case. One can explain the "irrelevant" judgments for the FF and the FT cases in line with the mental-models theory on the basis of the fact that in this initial model, the false-antecedent cases are not represented. Problematic for Johnson-Laird and Byrne, however, are the "false" judgments with respect to the TF case. How can reasoners come up with this judgment? The solution lies in the square brackets, which is the original notation of Johnson-Laird and Byrne to indicate that the antecedent was represented exhaustively: [A] C. In other words, there are no further models in which there is an A. Therefore, the TF case is impossible and thus false. The problem, however, is that in recent writings (e.g., Johnson-Laird & Byrne, 2002) the square brackets are dropped. The

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[1] Square brackets [ ] are the notation for an exhaustive representation. [A] C means that the antecedent is represented exhaustively.

[2] The symbol ¬ denotes an abstract mental symbol representing negation.

omission of these brackets in their latest publications is not explicitly accounted for, but if you drop them, then it is not possible anymore to explain the frequently observed defective truth table pattern TFII.

The high proportion of "irrelevant" judgments however has to be put into perspective. The question in the possibilities-based truth table task "is the following case possible or impossible with respect to the rule" is much more straightforward than the question in the truth-based truth table task: "does the following combination make the rule true, false or is it irrelevant with respect to the rule". How do people interpret the clause "to make the rule true"? In this context, Johnson-Laird (1990) argues that "adults are often puzzled by the truth or falsity of assertions containing connectives", since children first learn to use language to refer to possibilities and develop only later a meta-linguistic ability to use the concepts true or false. This means that reasoning about the truth or falsity of cases is at a higher level and therefore more difficult than reasoning about possibilities.

It is not unthinkable that when participants do not exactly know how to reason with the truth-question, they choose the response "irrelevant" as a synonym for "in fact, I don't know". And this is not the meaning an "irrelevant" judgment is supposed to have. This explanation could be ruled out in an experiment that compares the truth-based truth table task with a possibilities-based task that also provides three answer-possibilities to the participants: "possible", "impossible" and "irrelevant". Only when participants respond "irrelevant" to the straightforward question "is the case possible, impossible or irrelevant according to the rule", one can conclude that they did not choose this alternative due to a lack of comprehension. In the current study however, we made use of the possibilities-task with two answer-alternatives, since it is this format that is most frequently used in contrast to the truth-based task within the present research tradition.

Of course, one can debate the meaning of "irrelevant" in both variants of the truth table task: Saying that a case is "irrelevant according to a given rule" does not presuppose the same understanding of the word "irrelevant" than judging a case to be "irrelevant with respect to the truth of a given rule". Moreover, the difference in directionality has to be taken into account: from rule to instance in the possibilities-based truth table task vs. from instance to rule in the truth-based truth table task. Future work should therefore pin-point the exact meaning participants ascribe to this answer-alternative, in order to clarify the reason why they make an irrelevance-judgment, and asses whether the interpretation of "irrelevant" depends on the difference in directionality.

In psychological reasoning literature, many experiments using the truth table task methodology are reported, each of them administering either the possibilities-based truth table task or the truth-based variant. It was observed that both task-types yield different results, but a systematic comparison between them was never made. In the present study, we brought together those two separate methodologies within the same field of interest: the representation of conditional sentences. The most eye-catching difference between the two methodologies was an important shift from the "possible" answers in the possibilities-based task, to the "irrelevant" judgments in the truth-based task. This shift towards a defective truth table is an observation that does not immediately leans towards the popular and influential mental-models theory. As we discussed above however, our findings are challenging for the mental-models theory, but not insurmountable if Johnson-Laird and Byrne can substantiate that reasoners base their answer solely on the initial model "[A] C", with an exhaustively represented antecedent.

Acknowledgments

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