Why Children Make "Better" Estimates of Fractional Magnitude than Adults

John E. Opfer (opfer.7@osu.edu)
245 Psychology Building, 1835 Neil Ave. Mall
Columbus, OH 43210

Clarissa A. Thompson (thompson.1345@osu.edu)
125 Psychology Building, 1835 Neil Ave. Mall
Columbus, OH 43210

Jeffrey M. DeVries (devries.19@osu.edu)
255 Psychology Building, 1835 Neil Ave. Mall
Columbus, OH 43210

Abstract

Development of estimation has been ascribed to two sources: (1) a change from logarithmic to linear representations of number and (2) development of general estimation skills. To test the representational change hypothesis, we gave children and adults an estimation task in which an automatic, linear representation is less adaptive than the logarithmic representation: estimating the value of salaries given in fractional notation. The representational change hypothesis generated the surprising (and accurate) prediction that when estimating the magnitude of salaries given in fractional notation, young children would outperform adults, whereas when estimating the magnitude of the same salaries given in decimal notation, adults would outperform children. Further support for the representational change hypothesis came from a training study, in which children were given feedback on their estimates of numerical magnitude; as expected, children who improved their estimates of the value of whole numbers subsequently harmed their estimates of fractional value.

Keywords: cognitive development; numeric representations; magnitude estimation

Development of Estimation Accuracy

Numerical estimation is a pervasive process, both in school and in everyday life. Because it plays a central role in a wide range of activities in the lives of children and adults, educators have assigned a high priority to improving estimation for at least the past 25 years (e.g., Geary, 1994). Despite this prolonged effort, estimation remains a process that children find difficult. Whether estimating distance, amount of money, number of discrete objects, answers to arithmetic problems, or locations of numbers on number lines, 5- to 10-year-olds' estimates are highly inaccurate (see Booth & Siegler, 2006, for review).

Recent findings suggest that an important source of children's difficulty with numerical estimation is their use of an inappropriate representation of numerical value. Specifically, children often use an immature logarithmic representation in situations where accurate estimation requires use of a linear representation (Opfer & Siegler, in press; Siegler & Opfer, 2003). For example, when asked to estimate the locations of numbers on number lines with 0 at one end and 1,000 at the other, the large majority of second graders tested by Siegler and Opfer (2003) generated logarithmic distributions of estimates (e.g., estimating 150 to be closer to 1000 than to 1), suggesting that children initially use a representation of numerical value that is consistent with Fechner's Law and that is widespread among species, human infants, and time-pressured adults (Dehaene, Dehaene-Lamberts, & Cohen, 1998; Moyer & Landauer, 1967). In contrast, a large majority of sixth graders and adults in the same study produced linear distributions of estimates (e.g., estimating 150 to be closer to 1 than to 1000), and about half of fourth graders produced estimates that were best fit by each function (Siegler & Opfer, 2003), suggesting that the use of a linear representation of number becomes more widespread with age and experience.

Although evidence of young children's poor estimation accuracy and logarithmic representation of numeric value has been drawn from a wide range of tasks (see Siegler & Booth, 2005, for review), all of the tasks share an important property—accuracy could be attained either from representational changes (Siegler & Opfer, 2003) or from improving estimation strategies (Dowker et al., 1996; Hiebert & Wearne, 1986). To address this issue, we sought to provide a particularly strong test for the representational change hypothesis by examining development on an estimation task that favors the logarithmic representation at the expense of the linear one. Our reasoning was that if people learn to perform more accurately the task simply by improving general estimation skills (e.g., estimation strategies), estimation on such a task should also improve. However, if people learn to perform more accurately on the task by automatizing use of a linear representation, estimation on this task should suffer with age and experience.

One case in which an automatic linear representation would be misleading is the estimation of fractional magnitudes. Specifically, we propose that adults' success in using a linear representation of number has an unintended negative consequence: by automatizing that 150 is closer to 1 than to 1000, adults are subject to a powerful cognitive illusion in which 1/150 seems closer to 1/1 than to 1/1000; in contrast, children's belief that 150 is closer to 1000 than to 1 protects them from this illusion. Thus, if the representational change hypothesis is correct, young children—despite their poor understanding of fractions and poor estimation skills generally—would perform more accurately on a fractional magnitude task than would adults.

