Even Early Representations of Numerical Magnitude are Spatially Organized: Evidence for a Directional Magnitude Bias in Pre-Reading Preschoolers

John E. Opfer (opfer.7@osu.edu)
Department of Psychology, 1835 Neil Avenue
Columbus, OH 43210 USA

Clarissa A. Thompson (thompson.1345@osu.edu)
Department of Psychology, 1835 Neil Avenue
Columbus, OH 43210 USA

Abstract

Previous work has suggested that an important tool of adult numeric competence is a "mental number line" that codes increasing numeric magnitudes spatially and with a directional bias. Partly because the direction of this bias (left-to-right versus right-to-left) is culture-specific, it has been assumed that a directionally-biased mental number line is a late development. Our results, in contrast, suggest that nearly half of all preschoolers possess a directional magnitude bias and that those who possess this bias demonstrate competence in representing large numbers, a competence that is otherwise lacking in same-aged peers. Further, this directional bias does not appear to be related to pre-reading abilities, but instead seems to build on already existing preferences in counting direction.

Keywords: conceptual development, representation, numeric cognition, spatial cognition

Development of the Mental Number Line

Numeric competence is a fundamental prerequisite of many social, economic, and educational activities, and the importance of its acquisition is reflected in the considerable resources devoted to teaching children mathematics, to understanding the mental disorders that inhibit mathematical understanding, and to identifying the cognitive bases of mathematical understanding. A number of studies have indicated that an important tool of numeric competence is a "mental number line" (Dehaene, et al., 2004). Mental number lines are representations of numeric quantity in which numerals (e.g., integers) are associated with physical magnitudes and are ordered serially along the same dimension as the increasing physical magnitude. By mentally mapping a directionally-ordered list of integers onto a series of increasing linear magnitudes, people appear to gain proficiency in solving a number of quantitative tasks, including: (1) determining which of two integers denote greater magnitude (Göbel, Walsh, & Rushworth, 2001), (2) organizing semantic knowledge about the parity of numerals (Dehaene et al., 1993; Fischer, 2001), and (3) translating between numeric symbols and spatial magnitudes (Siegler & Opfer, 2003).

Adults' Mental Number Line

Three types of evidence suggest that adults represent knowledge about numeric values using mental number lines. The first type of evidence comes from adults' performance when comparing numerical magnitude (e.g., *Which is more, 5 or 6?*), where minimizing the distance in value between two symbols (*5 or 6* vs. *7 or 4*) or increasing the value of the two symbols (*8 or 9* vs. *1 or 2*) reduces solution times logarithmically (Göbel, Walsh, & Rushworth, 2001; Moyer & Landauer, 1967). An important parallel exists in psychophysics, where reducing the distance in length between two lines or increasing their individual lengths also reduces comparison times logarithmically (Johnson, 1939). These parallel findings of logarithmic decrement in comparison times suggest that when adults compare numerals, they automatically simulate the numeric quantities as linear magnitudes, which increase logarithmically with numeric value (Banks & Hill, 1974; Dehaene, 1997). In support of this interpretation, focal transcranial magnetic stimulation of brain regions involved in visuospatial search also disrupt performance in comparing numerosity while sparing performance on other tasks (Göbel, Walsh, & Rushworth, 2001).

A second type of evidence for adults' use of mental number lines is provided by their performance on a parity task, where subjects are asked whether a given number is odd or even (Dehaene et al., 1993; Fischer, 2001, 2003). Although knowledge of numeric magnitude is unnecessary to solve the task, if knowledge of parity is stored in semantic memory with the mental number line, Dehaene and colleagues (1993) reasoned, representation of numerals on a mental number line might intrude on task performance. Specifically, they found that when subjects were asked whether large numbers (e.g., 8 or 9) were even, they responded most quickly with their right hand, whereas when they were asked about small numbers (e.g., 1 or 2), they responded most quickly with their left hand. Interestingly, the direction of this effect—the "Spatial-Numerical Association of Response Codes (SNARC) effect"—was not affected by handedness or hemispheric dominance, but by the direction of writing in the culture of the subjects. When

Iranian subjects—who write right-to-left—were tested, the direction of the association was reversed. These results support the idea that the semantic representation of number is associated with a mental number line that is oriented left-to-right in societies that write left-to-right.

