From Unconscious to Conscious Insights

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Abstract

Whether insights arise consciously or unconsciously, and whether they arise suddenly or gradually, has been the subject of much speculation but little empirical research. Problem solving on the inversion problem presented an unusual opportunity to circumvent the methodological obstacles that have limited progress on these issues. This paper presents both empirical research on how children generate a simple mathematical insight and a computer simulation of how they do so. Both suggest that at least some insights arise first in unconscious form and that gradual shifts in attention play a large role in the insight process.

Keywords: cognitive development; insight; mathematical thinking; problem solving; strategies; unconscious.

Introduction

More than 2000 years after Archimedes stepped into the bath, saw the water rise, and exclaimed “Eureka,” his experience remains the prototypic insight: a sudden, conscious change from not knowing to knowing a problem’s solution. However, this is not the only view of how insights arise. Other accounts of famous scientific insights suggest that insights emerge first at an unconscious level. A famous example is Kekule’s dream of intertwined snakes that led him to “see” the structure of the benzene ring (Gruber, 1981). Both of these portrayals depict the insight process as sudden, but other portrayals depict it as gradual. Wittgenstein’s (1969) likening of scientific discoveries to sunrises, in which the amount of light slowly increases until the new idea “dawns” exemplifies this approach.

The two dimensions on which these accounts differ – consciousness and abruptness– are also at the center of psychological research regarding insight. Some theorists have depicted insights as conscious (e.g., Gick & Lockhart, 1995); others have depicted them as arising unconsciously (e.g., Karmiloff-Smith, 1992). The issue is difficult to resolve, because unconscious insights, by their nature, cannot be verbalized; without verbalization, how can we tell whether an insight has occurred? Similarly, some theorists have depicted insights as arising suddenly (e.g., Perkins, 1995), whereas others have depicted them as arising gradually (e.g., Isaak & Just, 1995). Again, methodological difficulties have limited investigation of whether gradual changes underlie novel strategies.

The Inversion Task

The inversion task offered an unusual opportunity to investigate these elusive issues. Inversion is the principle that adding and subtracting the same number does not change the original value. Understanding of this principle can be examined by contrasting performance on problems of the form A+B-B with performance on problems of the form A+B-C. People who understand the principle should solve A+B-B problems much faster than A+B-C ones.

First through fourth graders use several strategies to solve A+B-B problems (Bisanz & LeFevre, 1990; Klein & Bisanz, 2000.) The *computation strategy*, involves adding the first two numbers and then subtracting the third. The *negation strategy* involves adding A+B, typically by counting on one’s fingers, but then simultaneously putting down all of the fingers and saying “A.” The *shortcut strategy* involves the insight that inversion problems can be solved by just saying, “A.” Use of the shortcut increases between preschool and fourth grade, but even preschoolers have a nascent understanding of the inversion principle (Bisanz & LeFevre, 1990; Stern, 1992) and use the shortcut under favorable circumstances.

These data indicated the broad outlines of the development of the shortcut strategy, but not how children discover the approach. This led Siegler and Stern (1998) to examine the discovery process in an eight-session microgenetic study. The inversion task was particularly well suited for directly studying the issues of abrupt/gradual and conscious/unconscious insights. Whether insights occurred gradually or abruptly could be examined through the type of trial-by-trial assessment of strategy use characteristic of microgenetic studies. These assessments, based on examination of videotapes of ongoing problem solving, solution times, and immediately retrospective explanations, indicated whether intermediate forms incorporated parts of the insight before the shortcut strategy emerged.

The inversion task also allowed examination of whether the discovery occurred consciously or unconsciously. Conscious use of the shortcut could be assessed through asking children immediately after they solved the problem how they had done so. Such self-reports have been found to yield valid and non-reactive data on strategy use with children as young as 5-years (Siegler & Jenkins, 1989). What made the inversion task special, however, was that it also yielded a measure of implicit use of the shortcut strategy. Such implicit use could be inferred from children generating fast solution times -- too fast to be generated through adding and subtracting -- yet reporting that they added and subtracted all the numbers. Obtaining both the verbal report and the solution time on each trial allowed assessment of whether children ever used the shortcut strategy unconsciously, and if so, whether they did so especially often just before their first conscious use of the shortcut. The second graders who participated in Siegler and Stern (1998) were classified as using a) the computation strategy on each trial on which they required 8 s or more to answer and on which their ongoing behavior and verbal statements indicated that they added and subtracted all three numbers; b) the negation strategy when their verbalizations and overt behavior indicated that they added the first two numbers but answered without explicitly subtracting the third; c) the shortcut strategy on each trial on which said they did not add or subtract, did not show overt computation, and answered within 4 s; and d) the unconscious shortcut on each trial on which they answered within 4 s and showed no sign of overt computation, but said they computed the answer.