Estimation of Fractional Magnitude

Far from suggesting that children would perform more accurately than adults when estimating fractional magnitudes, previous work has indicated that fractions pose enormous difficulties for both children and young adults, leading them to provide grossly mistaken answers to simple addition and subtraction problems (Carpenter et al., 1981). One reason for this difficulty is that the majority of children and young adults fail to represent the whole magnitude denoted by fractions and instead represent only the magnitude of the denominator or numerator.

This interpretation of young adults' approach to fractions led us to make two predictions about how they might make estimates about the value of quantities expressed in fractional notation, such as when estimating the placement of a salary (e.g., $1/60 minutes) on a line that begins with one salary (e.g., $1/minute) and ends with another (e.g., $1/1440 minutes). The first prediction was that if adults ignore the fractional context and compare only the value of the denominators in the salary, their linear representation of the denominator would lead them to make radically incorrect estimates of monetary value. This inaccuracy is predicted by the fact that the relation between the numeral expressed in the denominator and the magnitude denoted by the whole fraction is provided by a power function rather than by a linear function. For example, although 60 is closer to 1 than to 1440, $k/60$ is closer to $k/1440$ than it is to $k/1$.

The second prediction was that if children also ignore the fractional context, their logarithmic representation of the denominator would have a correcting effect and thereby lead to more accurate performance on the task than performance of adults. This prediction stems from the fact that the power relation between the value of the denominator and the magnitude of the fraction is somewhat similar to that of a logarithmic function, where differences at the low end of the range are very high and differences at the high end of the range are very low. Thus, for example, the natural logarithm of 60 is closer to the natural logarithm of 1440 than to the natural logarithm of 1, much as $k/60$ is closer to $k/1440$ than it is $k/1$.

Although the combination of ignoring the fractional context and having a logarithmic representation of number could lead children to outperform adults, this is not a trivial prediction: "'Performance improves with age' is as close to a law as any generalization that has emerged from the study of cognitive development" (Siegler, 2004, p. 2). On the other hand, if shifting from logarithmic to linear representations is the source of improvements in estimation rather than the development of general estimation skills, performance in estimating fractional values would decline with age.

Study 1: Age Differences in Fraction Estimation

Method

Participants Participants were 24 second graders (M = 8.4 years, SD = 0.40) attending a suburban school in a middle-class neighborhood, and 24 undergraduates (M = 19.4 years, SD = .77) enrolled in an introductory psychology course at a large Midwestern university in the United States. Two research assistants (male and female) served as experimenters.

Tasks On two different tasks, participants were asked to estimate the total amount of money a person would make at a given salary (e.g., $1/hour) by placing a mark on a 20 cm "money line". The monetary value of the salaries to be estimated were held constant across the two tasks, but we varied the notation for expressing these salaries. On the *fractional units task*, both the value of the salaries to be estimated (e.g., $ 1/60 minutes) and the endpoints of the money line ($1/1 minute, $1/1440 minutes) were expressed in fractional units. On the *decimal units task*, salaries and endpoints were expressed in decimal units (e.g., $.0167 per minute, $1 per minute, $0.0007 per minute).

Design and Procedure Participants were told that they would receive a series of "money lines" and that values on the money line represented an amount of money that a person might make. For each problem, the experimenter indicated the two endpoints of the money line, indicated the target salary above the line, and asked the participant to make a hatch mark on the line to indicate where the target salary would go. For example, in the fractional units task, the participant was asked, "If this is $1 per minute and this is $1 per 1440 minutes, where would $1 per 60 minutes go?" The order of tasks was counterbalanced over subjects, and the order of problems within each task was randomized. Half of the participants solved problems with the smaller salary on the left; the other half solved the same problems with the smaller salary on the right.

