The Development of Analogical Reasoning in Children: A Computational Account

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Abstract

We have previously reported results showing that when children can identify the critical structural relations in a scene analogy problem, development of their ability to reason analogically interacts with both relational complexity and featural distraction (Richland, Morrison & Holyoak, 2004, in press). In this paper we present computer simulations in LISA (Hummel & Holyoak, 1997, 2003) demonstrating that both relational complexity and featural distraction effects can be parsimoniously accounted for by a simple change in inhibition in the model. This result is similar to data and simulations of analogy performance in patients with damage to prefrontal cortex (Morrison et al., 2004) and older adults (Viskontas et al., 2004), two other populations whose cognitive performance is associated with decreases in inhibitory control in working memory. These results lend support to the hypothesis that the development of inhibitory control in working memory is a critical factor in children's ability to perform relational reasoning.

Children's development of analogical reasoning allows them to notice correspondences and make inferences about relationally similar phenomena across contexts. This greatly enriches children's capacity for transfer of learning and schema abstraction, two essential aspects of children's learning and cognitive development (Chen, Sanchez & Campbell, 1997; Gentner, 1977; Goswami, 2001; Halford, 1993; Holyoak, Junn & Billman, 1984). While many have argued that analogy is important for children's cognitive development, there is considerable disagreement on the mechanisms of development of this important form of reasoning.

Developmental Change in Analogy

Historically, three primary hypotheses have been developed to explain age-related differences in analogical reasoning: changes in domain knowledge, a relational shift from object similarity to relational similarity, and increased working memory capacity for manipulating relations.

Increased Domain Knowledge

The relational primacy hypothesis as advanced by Goswami and colleagues argues that analogical reasoning is available as a capacity from early infancy, but that children's analogical performance increases with age due to the accretion of knowledge about relevant relations (Goswami, 1992, 2001; Goswami & Brown, 1989). Piaget conducted early developmental research that indicated children were unable to reason analogically prior to achieving formal operations, approximately at age 13 or 14 (Piaget, Montangero & Billeter, 1977). Piaget's tasks, however, frequently involved uncommon relations, such as "steering mechanism," which would likely have been unfamiliar to younger children. When Goswami and Brown (1989) replaced such high content knowledge relations with simpler causal relations, they found children as young as 3 years old could be successful on some analogical reasoning tasks when they demonstrated the relevant knowledge about the particular task relations. In spite of their success, these children still performed lower than children at higher ages. So, as noted by the authors, the knowledge-based account cannot fully account for age-related effects in young children's performance on analogical reasoning tasks. In particular, these authors pointed out that children seem to fail on analogies in systematic ways even when the children possess relational knowledge relevant to the task.

Relational Shift

Alternatively, Gentner and Rattermann (1991; Rattermann & Gentner, 1998) hypothesized that a domain-specific "relational shift" occurs. They suggest that as children build knowledge in a domain, they move from considering similarity based on perceptual features to considering similarity based on relations. Thus prior to the relational shift, children primarily attend to featural similarity between objects. Following the relational shift, children can and will reason on the basis of relational features,, making them successful on analogical reasoning tasks. Gentner and Rattermann have empirically demonstrated and replicated this effect. While these authors argue domain knowledge drives children's transition through the relational shift, the mechanisms underlying the observed reasoning patterns remain unclear.

Relational Complexity

Halford (1993) has proposed a third explanation for children's development of analogical reasoning based on working memory capacity. Halford and colleagues (Andrews & Halford, 2002; Halford, Andrews, Dalton, Boag & Zielinski, 2002) have argued that limits in children's working memory capacity affect their ability to process multiple relations simultaneously. Specifically, they argue that young children can processes only specific levels of relational complexity, defined as the number of sources of variation that are related and must be processed in parallel. For example, the simplest level of relational complexity, a binary relation, is defined as a relationship between two arguments, both of which are sources of variation. Thus "boy chases girl" specifies a single relation (chase) between two arguments (boy and girl). A reasoner would have to hold both arguments and the relevant relation in mind to reason on the basis of this relationship. The next level of relational complexity, a ternary relation, includes three arguments as sources of variation. A special case of a ternary relationship is formed by two integrated binary relations with three arguments, such as "mom chases a boy who chases a girl." Using this metric of relational complexity, Halford (1993) argued for a developmental continuum in children's working memory capacity, such that after age two children can process binary relations (a relationship between two objects), and after age five they can process ternary relations. Thus, children will be unable to systematically solve analogy problems with relational complexity above their current level of working memory capacity.