Discovery of the Shortcut Strategy

Siegler and Stern (1998) examined discovery of the shortcut strategy by presenting German second graders with 3-term arithmetic problems one session per week over an eight-week period. Session 1 was a pretest, in which 10 inversion (A+B-B) and 10 standard (A+B-C) problems were presented. Children who did not use the shortcut in Session 1 (31 of the 39 children tested) were randomly assigned to either the blocked or the mixed problems condition. The two conditions differed in the problems presented in Sessions 2, 3, 4, and 6. Children in the *blocked problems condition* received 20 inversion problems in each of those sessions; children in the *mixed problems condition* received 10 inversion and 10 standard problems in each of them. In Sessions 1, 5, and 7, children in both groups received 10 inversion and 10 standard problems; the purpose of these sessions was to trace the effects of the experimental manipulation as the children proceeded through the study. In Session 8, all children were presented transfer problems that superficially resembled the inversion problems. Some transfer problems could be solved via the shortcut (A-B+B), others could not (e.g., A+B+B problems).

This design allowed us to test the *unconscious activation hypothesis*, the idea that increasing activation of a strategy leads first to unconscious use of it, and then, with further increases in activation, to conscious use. The hypothesis also implied that the blocked problems condition, in which children received 100% inversion problems in four sessions, would lead to 1) earlier use of both the unconscious and conscious versions of the shortcut, 2) a smaller number of trials between discovery of the unconscious and conscious shortcut strategies, 3) more frequent use of the shortcut on both inversion and transfer problems.

Results of Siegler and Stern (1998) supported each of these predictions, and thus supported the unconscious activation hypothesis that led to them. The tests of the hypotheses were based on comparisons of performance on the first 10 inversion problems encountered by children in each group (the only 10 problems encountered by children in the mixed problems condition.) Almost 90% of children discovered the shortcut strategy at an unconscious level before they discovered it at a conscious level. Children in the blocked problems condition discovered the shortcut strategy earlier, and used it more often, than did children in the mixed problems condition. A smaller number of trials separated first use of the unconscious and conscious shortcut in the blocked problems condition. Children in that condition also transferred the shortcut strategy more often, both to novel problems on which it was applicable and to novel problems on which it was inapplicable.

Strategy use immediately before and after the first use of the shortcut strategy provided particularly direct support for the unconscious activation hypothesis. The strategies used by children in the blocked problems condition just before and after discovery of the shortcut strategy are illustrated in Figure 1. In this Figure, the “0” on the X-axis indicates the trial on which each child first used the shortcut; use of the shortcut is, by definition, 100% on that trial. The –1 trial for a given child is the trial immediately before the trial of discovery for that child, the –2 trial for the child is the trial just before that, and so on.

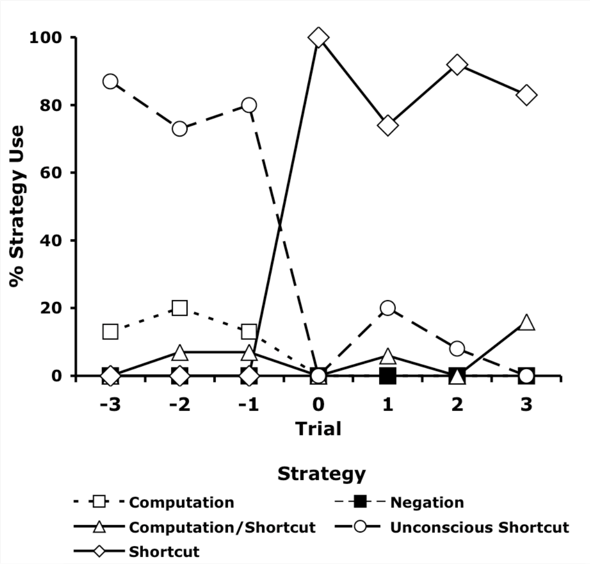

Figure 1: Children’s strategy use on trials immediately before and after first use of shortcut strategy in blocked problems condition (data from Siegler & Stern, 1998)

