Development of Partitioning Skills

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Abstract

When introducing any new mathematical instruction to children, it is important to consider their cognitive development. Therefore, for teaching fractions to children a clear understanding is necessary of how young children develop partitioning skills. In this study, we presented incorrect solutions to young children and asked whether they were the right solutions to examine the development of the cognitive skill involved in mastering fraction skills. If children rejected the solutions, they were then encouraged to solve the problems by themselves. Based on preliminary observations, two types of partitioning problems were designed: partitioning of continuous (dividing a lump of clay) and discrete (marbles) quantities. Testing 80 young children from four to seven years old, we found different development trends for discrete and continuous quantities. Children showed marked development of partitioning skills for continuous quantity from five to six years old. In contrast, their development of partitioning skills for discrete quantities accelerated between the age of six and seven.

Introduction

Motivated by widely expressed concerns about declining mathematical skills in Japan, especially among university students, we studied the perception of fractions in university students and compared their performance and production of fractions with primary school children (Baba & Iwasaki 2003). The findings prompted a historical survey of fractions in Japan since the end of World War II. Although the general content of such fraction instructions has remained the same, the grades at which the subject had been introduced had clearly and considerably varied over the years. In addition, the number of years they study fractions as well as the number of instruction hours had declined dramatically with successive changes in instruction guidelines provided by the Ministry of Education.

One reason for the inconsistency of the school grade at which fraction instructions are first introduced may reflect the lack of precise knowledge concerning the development of partitioning skills. We only have a limited amount of empirical data on their development. Most previous studies specified the partitioning of objects such as dolls or plates. Children were often explicitly instructed to "divide items in equal quantities" in previous research. Young children may be able to allocate items to dolls or place them on plates one by one without clearly understanding the principles of partitioning. Because allotting items to dolls or placing them on plates can be achieved on a one-to-one basis, successfully performing these tasks does not necessarily reflect an understanding of the principles of partitioning. Therefore, we studied the development of partitioning skills among young children between four to seven without using dolls or plates and without instructing them to "divide equally." Based on the findings we analyzed the developmental stages of partitioning skills and propose an optimum time for the introduction of fractions.

Experiments in partitioning skills

The arithmetic concept of fractions implies that quantities can be subdivided in equal fractions. In a preliminary experiment, kindergarten children were asked to solve partitioning problems with discrete (marbles) and continuous (clay) quantities, and we found that their error patterns differed depending on the type of problem. As it is possible that even those children who have implicit understanding of a correct solution might actually fail to solve the task due to insufficient skill or lack of practical knowledge of manipulation, we first presented erroneous solutions to children to see whether they could perceive the problems involved in them. These mistakes were ones we frequently observed during preliminary tests. Then we asked the children whether these answers were correct. If they rejected them as wrong, they were asked to solve the problems themselves. In this way, 80 young children (20 for each age group) between four and seven were tested. The children were exposed to the same solutions for the sake of consistency. Partitioning was done for both continuous and discrete quantities. The observations suggested different developmental trends for continuous and discrete quantities. A clear developmental spur was observed for continuous quantity from age five to six, whereas the development of discrete quantity lagged behind continuous quantity with a leap between six and seven year olds. Statistical tests with ANOVA on the arcsine transformed scores and additional multiple comparison tests with Ryan's procedure confirmed these impressions.

Development process of partitioning skills

The following are our proposed developmental stages for partitioning continuous and discrete quantities.

Continuous quantities (clay)

Level 1: a child does not understand what it means to divide something.
Level 2A: a child can divide a given amount, but shows no concern about the remainder and does not understand that a whole must be partitioned without leaving some portion undivided.

Level 2B: a child can divide a given amount, but he/she still leaves a portion of the clay. He/she has begun to grasp the importance of wholeness, but appears unable to discover a solution.
Level 3: a child shows clear understanding of wholeness, but still considers it acceptable to divide the clay unequally. At seven years old, children divide the clay equally when dividing it for themselves.
Level 4: a child understands that partitioning means dividing a given amount without any remainder and divides it into equal parts. Until the age of about six, the primary reason mentioned by children for the unacceptability of unequal partitioning was because it would "cause a fight" among the recipients. At seven, they focus on the size itself, saying that because they are "different in size" they are not good ways of partitioning. At the age of seven, 83% of the children could evenly divide the clay.