Results and Discussion

We first converted all estimates (i.e., hatch marks) to a numeric code (0 – 1) by measuring the distance between the endpoint of the scale and the student's estimate (0 – 20 cm) and dividing by the total length of the scale (20 cm). To assess estimation performance across the two tasks, we examined the mean absolute error for each subject, which was calculated using the formula,

$$MAbsErr = \sum_{i=1}^{7} \frac{|\, actual_i - estimate_i \,|}{7}$$

We then conducted a 2 (age group: 8-year-olds, adults) X 2 (task: fractional units, decimal units) repeated-measures ANOVA on the mean absolute error scores for each subject (see Figure 1, where accuracy reflects the inverse of the mean absolute error). As predicted, the decimal units task elicited fewer errors (MAbsErr, $M = .27$, $SD = .16$) than the fractional units task (MAbsErr, $M = .57$, $SD = .17$), F (1, 138) = 64.58, $p < .001$. Age and task also produced interactive effects on mean absolute errors, F (1, 138) = 21.0, $p < .001$, leading us to analyze each task separately.

On the *fractional units task*, both adults and children were less accurate (p's < .05) than random responding (modeled in three Monte Carlo simulations), but adults' overall errors (MAbsErr, $M = 0.65$, $SD = 0.16$) were even greater than children's (MAbsErr, $M = .48$, $SD = .13$), $t(46) = 4.16$, $p < 0.001$, $d = 1.23$. To determine whether the large errors in both age groups came from ignoring fractional context, we first compared estimates on the present 1/1-1/1400 fraction line to estimates provided on a number line task without any fractional context (i.e., the 0-1000 number line used by Siegler and Opfer [2003]). As Figure 2a illustrates, the fractional context had no effect on estimates, whereas the value of the denominator had a large effect on estimates. To examine this more closely, we regressed median estimates against the value of the denominator using the logarithmic and linear regression functions. As predicted, the logarithmic function ($R^2 = .99$) provided a much better fit to children's estimates than to all three sets of randomly generated data (average log $R^2 = .18$), and the logarithmic function also provided a better fit than did the linear function ($R^2 = .75$). In contrast, the linear function ($R^2 = .94$) provided a better fit to adults' estimates than to randomly generated data (lin $R^2 = .12$), and the linear function provided a slightly better fit than did the logarithmic function (log $R^2 = .90$).

To determine the link between the fit of the two functions and overall accuracy, we fit each function to individual participants' estimates, and then compared the performance of those participants (regardless of age) who used a logarithmic representation of the denominator (71% of participants in total) versus those who used a linear representation. As expected, the linear group (71% adults) provided substantially more errors (MAbsErr, $M = .68$, $SD = .13$) than the logarithmic group (41% adults) (MAbsErr, $M = .52$, $SD = .13$), $t(46) = 3.28$, $p < 0.001$, $d = .97$.

Thus, when making estimates of fractional magnitude, both children and adults tended to ignore the fractional context and to focus exclusively on the value of the denominator. This focus on the denominator, however, had different effects on accuracy, depending on whether participants' underlying representation of the denominator was logarithmic (mostly children) or linear (mostly adults). This difference in underlying representation explains why children provided more accurate estimates than did adults on this task.

A number of alternative explanations for the superior performance of children in the previous task might be proposed. Perhaps the children we found just happened to be more mathematically capable, more attentive, or more motivated than the adults we recruited, thereby undermining the developmental trajectory we proposed. Alternatively, children may have been better at reasoning about money, rates, time, or the specific salaries involved than were adults, thereby limiting the developmental trajectory we proposed to one concerning these variables rather than to one concerning fractional magnitudes. To rule out these alternative explanations, we examined performance on the decimal units task, which also involved money, rates, time, the same specific magnitudes, and the same subjects, thereby offering children all of the possible benefits that they might have obtained on the previous task save one: the fractional notation, which was the variable we hypothesized to elicit their superior performance.