As shown in Figure 1, on the three trials just before the first use of the shortcut, children in the blocked problems condition used the unconscious shortcut on 80% of trials, far more than the 9% of trials on which they used the unconscious shortcut in the experiment as a whole. After the children first used the shortcut, they used it consistently on the remaining trials in the session (as indicated by the data for Trials 1, 2, and 3 in Figure 1). However, when children returned a week later for the next session, they regressed to strategies involving computation; the shortcut was used by fewer than 35% of children on each of the first four trials in the session immediately after the one in which the child discovered the shortcut. On the remaining trials in the session after the discovery, children rediscovered the shortcut; by the final trial of the session, more than 90% of children were again using it.

The unconscious shortcut appeared to reflect an abrupt change in thinking. On the three trials immediately before the first use of the unconscious shortcut, the mean solution times was 12 s; on the first use of the unconscious shortcut, the mean solution time was less than 3 s. Thus, it appears that the insight first arose at an unconscious level.

Constraints on Models of Strategy Discovery

These and other findings that emerged from Siegler and Stern (1998) can be summarized in terms of nine constraints that a satisfactory model of strategy discovery on this task would need to generate:

1) Five strategies were used: the computation, unconscious shortcut, shortcut, negation, and computation/shortcut strategies. The first four approaches have already been described. The fifth, the computation/shortcut strategy, occurred when children reported solving a problem via the shortcut, but the solution took more than 4 s. This relatively uncommon strategy usually emerged when children started to add the first two numbers but answered "A" before they finished doing so.
2) Over the eight sessions, children solved problems increasingly accurately and quickly, and shifted from usually using the computation or negation strategy to usually using the shortcut.
3) Almost all children in both experimental groups discovered both the unconscious shortcut and shortcut strategies.
4) Most children first used the computation strategy, then negation, then the unconscious shortcut, and then the shortcut.
5) Children in the blocked problems condition started using the shortcut earlier, subsequently used it more often on inversion problems, and transferred it more frequently both appropriately and inappropriately.
6) Even after discovering the shortcut, children continued to use other, less efficient strategies.
7) Use of the unconscious shortcut was most frequent just before the first use of the shortcut.
8) Children in the mixed problems condition used the negation and computation strategies more often than did children in the blocked problems condition.
9) When presented similar problems (in Sessions 1, 5, and 7), children in the two conditions generated similar distributions of strategies.

These were the data that Siegler and Araya's (2005) SCADS* model of strategy discovery attempted to generate.

SCADS* and Prior Models

SCADS* was an extension of Shrager and Siegler's (1998) SCADS model, which was an extension of Siegler and Shipley's (1995) ASCM model. The philosophy underlying all three was to keep the model as lean as possible, that is, to include only those mechanisms needed to generate the experimental data. The reason was not that we believed that the mechanisms included in the models were the only important ones, but rather to highlight the importance of those mechanisms that seemed essential for generating the data that each simulation was modeling. In this section, we briefly describe the earlier simulations and then describe the innovations within SCADS*.

ASCM (Adaptive Strategy Choice Simulation) embodied ideas about how basic associative processes could lead to improvements in speed, accuracy, and strategy choices on single-digit addition problems (Siegler & Shipley, 1995). Within ASCM, the strategies used to solve problems produced data about the speed and accuracy of the strategies on all problems, problems with specific features, and individual problems, which together determined each strategy's strength. The answers generated by the solution process also became associated with the problems on which the answers were produced. These data on strategies, problems, and answers were used to select strategies and answers on subsequent problems. The result was increasing use of retrieval, decreasing use of counting strategies, increasingly adaptive choices of when to use each strategy, and increasingly fast and accurate performance. Unlike children, however, ASCM did not discover new strategies.