A third type of evidence of adults' use of mental number lines is provided by their performance on number-line estimation tasks, where adults were given a numeral and asked to estimate its value by marking its position on an unmarked line that began with a 0 and ended with a 100 or 1000 (Siegler & Opfer, 2003). Hypothesizing that adults' understanding of the decimal system would lead them to represent numeric value in terms of linearly increasing magnitudes, Siegler and Opfer tested whether adults' estimates increased linearly, logarithmically, or exponentially with increasing numeric value. On both the 0-100 and 0-1000 number-line estimation tasks, there was a linear relation between estimates and numeric value (R^2s = 1), with 100% of adults providing more linear than logarithmic or exponential series of estimates.

Children's Mental Number Line

Some of the types of evidence indicating adult use of mental number lines—size and distance effects on numeric magnitude comparison tasks, the SNARC effect on parity judgment tasks, and the linearity of estimates on a number-line estimation task—have also been seen in school-aged children (Berch et al., 1999; Opfer & Siegler, 2005; Sekuler & Mierkiewicz, 1977; Siegler & Opfer, 2003; Siegler & Robinson, 1982); however, it is unclear whether still younger children also possess a mental number line or how it might develop.

In this paper, we addressed two questions about the mental number line. First, what is its origin? That is, when do children—who have not yet learned about numeric parity—first begin to show evidence of a directional bias in representing increasing numerical magnitudes? Second, what numeric competences does it initially serve? To address these questions, we first examined whether children showed adult-like directional biases in their adding and subtracting of elements to a set (Study One). We next examined whether individual differences in these directional biases predicted competence in translating the symbolic code of numbers into numerical quantities (Study Two).

Study 1: When Are Representations of Increasing Magnitude Directionally Biased?

Much of the behavioral evidence in favor of a mental number line comes from the SNARC effect on the parity task (Dehaene et al., 1993). Given the strong influence of participants' writing system (Dehaene et al., 1993) and the late emergence of the effect (typically around 3rd grade; Berch et al., 1999), it seems reasonable to suspect that children associate large numbers with the right and small numbers with the left rather late in development, well after learning to read (Dehaene et al., 1993; Berch et al., 1999; Fischer, 2001). Another possibility, however, is that the parity judgment task typically used to elicit the SNARC effect presents an extraneous task demand on children who do not yet know the odd-even distinction very well (Miller & Gelman, 1983), leading researchers to underestimate how early children possess directionally-biased representations of increasing magnitude. Furthermore, whether the direction of the mental number line runs *rightward or leftward* may be influenced by the writing system of the society in which children grow up, but the tendency to develop *some directional bias* could emerge well before children show any sensitivity to this aspect of their writing system.

To test these hypotheses, we presented children with three numerical tasks: a counting task, in which children were asked to serially tag the items in a linear array; an adding task, in which children were asked to add an object to a linear array; and a subtracting task, in which children were asked to subtract an object from a linear array.

If participants represent increasing magnitudes as following a left-to-right or right-to-left orientation, a consistent pattern of behavior should be evident across all three tasks. For example, in the counting task, participants should begin counting from the leftmost end and count sequentially to the rightmost end; in the adding task, participants should add objects from left to right; in the subtracting task, participants should remove objects from right to left.

Method

Participants. Participants included 58 English-speaking, American preschoolers drawn from 3 age groups: 2.5-3.5-year-olds (N = 12; M = 3.21, SD = .32; 6 males, 6 females), 3.5-4.5-year-olds (N = 25, M = 3.98, SD = .29; 10 males; 15 females), and 4.5-5.5-year-olds (N = 21, M = 5.18, SD = .47; 13 males, 8 females). Additionally, 22 adults (M = 19.96, SD = 2.56; 10 males, 12 females) participated as a comparison group.

Stimuli and Procedure. Participants were presented with three tasks—counting, adding, and subtracting. In the counting task, participants were shown a linear array of 9 poker chips, centered on the participants' midline, and were asked to count the number of poker chips in the set aloud and to touch each object that they counted. In the adding task, participants were presented with a linear array of 3 objects, also centered on the participants' midline, and asked to add 1 poker chip to the set "to make it 4". In the subtracting task, participants were presented with a linear array of 4 objects, similarly centered, and asked to take 1 object away from the set "to make it 3 again". Performance was videotaped for coding.