Multiple Factors in Analogical Development

We believe it is necessary to consider multiple factors to completely understand the dynamics of the development of analogical reasoning in children. In particular, we believe that while acquisition of relational knowledge doubtless is essential, changes in processing capacity with development are also important. Constructing an analogy requires a reasoner to represent source and target analogs and construct a mapping between elements of the source and target based upon correspondences between relations in each (Gentner, 1983; Gick & Holyoak, 1980). Empirical work has supported Halford's (1993) claim that these processes are dependent on working-memory functions (Morrison, 2005; Morrison & Holyoak, 2006; Morrison, Holyoak & Truong, 2001; Waltz et al., 2000). In children, these capacities are in turn dependent on developmental changes in prefrontal cortex (see Diamond, 2002). Using an analogy frequently involves mapping multiple relations, a process that has been shown to critically depend on areas of the prefrontal cortex associated with working memory (Christoff et al., 2001; Kroger et al., 2002; Prabhakaran et al., 1997; Waltz et al., 1999). Thus it follows that increases in capacity to cope with relational complexity (Halford, 1993) would be expected to lead to increased analogical ability.

As noted by Gentner and colleagues, a second factor of importance in the development of analogical reasoning is the challenge of reasoning on the basis of relational correspondences as opposed to perceptual/ object-based cues. As demonstrated by studies examining the relational shift, relational correspondences may compete with tendencies to respond on the basis of more superficial featural or semantic similarities between individual objects (Gentner & Toupin, 1986). Children's developmental increases in ability to successfully make relational decisions in spite of competition may be explained by improvments in inhibitory control in working memory. Inhibitory control is of particular importance in managing working memory when relational and more superficial responses conflict. Inhibitory control has not been previously discussed directly as a factor in the development of analogical reasoning, but this hypothesis is consistent with results from other cognitive tasks that explore developmental changes in children's ability to use inhibitory control (e.g., Diamond, Kirkham & Amso, 2002). Accordingly, acquisition of fully developed analogical reasoning seems likely to require both the working memory capacity to integrate multiple relations, and the ability to inhibit tendencies to respond on the basis of competing superficial similarities (see Morrison, 2005, for a review).

A Computational Account of Analogy

Learning and Inference with Schemas and Analogies (LISA; Hummel & Holyoak, 1997, 2003) is a neurally-plausible symbolic-connectionist model of analogical reasoning which uses synchrony of firing to bind distributed representations of relational roles to distributed representations of their fillers. The process of "thinking about" a proposition entails keeping separate role-filler bindings firing *out* of synchrony with one another. According to LISA, working memory is therefore necessarily capacity-limited: It is only possible to keep a finite number of role-filler bindings simultaneously active and out of synchrony with one another (see Hummel & Holyoak, 2003, Appendix A). The synchronized (and de-synchronized) patterns of activation representing propositions in LISA serve as the basis for memory retrieval, analogical mapping, analogical inference and schema induction.

LISA represents propositions using a hierarchy of distributed and localist units (see Figure 1 for a schematic representation of LISA's architecture as applied to the Scene Analogy Problems presented in this study). At the bottom of the hierarchy, *semantic* units (small circles in Figure 1) represent objects and relational roles in a distributed fashion. For example, consider the proposition *chase* (cat, mouse). Each role of the *chase* relation would be represented by units coding for its semantic content (e.g.,