SCADS surmounted this problem by adding a metacognitive system to ASCM's associative one. This metacognitive system included three components: the attentional spotlight, strategy-change heuristics, and goal sketch filters. The attentional spotlight focused cognitive resources on execution of strategies that were not fully mastered, which increased the likelihood of correct execution. As strategies became automated, the attentional spotlight was increasingly focused on the strategy change heuristics, to determine if more effective strategies could be generated. These heuristics operated on the trace of the operations that were used to solve the immediately previous problem. SCADS included two strategy-change heuristics: 1) If a redundant sequence of behaviors is present, delete one of the sets of operators that produced the redundant sequence, and 2) If a strategy shows greater success when its operations are executed in a particular order, create a version of the strategy that always operates in that order. These heuristics led to SCADS generating a number of strategies for consideration, some legitimate and some conceptually flawed (e.g., counting the first addend twice.)

A third metacognitive mechanism, the goal sketch filter, evaluated potential strategies to ensure that they did not violate the system's conceptual understanding of the requirements of a legal addition strategy. In particular, the goal sketch filters examined whether each proposed strategy represented both addends and whether the strategy included representations of both in the sum. Potential strategies that did not meet these criteria were eliminated without being tried. The data that motivated the goal sketch filters were Siegler and Jenkins' (1989) findings that preschoolers discovered a variety of legal addition strategies, but no illegal ones, over the course of 30 sessions of addition practice, and Siegler and Crowley's (1994) finding that 5-year-olds possess conceptual understanding akin to the goal sketch filters that allows them to evaluate both familiar and novel strategies. The three metacognitive mechanisms allowed SCADS to discover new, useful strategies and rule out flawed ones without trying them.

New Features of SCADS*

Because the strategies needed to solve inversion problems are more complex than those needed to solve single-digit addition problems, SCADS* required several mechanisms beyond those included in SCADS. All of these mechanisms were well-documented features of human cognition.

Controlled attention. Strategies within SCADS* include attention shifts, such as moving attention from A to B or C, and arithmetic operations. Such attention shifts are crucial to discovery of the shortcut strategy, because the shortcut requires a different sequence of attentional foci than does computation. Whereas computation ordinarily involves shifting attention from A to B to C, the shortcut requires focusing attention on, and comparing, B and C before any arithmetic operation involving A is performed. Only by examining B, C, and the arithmetic operator between them can the applicability of the shortcut be evaluated. Thus, controlled attention is essential on the inversion task.

Interruption of procedures. Siegler and Stern (1998) concluded that children used five strategies: computation, negation, computation/shortcut, unconscious shortcut, and shortcut. Formulating SCADS*, however, changed our perspective on the number of distinct strategies that were present. SCADS* generates the same five behavioral patterns observed by Siegler and Stern. However, at the level of mechanisms, there are only two strategies: computation and the shortcut. Increasingly early interruptions of computation by the shortcut produced the other three behavioral patterns. It was unclear how the other three behavioral patterns could emerge mechanistically other than as interruptions of computation.

Verbalization. Within SCADS*, a strategy's verbalization activation is the product of the number of times the strategy has been used and the strategy's mean execution time. The strategy can be verbally described only when its activation exceeds a threshold. This view helps explain the presence of both unconscious and conscious versions of the shortcut. Initially, the shortcut is unconscious, because both variables determining verbalization activation have low values. As number of uses of the strategy increase, it becomes possible to verbally describe the strategy. In contrast, the other strategies take longer to execute, thus allowing their verbalization activation to exceed the threshold from their first use onward.

Priming. SCADS* includes priming both from the previous trial and from other foci of attention. The priming is somewhat location specific. Thus, if on one trial, the shortcut interrupts computation late in its execution, when attention is on the rightmost number, attention to that rightmost location on the next trial provides greater activation to the shortcut on that trial. Over trials, priming of the shortcut gradually generalizes leftward, leading to earlier interruptions of computation (and thus to use of the computation/shortcut, and eventually the unconscious shortcut and shortcut strategies).