Results and Discussion

We first analyzed directionality of counting. On our counting task, 98% of all children and 100% of adults counted from left to right. This finding is consistent with previous findings that most 3- to 5-year-olds have a strong preference for a left-to-right direction in counting, though most realize that a set can be counted in any order (Briars & Siegler, 1984) Interestingly, in the same research, the proportion of children who erroneously stated that starting from an end is an *essential* element of counting increased from 10% (of 3-year-olds) to 50% (of 4- and 5-year-olds) (Briars & Siegler, 1984), an intriguing trend that initially suggested to us the beginnings of a mental number line.

To examine the issue more directly, we next examined how children added/subtracted elements to a set (e.g., by placing the new element above the set, below the set, to/from the right of the set, or to/from the left of the set). We were particularly interested in finding participants who both added from left to right and subtracted from right to left ("MNL Group" henceforth). As indicated in Figure 1, the number of participants who fell in this group increased substantially with age. The average age of all participants in the MNL group (M = 11.9) was greater than that in the No MNL group (M = 6.36), $t(50) = 3.36$, $p < .01$, even with adults excluded (MNL, M = 4.64; No MNL, M = 4.1; $t(30) = 2.27$, $p < .05$). This finding is remarkable in that the spatial-numeric association appeared much earlier than in other studies (e.g., Berch et al., 1999) with the majority of change appearing long before children had any experience reading, which had been the presumed source of this association.

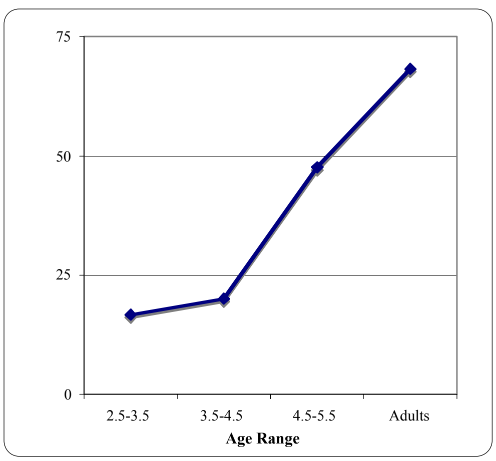

Figure 1. Proportion of subjects who created a set of four elements by adding to the right and who created a set of three elements by subtracting from the right

Study 2: How Are Spatial-Numeric Associations Related to Numeric Competence?

If directional biases in adding and subtracting were a feature of mental number lines, we would expect children with these biases to have a better understanding of the relation between numeric symbols and linearly-increasing physical quantities than children who do not possess these biases. To test this hypothesis, we presented children with four tasks in which they had to translate between numeric symbols and physical quantities. The main prediction was that children in the MNL group would be more likely to provide a correct physical quantity for a verbal numeral than children in the No MNL group. This difference in performance was expected to increase with increasing numeric value (e.g., to be greatest for numerosities greater than 4, where children could not represent quantities using subitization). For lesser numerosities, all children were expected to perform at ceiling.

Method

Participants. Participants were the same preschoolers who participated in Study One.

Stimuli and Procedure. For the object-set matching tasks, participants were shown a linear array of white poker chips arranged on a strip of black poster board (all numerosities 1-9). Children were provided with a pile of twenty-five white poker chips to use in the object-set matching tasks.

Procedure. All groups were given four tasks: two integer matching tasks--Numerals to Numerals (NN) and Numerals to Chips (NC), and two object-set matching tasks-- Chips to Numerals (CN), and Chips to Chips (CC). In each case they were asked to match one kind of numeric magnitude with either the same kind of numeric magnitude or another kind. For each task, children were asked to match to all numerosities 1 - 9. The four tasks were randomized as were the presentation of numerosities within each task. All experimental sessions were videotaped for later coding.

On the integer matching tasks, children were asked, "How many is four?" and the child was to give the experimenter four poker chips from their pile of twenty-five poker chips (NC) or to say 'four' (NN). On the object-set matching tasks, children were asked, "How many is this (experimenter shows the child four poker chips)?" and the child was asked to say 'four' (CN) or to give the experimenter four poker chips (CC) from their pile of twenty-five poker chips. This matching paradigm has been used successfully in prior studies to explore children's representations of integers (the counting and "give-a-number tasks" used by Wynn, 1990) and object sets (the "matching task" used by Huttenlocher, Jordan, & Levine, 1994).