Discrete quantities (marbles)

Level 1: a child only plays with marbles and does not try to divide them.
Level 2: a child cannot distinguish between taking away a portion of the marbles and dividing them into equal amounts. She/he just removes some of the marbles.
Level 3A: a child carries out measurement division (i.e., taking away N marbles rather than dividing them into N portions) without worrying about the remainder.
Level 3B: a child carries out measurement division and worries about the result when realizing that some marbles are left over. Also even some seven-year olds carried out measurement division, normally six is the peak age at which measurement division is performed. After that age it rapidly declines (see Fig. 2).
Level 4: a child understands the concept of wholeness and has learned to divide a given amount into equal sets. Some advanced children can do so at four; 50% of seven-year olds can do so.

In comparison with continuous quantities, partitioning skills for discrete materials develop more slowly.

Conclusion

As a result of experiments on the partitioning skills of children between four and seven years old, the difference in the rate of the development of skills for evenly dividing countable and uncountable quantities became clear. By the age of seven (second graders), 83% of children have grasped the concept of evenly dividing uncountable quantities. Even those children who accepted the initial wrong solutions presented by the experimenter could divide a given quantity into equal parts when allowed to try by themselves after the experiment. Teachers believe that when about 60% of the pupils can divide things in equal portions it is possible to teach them fractions [S.Satoh, personal communication, 2005]. Presently in Japan, fractions are introduced in the final semester of fourth grade. However, because 83% of second graders understand equal division of continuous quantities, it may be possible to introduce the fractions of continuous quantities in second grade. Moreover, in the present curriculum, third graders are taught division. In introducing division, pupils are also taught measurement division. It is our opinion that we can introduce fractions when division is taught. It will help them understand the differences between measurement division and equal partitioning.

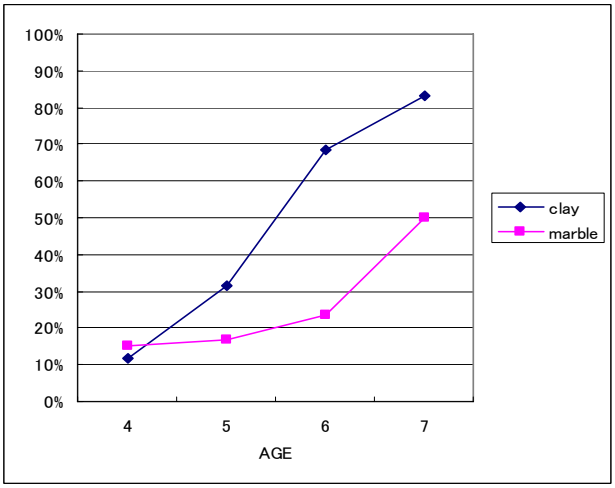

Figure 1: Development of equal partitioning

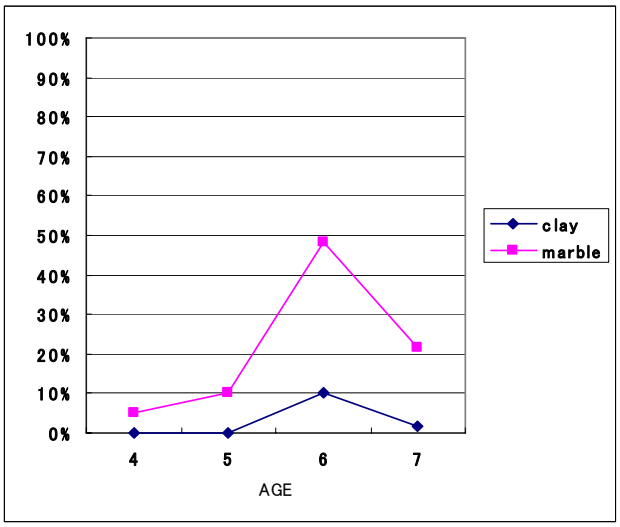

Figure 2: Development of measurement division

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