Forgetting. The substantial decrease in use of the shortcut strategy in the week between sessions is assumed to reflect forgetting. SCADS*'s forgetting mechanism operates in the same way as priming; indeed, its forgetting could be described as decay of priming. Over time, memories of each strategy's effectiveness blur, such that the most effective strategies lose activation and the least effective ones gain it. This leads to the fall-off in use of the shortcut from one session to the next. However, some of the change in activation within each session is retained, thus leading to the quicker re-learning of the shortcut in subsequent sessions.

Dynamic feature selection. SCADS* encodes both features relevant to inversion problems, such as whether any two numbers in the problem are equal, and features that are irrelevant, such as the color of the type. The simulation keeps track of two types of data. One is the proportion of trials on which a feature is present and the strategy generates atypically good performance relative to the total proportion of trials on which the feature is present. The other is the proportion of trials on which the feature is absent and the strategy generates atypically good performance relative to the proportion of trials on which the feature is absent. If the difference between the two proportions remains sufficiently great for several trials, presence of the feature begins to be used on all trials to calculate the strength of the strategy (and therefore its probability of use.) This aspect of the simulation is crucial for use of the shortcut strategy; the shortcut is highly useful when the feature "B=C" is present, but useless when that feature is absent.

Functioning of SCADS*

Overview. SCADS* begins with two types of knowledge. It knows how to add and subtract, and thus can execute the computation strategy. It also knows that N-N=0 and generates that answer very quickly. Both assumptions are well supported by empirical data on children of the age whose performance was modeled, second graders.

At the outset of the simulation's run, SCADS*'s attention is always focused on the leftmost number in the problem, the typical start point on horizontally written arithmetic problems. This attentional focus, together with the system lacking the cognitive resources to interrupt execution of the computation strategy once it starts, leads to consistent use of that approach at the outset.

Practice on the three-term problems leads to the system soon gaining sufficient cognitive resources to interrupt the computation strategy after A and B have been added. When the attentional focus moves to the second B, the simulation makes another strategy choice, which is often to solve the problem as "B–B=0, 0+A=A." The behavior on such trials would be classified as reflecting the negation strategy, though from the simulation's perspective, the shortcut generated the answer. The computation/shortcut reflects an even earlier interruption, one that occurs while adding A+B.

With further practice, the system starts attending to the rightmost two terms and checking whether they are equal before performing any computation. This at first gives rise to the unconscious shortcut, because the shortcut's verbalization activation does not exceed the threshold. Use of the shortcut leads to increases in its verbalization strength, until verbalization is possible. This occurs more rapidly in the blocked problems condition, due to verbalization activation building continuously in that condition. However, between-session forgetting leads to use of strategies other than the shortcut at the beginning of each new session.

Strategy Choice. SCADS* maintains the basic strategy choice process used in SCADS and ASCM. The probability of choosing any given strategy depends on the strength of that strategy relative to those of competing approaches. A strategy's strength is in large part determined by the accuracy and speed it has produced previously.

Strategy selection within SCADS* also reflects features that were not considered in the previous models. SCADS* encodes each problem in terms of the feature detectors that are active at the time. These always include the numbers and arithmetic operations, and also include on a probabilistic basis other features such as the magnitudes, colors, and physical sizes of the numbers; whether any numbers in the problem are identical; and whether all numbers are odd or even. Strengths of the strategies also vary with the focus of attention – for example, the computation strategy is strongest when attention focuses on the "A" term – and with priming from the previous problem.

Executing strategies requires cognitive resources. As in SCADS, free resources increase with experience executing a strategy; they also increase within a trial as strategy execution proceeds. The freed resources can then be used for other purposes, including checking whether another strategy is stronger at the current focus of attention. If it is, execution of the original strategy can be interrupted and execution of a different strategy begun. This second strategy choice often results in the shortcut interrupting execution of the computation strategy as attention shifts rightward.

Strategy Discovery. Like the children in Siegler and Stern (1998), SCADS* generates the negation, computation/shortcut, unconscious shortcut, and shortcut patterns. Discovery becomes possible when the system possesses sufficient cognitive resources to allow interruption of strategies that are being executed. When the model tries to discover a new strategy, it produces a sequence of visual attention and arithmetic operators and tries them from the point of the interruption. The effect is to change the order of attention to the numbers and the order in which operations are executed. SCADS* then applies the redundancy elimination mechanism that is part of SCADS' strategy change heuristics and also applies SCADS' goal sketch filters to assure that the proposed strategy uses each number in the problem once and only once.