On the *decimal units task*, children's overall errors (MAbsErr, $M = 0.35$, $SD = 0.13$) were indeed much greater than adults' (MAbsErr, $M = .19$, $SD = .15$), $t(46) = 4.03$, $p < 0.001$, $d = 1.19$, indicating that previous effects did not result from our finding extraordinary children or children being generally better at reasoning about money, rates, time, or the specific magnitudes tested. Further, adults' accuracy was better than that of randomly generated responses, whereas children's accuracy was not. To determine whether adults' superior performance on this task resulted from their hypothesized use of a linear representation of the decimal units, we regressed their median estimates against the actual value (Figure 2b). As predicted, the fit of the linear function ($R^2 = .92$) was better than the fit of the logarithmic function ($R^2 = .86$), and the linear function also provided a better fit to adults' estimates than to random responses (average $R^2 = .51$). We performed the same analysis on children's estimates by regressing their median estimates against the value of decimal units. As predicted, the fit of the logarithmic function ($R^2 = .54$) was slightly better than the fit of the linear function ($R^2 = .51$), though clearly neither function provided a very good fit, with only the logarithmic providing a better fit to children's estimates than to randomly generated data (average log $R^2 = .38$, average lin $R^2 = .51$). (Possibly some children interpreted decimal values as analogous to whole numbers; normally children do not encounter decimals until well after first and second grade.) Finally, when we compared the performance of those participants (regardless of age) who used a linear representation of the decimal versus those who used a logarithmic representation, participants who used a linear representation provided substantially fewer overall errors (MAbsErr, $M = .19$, $SD = .15$) than those who used a logarithmic one (MAbsErr, $M = .34$, $SD = .13$), $t(46) = 3.66$, $p < 0.001$, $d = 1.08$.

Last, we attempted to examine the processes by which participants generated their estimates. We had hypothesized that adults' poor estimation in the fractional units task would be produced by an automatic linear representation of the denominator, but an alternative view is that it was produced by a poor strategy. Specifically, in Siegler and Opfer (2003), adults' linear estimates were seemingly produced by their use of a landmark strategy, in which they mentally subdivided the line into quartiles (e.g., from 0-250, 250-500, 500-750, and 750-1000). Consistent with this strategy use, adults' estimates in Siegler and Opfer (2003) became increasingly variable with the distance between quartiles and the numbers-to-be-estimated ($r = .63$), whereas no such relation was observed for children's estimates. To examine whether children and adults used a similar strategy in our tasks, we also regressed the standard deviation in estimates against the distance of the number in the denominator from the nearest quartile. As in Siegler and Opfer (2003), children did not appear to use the quartile strategy on either of our tasks (rs $< .24$, ns). For adults on the fractional task, distance from quartile also did not account for a significant amount of variability in estimates ($r = .00$); however, distance from quartile did account for a significant amount of variability in adults' estimates on the decimal units task ($r = .79$, $p < .01$). Thus, while adult accuracy on the decimal units may have stemmed from strategic performance rather than automatic use of a linear representation, their inaccurate performance on the fractional task did not seem to be strategic.

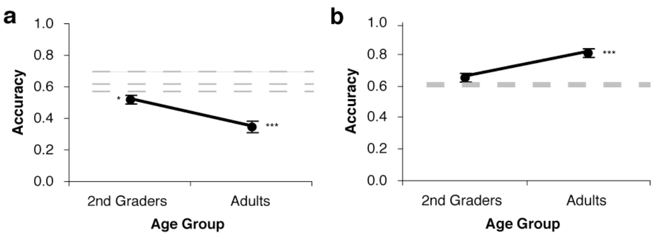

Figure 1: Accuracy for both age groups on each task: a) fractional units and b) decimal units. Dashed lines represent the levels of accuracy obtained in three Monte Carlo simulations. Asterisks represent the number of simulations differing reliably ($p < .05$) from observed responses.