Results and Discussion

We first examined performance on the NN and CC tasks to ensure that the MNL and No MNL groups were equally capable of attending to and encoding the numerals and set sizes that we presented in the NC and CN tasks. To do so, we regressed children's answers against the actual number provided. As expected, both groups did very well. On the NN task, the R^2 of the linear function was nearly 1.00 for numerosities 1-4 and 5-9 (MNL: 1-4, $R^2 = .998$, 5-9, $R^2 = .971$; No MNL: 1-4, $R^2 = .999$, 5-9, $R^2 = .941$). On the CC task, the R^2 of the linear function was also nearly 1.00 for numerosities 1-4 and 5-9 (MNL: 1-4, $R^2 = .998$, 5-9, $R^2 = .971$; No MNL: 1-4, $R^2 = .999$, 5-9, $R^2 = .941$).

We next examined performance on the NC and CN tasks, which require children to map a symbolic to a non-symbolic numerosity. On the CN task, where children could simply recite the integer list while tagging each chip and provide the last numeral as the value of the set, both the MNL and No MNL groups did equally well. The R^2 of the linear function was again nearly 1.00 for numerosities 1-4 and 5-9 (MNL: 1-4, R^2 = 1, 5-9, R^2 = .991; No MNL: 1-4, R^2 = .995, 5-9, R^2 = .987). This performance suggests either that children in both groups understood the numerals they provided to denote the cardinal value of the set or that the children simply used a "last word rule" (Fuson, 1988), but did not actually represent the cardinal value of the numerals.

To test this, we examined performance on the NC task, where children were given a numeral and asked to provide that number of objects. If children used numerals to represent cardinal value (as implied by a mental number line), performance on the NC task should be as good as performance on the CN task for the MNL group but not the No MNL group. As hypothesized the MNL and No MNL groups differed dramatically (see Figures 2 and 3). For the MNL group, the R^2 of the linear function was again nearly 1.00 for numerosities 1-4 and 5-9 (1-4, R^2 = .965, 5-9, R^2 = .948). In contrast, for the No MNL group, the R^2 value for numerosities 1-4 was 1.00, whereas for larger numerosities (5-9), R^2 = .419.

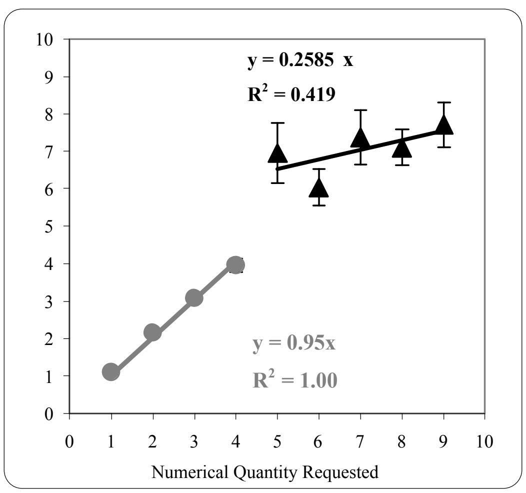

Figure 2. No MNL group performance on NC task.

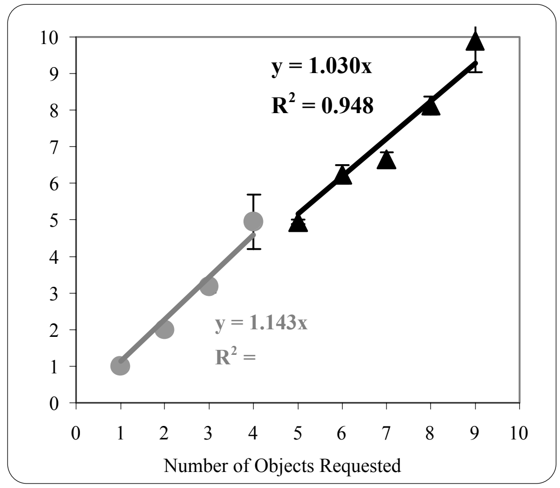

Figure 3. MNL group performance on NC task.

One possible (albeit uninteresting) reason for the difference between the two groups' performance would be that some number of outliers adversely affected the R^2 values of the No MNL group. To ensure that this was not the case, we next fit linear regression models to each individual subject and performed an ANOVA to compare the mean R^2 values as a function of numeric range given (1-4 vs. 5-9) and MNL status (MNL vs. No MNL). As expected, there was a strong interaction between numeric range and MNL status, $F(1, 56) = 9.62$, $p < .005$ (see Figure 4).