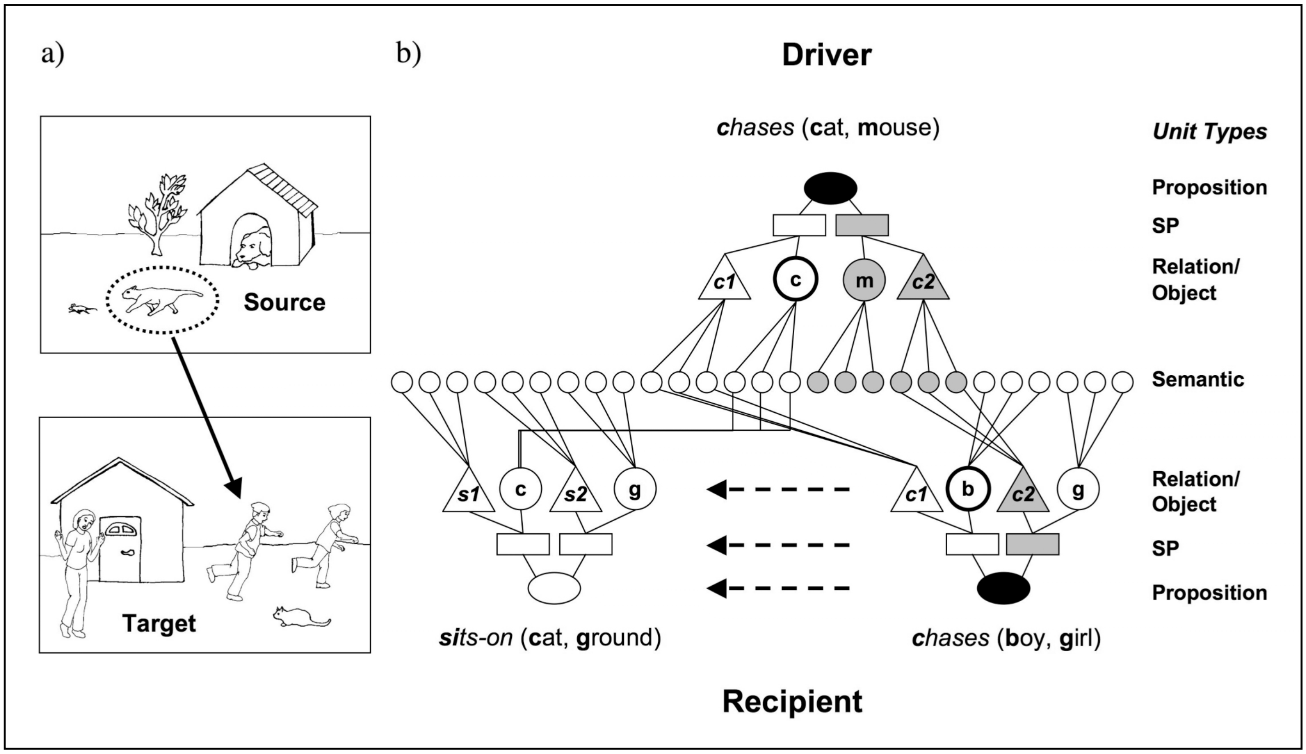

Figure 1: a) Example of 1-Relation/Distractor Scene Analogy Problem (Richland, Morrison & Holyoak, 2004, in press), b) LISA architecture for 1-Relation/Distractor Scene Analogy Problem. In order for a reasoner to select the boy in the target as the correct analogical mapping to the cat in the source, units in the recipient representing the *chases* (boy, girl) must inhibit corresponding units in the propositional structure containing the featurally similar "sitting cat" distractor.

among others, *aggressor* for the first role, *victim* for the second, and *pursuit* for both). Similarly, the arguments "cat" and "mouse" would be represented by units specifying their meaning (e.g., cat: *animal, pet, soft*). *Predicate* and *object* units (triangles and large circles, respectively, in Figure 1) represent relational roles and their fillers in a localist fashion, and have bi-directional excitatory connections to the corresponding semantic units. Sub-proposition (SP) units (rectangles in Figure 1) bind roles to their arguments, and have bidirectional connections to the corresponding predicate and object units. In the case of *chase* (cat, mouse), one SP would bind "cat" to the first role of *chase*, and another would bind "mouse" to the second. At the top of the hierarchy, proposition (P) units bind role-filler bindings into complete propositions via excitatory connections to the corresponding SPs. A complete analog (i.e., situation, story or event) is represented by the collection of semantic, predicate, object, SP and P units that collectively code the propositions in that analog. Separate analogs do not share object, predicate, SP or P units. However, all analogs are connected to the same set of semantic units. The semantic units thus permit the units in one analog to communicate with the units in others.

For the purposes of memory retrieval and analogical mapping (Hummel & Holyoak, 1997) as well as analogical inference and schema induction (Hummel & Holyoak, 2003), analogs are divided into two mutually exclusive sets: a *driver* and one or more *recipients*. The sequence of events is controlled by the driver: One (or at most three) at a time, propositions in the driver become active (i.e., enter working memory). When a proposition enters working memory, the binding of its roles to their arguments is represented by synchrony of firing: All the units under a given SP fire in synchrony with one another, and separate SPs fire *out* of synchrony with one another. The result on the semantic units is a set of mutually desynchronized patterns of activation: one pattern for each active SP (i.e., role binding) in the driver. In the case of *chase* (cat, mouse), the semantic features of "cat" (e.g., *animal, pet, soft*) would fire in synchrony with the features of the first role of *chase (*ie., *chase1)*, while "mouse" fires in synchrony with the second. In order to represent the proposition *chase* (mouse, cat), LISA would activate exactly the same semantic units, but their synchrony relations would be reversed, with "mouse" firing in synchrony with the *chase1*, and "cat" firing with the second. The resulting patterns of activation on the semantic units drive the activation of propositions in the various recipient analogs, and serve as the basis for analogical mapping, inference, schema induction, and the other functions LISA performs (Hummel & Holyoak, 1997, 2003).

