College Students' Representation of Introductory Calculus Problems

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Introduction

Experts and novices have different problem representations (i.e., interpretations or understandings of the problems). Experts abstract principles from problems whereas novices represent superficial problem features that are irrelevant to problem solution (Larkin, 1983; Silver, 1981). While quite a bit is known about problem representations at either extreme of the expertise spectrum, information about intermediate representations is lacking.

In this study, the goal was to examine the problem representations of novices (students with limited experience with calculus such as undergraduates) and more advanced novices or intermediates (graduate students with 6-8 years of experience) to explore similarities and differences between two different but closely related levels of expertise.

Method

43 undergraduate non-math majors (14 males, 26 females) and 2 math graduate students (both females) participated in the study. Their task was to (a) sort 9 calculus problems differing in structural features (i.e., optimization, implicit differentiation, and exponential functions) and superficial features, with one surface feature (i.e., graphs) more salient than the other two features (i.e., volume and price) and (b) to document the problems that were sorted together and explain in writing the rationale for each grouping. Students performed the task during a regularly scheduled lab session.

Each student was assigned an *explanation score* (i.e., whether the correct feature was identified in the explanation) and a *sorting performance score* (i.e., whether problems comprising a sort contained the correct feature) in structural features and in surface features. The maximum score for each category was 12. Inter-rater agreement on the scoring was 98.5%.

Results

Problem Representation

Graduate students sorted the calculus problems mainly based on structural features while undergraduate students tended to sort problems based on surface features. Graduate students identified all the calculus principles and only one surface feature (the graph) in their explanations, receiving a structure-based explanation score of 3 (SD=0) and a surface-based explanation score of 1 (SD=0). The mean score for graduate students' structure-based performance was 5.5 (SD=0.7) and surfaced-based performance was 2.5 (SD=0.7). Undergraduate students' structure-based explanation score (M=0.63, SD=1.05) was much lower than the surface-based explanation score (M=1.65, SD=1.19). In addition, undergraduate students had a much higher surface-based performance score (M=4.81, SD=3.58) compared to the structure-based performance score (M=0.95, SD=1.80). Finally, most undergraduate students did not identify principles in the explanations (N=29, 67.4%). Of those who did, 6 students identified 1 principle, 3 students identified 2 principles, and 5 students identified all three principles for their groupings. This finding suggests that there might be differences in the breadth (i.e., grasp something about all principles) or depth (i.e., understand one principle quite well) of students' understanding of calculus principles.

Salience of the Surface Features

When asked if it was possible to sort the problems in other ways most undergraduate students (N=26, 66.7%) acknowledged that there were other ways. However, they explained that they chose to sort the problems based on what "came to them first". In this situation, the salience of surface features plays a critical role on how problems are sorted. Moreover, certain types of surface features might have a larger impact than others. For instance, more undergraduate students grouped all the graph problems together (70%) than problems with other surface features (volume=44%, price=37%). The two graduate students also grouped the graph problems together even though they represented other problems structurally.

Discussion

The results of the study suggest that our sample of college students had a naïve representation of calculus problems, and that the breadth and depth of their understanding may have varied. Further studies are needed to examine how this understanding impacts the development of problem representations and when the salience of surface features decreases. The fact that more advanced calculus learners are also susceptible to these features points to the potential value of investigating trajectories of expertise.

References

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