The finding that small numerals are better represented than large numerals is consistent with previous claims that children possess at least two different systems for representing numerical quantity—an "object file" system that provides young children with a direct, linear mapping between small numerals (1-4) and small set sizes, and

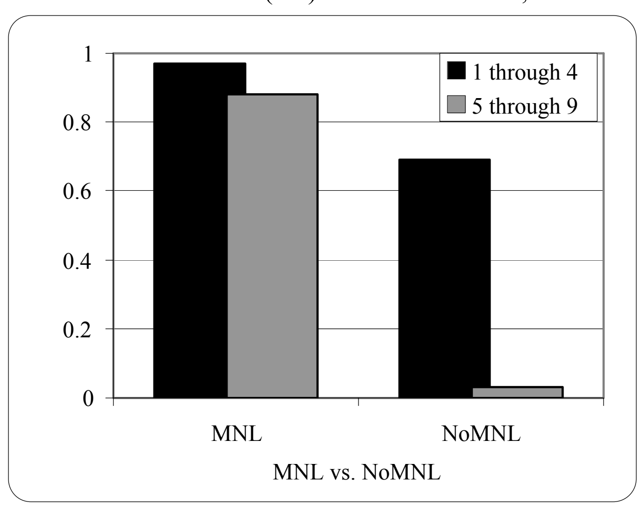

Figure 4. Mean R^2 values for MNL and No MNL groups' performance for numerosities 1-4 versus 5-9.

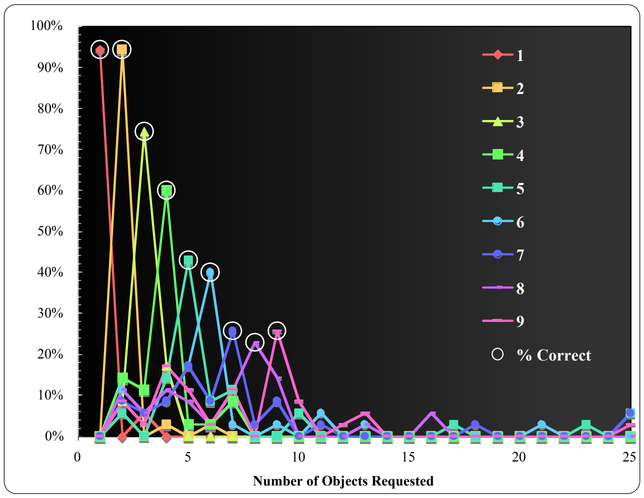

Figure 5. Probability function for No MNL group.

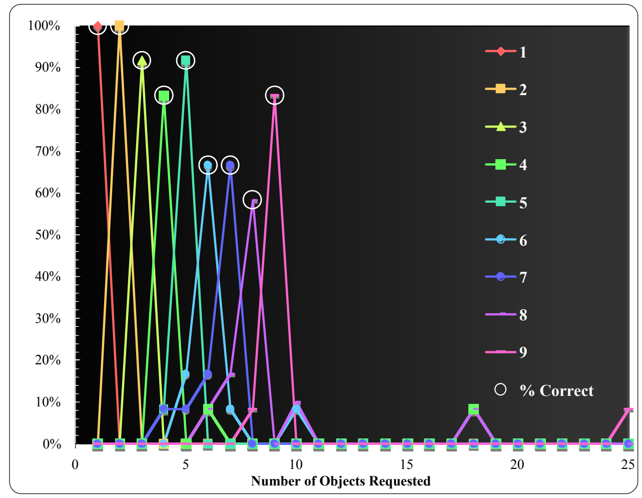

Figure 6. Probability function for MNL group

another system that provides older children with an analog mapping between larger numerals (5-9) and larger set sizes (Feigenson, Dehaene, & Spelke, 2004).

Subsequent analyses, however, tend to militate against the hypothesis that children initially use two different systems for representing the meaning of small versus large numerals. Rather than observing a break in the response patterns for 1-4 versus 5-9, we found that children's accuracy was better and more parsimoniously described as a single function between numeric value and accuracy. For children the in the No MNL group, for example, accuracy decreased exponentially with numeric value (R^2 = .95), whereas the fit of the two linear functions was poorer for both small (R^2 = .89) and large numbers (R^2 = .77) (see Figure 5).