The final component of the LISA architecture is a set of *mapping connections* between units of the same type (e.g., object, predicate, etc.) in separate analogs. These connections grow whenever corresponding units in the driver and recipient are active simultaneously. They permit

LISA to learn the correspondences (i.e., mappings) between corresponding structures in separate analogs. They also permit correspondences learned early in mapping to influence the correspondences learned later.

The Role of Inhibition

In LISA, inhibition is critical to the selection of information for processing in working memory. Specifically, inhibition determines LISA's working memory capacity (see Hummel & Holyoak, 2003, Appendix A), controls its ability to select items for placement into working memory and also regulates its ability to control the spreading of activation in the recipient.

Of particular importance to the present simulations, inhibition plays a role in the selection of items to enter working memory because selection is a competitive process: Propositions in the driver compete to be entered into working memory on the basis of several factors, including their pragmatic centrality or importance, support from other propositions that have recently fired, and the recency with which they themselves have fired. Reduced inhibition results in reduced competition and more random selection of propositions to fire. The selection of which propositions are chosen to fire, and in what order, can have substantial effects on LISA's ability to find a structurally consistent mapping between analogs. It follows that reduced inhibition, resulting in more random selection of propositions into working memory, can likewise affect LISA's ability to discover a structurally-consistent mapping.

The role of inhibition in the activity of a recipient analog is directly analogous to its role in the activity in the driver. Inhibition causes units in the recipient to compete to respond to the semantic patterns generated by activity in the driver. If LISA's capacity to inhibit units in the recipient is compromised, then the result is a loss of competition, with many units in the recipient responding to any given pattern generated by the driver. The resulting chaos hampers (in the limit, completely destroys) LISA's ability to discover which units in the recipient map to which in the driver.

Scene Analogy Problems

Task Description

Richland, Morrison and Holyoak (2002, in press) developed Scene Analogy Problems to investigate relational complexity and featural distraction within a single analogical reasoning task based on a paradigm originated by Markman and Gentner (1993). The relations and the objects used to represent them were familiar to preschool age children.

Figure 1a depicts an example of one of the four counterbalanced versions that were created for each of the 20 picture sets in the Scene Analogy Problems. Each set of problems factorially varied (1) the number of instances of the relevant relation that needed to be mapped (1-Relation or 2-Relation), and (2) the presence of an object in the target scene that was either featurally similar (Distractor) or dissimilar (No Distractor) to the object to be mapped in the source scene. 2-Relation problems were created by having one object that was not involved in the principal relation (dog in Figure 1a) in the 1-Relation problems participate in the principle relation for the 2-Relation version (*chase* (dog, cat). Distractor and No-Distractor versions were created by having an extra object in the same picture that was either similar (sitting cat in Figure 1a) or dissimilar (sandbox) to the item to be mapped in the source picture (running cat).

Summary of Experimental Results

In a series of experiments, Richland, Morrison and Holyoak (2002, in press) found reliable effects of both relational complexity and featural distraction on children's analogical reasoning ability (see Figure 2, solid lines). Specifically, 3-4 year olds showed strong effects of both distraction and relational complexity that interacted to reveal the highest accuracy in the 1-Relation/No Distractor condition and the lowest accuracy in the 2-Relation/Distractor condition. This pattern was similar for the 6-7 year olds, with main effects of both relational complexity and distraction. In contrast, the 13-14 year olds showed a main effect of relational complexity but no effect of distraction. In a second experiment Richland, Morrison & Holyoak (in press), demonstrated these effects in young children were not due to problems in identifying the relevant relations.

Simulations

Methods

LISA simulations were performed for the Scene Analogy Problems. Our intent was to demonstrate that a simple change in inhibition levels in LISA can account for age-related performance changes in analogical reasoning as characterized by relational complexity and distraction in the stimuli.

To model the Scene Analogy Problems we constructed LISA representations of the four problem types (Figure 1b depicts a LISA representation of the 1-Relation/Distractor problem). For 2-Relation problems both relations were represented in LISA's WM together (Hummel & Holyoak, 1997). In LISA units of the same type in the driver and recipient inhibit one another (ie., SPs inhibit other SPs, Ps inhibit other Ps, etc). To simulate each age group we changed the inhibition level between corresponding units in the recipient. Younger age groups tended to have lower inhibition levels. Recipient inhibition levels for each age group are shown in Table 1.