The first approach discovered by the simulation (and by children) is negation. This strategy is generated when SCADS* has added the leftmost two numbers, attends to the rightmost number, interrupts the procedure, and chooses the shortcut. This behavioral pattern arises first because cognitive resources gradually increase during the left-to-right execution of the computation strategy, thus making interruption and choice of a new strategy most likely near the end of computation. As computation requires fewer resources, and as priming diffuses from the rightmost to the middle number, the computation/shortcut arises.

Shortly before or after the computation/shortcut is generated, the shortcut begins to be chosen at the beginning of trials and attention immediately shifted to checking if the middle and rightmost numbers are equal. This leads to creation of the shortcut, in which the system first checks whether B and C are equal, responds "A" if they are, and shifts attention leftward and uses the computation strategy if not. The first uses of the shortcut are unconscious, because its verbalization strength is weak. With increasing use of the shortcut, its verbalization strength becomes sufficiently great for the system to report using it.

SCADS*'s performance. Siegler and Araya (2005) reported the results of 50 runs of SCADS*, each varying randomly on several of the model's parameters. Thus, each run can be thought of as representing a different child with differing capabilities. The problems were identical to those presented in either the blocked or the mixed condition of Siegler and Stern (1998.) Also as in that experiment, the model's strategy use on each trial was classified on the basis of overt behavior, solution time, and, verbalization.

SCADS* generated all nine main characteristics of the behavior of children in Siegler and Stern (1998). It produced the five strategies observed among children. Over sessions, performance became faster and more accurate, and use of the shortcut became more common. The shortcut was discovered on 100% of runs, and the unconscious shortcut on 81%. Strategies were discovered in the same order, except for the shortcut being generated somewhat more often (30% of runs) without prior use of the unconscious shortcut. The blocked problems condition elicited earlier, more frequent, and more widely generalized use of the shortcut. Strategy use remained variable after discovery of the shortcut, especially at the beginning of new sessions. Use of the unconscious shortcut was especially common just before the first use of the shortcut – 60% of trials on the three problems just before generation of the shortcut, versus 7% in the experiment as a whole. Comparing Figures 1 and 2 illustrates parallels between the children's and model's behavior around the trial of discovery of the shortcut.

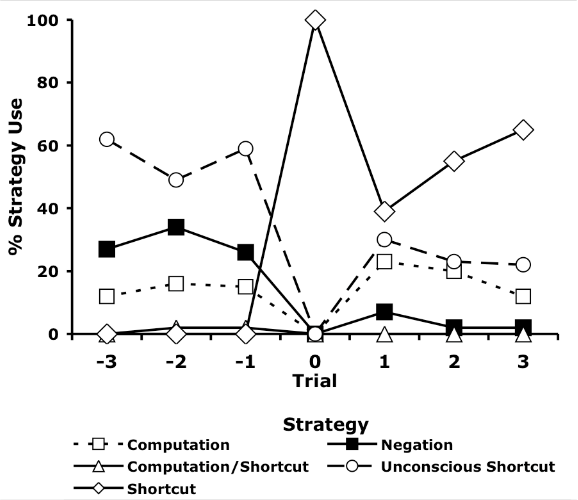

Figure 2. SCADS*'s strategy use on trials immediately before and after first use of shortcut strategy in blocked problems condition (data from Siegler & Araya, 2005).

These results demonstrate the sufficiency of SCADS* to produce many aspects of children's insightful problem solving. The data also provide a useful perspective on the questions that motivated the research. Insights clearly can arise at an unconscious level. Whether they appear sudden or gradual depends on the level of analysis. At a behavioral level, the shortcut arose suddenly, as indicated by the dramatic reduction in solution times on successive trials in both the simulation's and the children's behavior. At a mechanistic level, the shortcut was the culmination of slowly changing activations and attentional patterns. This phenomenon of qualitative behavioral changes arising through quantitative mechanistic changes may be a frequent feature of learning and development.

Acknowledgments

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