This overall pattern of errors is consistent with previous reports (Gallistel & Gleman, 2000; Dehaene, 2003) that children's numeric representations become exponentially less discriminable with increasing numeric value, suggesting either (1) that the quantities represented by children increase linearly with numeric value but become more variable (Gallistel & Gelman, 2000) or (2) that the quantities represented increase logarithmically with constant variability (Dehaene, 2003). By inspection, our results seem to favor the Gallistel and Gelman (2000) model: the larger the number of chips requested, the more variable the responses (Figure 4). Two additional analyses, however, favor the logarithmic scaling account. First, the variability in responses did not assume a Gaussian distribution around a linearly increasing median. Rather, around each correct (modal) response, the distribution of estimates was asymmetric, with the ratio of under- to over-estimates increasing with numeric value, $r(9) = .88$, $F = 25.19$, $p < .005$, as one would expect if subjective magnitude increases logarithmically with objective magnitude and the variability around each subjective magnitude is fixed. Second, the actual median number of chips provided did not increase linearly. Rather, the number of chips provided increased logarithmically with the number of chips requested (log R^2 = .91), and the logarithmic function accounted for more variance than even the two linear functions combined (lin R^2s = 1.0 for 1 - 4 and .42 for 5 - 9).

Intriguingly, the same analyses performed for the MNL group indicated a representation in which magnitude increased linearly with increasing numeric value, with the distribution of errors assuming a Gaussian pattern (see Figure 6). That is, the ratio of under- to over-estimates did not covary with numeric value, $r(7) = .50$, *ns*, and (more importantly) the number of chips provided increased linearly with the number of chips requested (lin R^2 = .98).

In sum, we found that individual differences in spatial-numeric associations mirrored prior findings of age differences in numeric representation (e.g., Wynn, 1990; Siegler & Opfer, 2003). First, paralleling the developmental finding in Wynn (1990), children in the MNL group provided more accurate estimates of large numbers than did children in the No MNL group. Second, subsequent analyses indicated that much of the inaccuracy in the estimates of children in the No MNL group came from their use of a logarithmic representation of number, whereas children's errors in the MNL group came from random noise distributed around an accurate midpoint. These findings of a logarithmic versus linear representation of number also parallel the developmental findings of Siegler and Opfer (2003), who observed a similar transition for estimates of the values 0-1000.

General Discussion

Across two experiments we examined the development of directional biases in the representation of numeric magnitude (e.g., representing numerical magnitudes as increasing in a left-to-right direction). Although previous work had indicated that this directional bias originated in reading practices (Dehaene et al., 1993), our results indicated that this "mental number line" develops before children learn to read and before they show any awareness that symbols are decoded in a left-to-right orientation. Specifically, we found that (1) the consistency of counting and adding in a left-to-right manner and subtracting in a right-to-left manner increased with age (from 17% of 2.5- to 3.5-year-olds to 68% of adults) and (2) the majority of developmental change in this bias (58%) occurred between the ages of 3.5- to 5.5- years, well before children typically learn to read.

To test the hypothesis that this directional bias indicated a "mental number line" by which children could estimate the quantities associated with symbolic numerals, we asked the same children participating in Experiment 1 to translate between numeric symbols and physical quantities. As expected, all children performed nearly perfectly for the lesser numerosities (1-4) in the NC task, indicating that the task could be understood by all children. For numerosities outside the subitizing range, however, children with consistent directional biases greatly outperformed children who did not have consistent directional biases, even with age controlled.

Taken together, results suggest that one source of developmental change in estimation may be the development of spatial-numeric associations, which could allow for the excellent discrimination of numeric values seen in the MNL group and older children. In other studies, for example, well-discriminated concepts (like *good* versus *bad*) are often associated with spatial dimensions (Lakoff & Johnson, 1980), and it may be that abstract concepts are generally represented spatially (Boroditsky, 2000). In the current case, of course, our proposal is based only on correlational data, and it remains possible that spatial-numeric associations develop from improvements in numeric discrimination or even that some third factor (e.g., maturation, intelligence, or experience) accounts for the covariation observed. Further evidence is needed to test the direction of causation.

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