Table 1: Inhibition Levels in LISA.

Age Group Simulated	Mean Recipient Inhibition Level*
3-4 year olds	0.3
6-7 year olds	0.6
13-14 year olds	0.9

*Note: Value sampled from a normal distribution with SD = .1

Each simulation run consisted of firing three phase sets in LISA's working memory, "randomly" assigned by LISA. On each simulation an inhibition level for units in the recipient was sampled for a normal distribution with the means listed in Table 1 and a SD of .1. The inhibition between corresponding units in the recipient was set to the inhibition level. We ran 40 simulations of each problem type for each age group. When LISA failed to determine a stable mapping after firing three phase sets, an answer was selected based on Equation 1, where mapWeight was unit *i's* maximum mapping weight, and max(mapWeight) was the highest mapping weight into any recipient Predicate or Object unit.

$$P_{select} = \begin{cases} \frac{1+\max(w_{ij})}{n} & w_{ij} > \max(w_{kj}) \\ \frac{1-\max(mapWeight)}{n} & otherwise \end{cases} \qquad \text{(eq. 1)}$$

Results

The simulation results along with the experimental results from Richland, Morrison & Holyoak (2002, in press) are presented in Figure 2. LISA's performance mirrored experimental results for each age group across conditions. Specifically, 1) LISA showed a main effect of age, 2) for 3-4 year olds LISA showed an effect for both relational complexity and distraction, 2) for 6-7 year olds LISA showed an effect for both relational complexity and distraction, but smaller than that for 3-4 year olds, and finally 4) for 13-14 showed a mild effect for relational complexity, but no effect for distraction. Lastly, as in the experimental results, when LISA did not select the correct analogical mapping in the distractor conditions, the model preferentially choose the featurally similar distractor object.

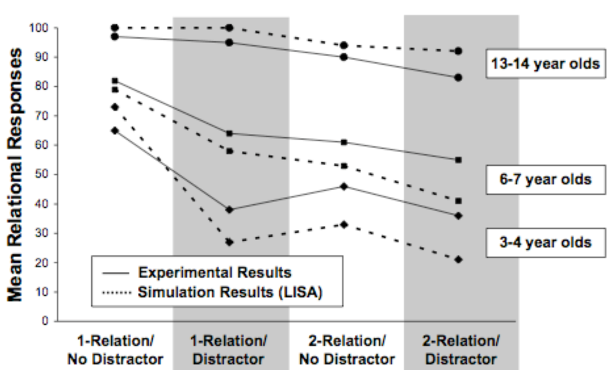

Figure 2: Experimental (Richland, Morrison & Holyoak, 2002, in press, Experiment 1) and Simulation results.

General Discussion

In this paper we presented simulations in LISA that support the role of inhibition in explaining age-related changes in analogical reasoning. We demonstrated that simple changes in recipient inhibition levels in LISA (i.e., inhibition between elements of competing relational representations in working memory) could account for both relational complexity and featural distraction effects in children's analogical reasoning performance from age 3 to 14 (Richland, Morrison & Holyoak, 2002, in press). This account is consistent with previous simulations of results from frontal patients (Morrison et al., 2004) and older adults (Viskontas et al., 2004), whose analogical reasoning performance also suffered under increases in relational complexity and featural or relational distraction.

It is our contention, that both long-term relational knowledge and processing capacity determine an individual's reasoning performance. We suggest a useful way to conceptualize the development of reasoning in children is an equilibrium between relational knowledge and processing capacity. As children age, their knowledge about relations advances while their working memory capacity as modulated by inhibitory control also advances. At a given time during development, the child is able to perform an analogical task based on both their level of relational knowledge and their working memory resources. Specifically, the equilibrium operates such that greater relational knowledge imposes fewer processing demands, while less knowledge imposes higher demands. Thus, as relational knowledge increases in a domain, the demands on a working memory decline, allowing for more complex reasoning. This pattern in cognitive development builds on an understanding of working memory effects in expertise (e.g., Chase & Simon, 1973), once again, a situation where advanced relational knowledge can decrease processing demands and thereby allow experts to accomplish cognitive tasks.

We believe that to truly understand the development of relational reasoning in children, future experimental and computational studies must take into account both advances in relational knowledge and changes in processing capability, and importantly, studying how these two aspects of development interact.

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