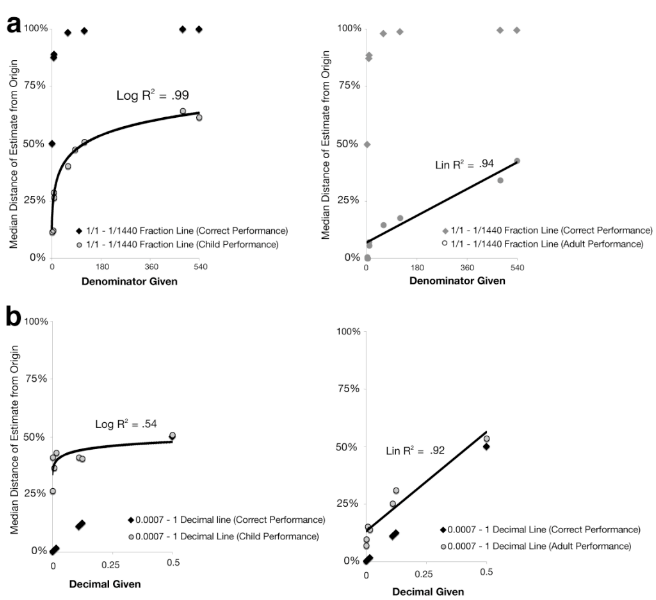

Figure 2: Median estimates for children (left) and adults (right) on each task: a) fractional units, shown with Opfer & Siegler's (2003) 0-1000 number line data and b) decimal units.

Study 2: Relation between Integer Estimation and Fraction Estimation

Results of Study 1 suggested that learning to generate accurate, linear estimates of numerical quantity (as adults do) comes at the cost of accurately estimating fractional values (as children do). We tested this hypothesis directly in Study 2, where we trained children to generate accurate estimates on a number line task and examined their subsequent performance on a fraction line task.

Method

Participants Participants were 32 second and third graders (*mean* age = 7.98 years, $SD = 0.85$) attending a suburban school in a middle-class neighborhood. Twelve of these students (38%) were excluded from further training because their pretest estimates were already best fit by the linear function. There were 7 boys and 13 girls ($M = 7.66$, $SD = 0.61$) who received training. Two female research assistants served as experimenters.

Number Line Estimation Task All number line problems consisted of a 20 cm line with the left endpoint labeled "0," the right endpoint labeled "1000," and with the number to be estimated appearing 2 cm above the midpoint of the number line. Participants were asked to place the following numbers on a number line by making a hatch mark: 2, 5, 18, 27, 34, 42, 56, 78, 100, 111, 122, 133, 147, 150, 156, 162, 163, 172, 179, 187, 246, 306, 366, 426, 486, 546, 606, 666, 722, 725, 738, 754, 818, 878, and 938. These numbers maximized the discriminability of logarithmic and linear functions by oversampling the low end of the range, minimized the influence of specific knowledge (that 500 is halfway between 0 and 1000), and tested predictions about the range of numbers where estimates would most differ between pretest and posttest.

Fractional Units Task The fractional units task used in Study 2 was the same as the fractional units task used in Study 1.

Design and Procedure Children were randomly assigned to one of two groups: one group received feedback during training, whereas the other group did not receive feedback.

Children in both groups completed the number line estimation task at pretest, over three training trial blocks, and at posttest. On pretest and posttest, children in both groups were presented the same 22 problems without feedback. For children in the feedback group, each training trial block included a feedback phase and a test phase. The feedback phase included either one item on which children received feedback (Trial Block 1) or three items on which they received feedback (Trial Blocks 2 and 3). The test phase included 10 items on which children did not receive feedback; this test phase occurred immediately after the feedback phase in each training trial block. Children in the no feedback group received the same number of estimation problems, but they never received feedback.

On the feedback problems, children were told to make a hatch mark indicating where they believed the numerosity was supposed to go, and then the experimenter would show him/her how close the mark was to the actual location of the numerosity by making a second hatch mark on the number line. Children's answers that deviated from the correct hatch mark by less than 10% were described to the children as being "really quite close", whereas children's answers that deviated by more than 10% were described as "a bit too high/too low".

Results and Discussion

Our principal analysis was to compare the performance of "learners," those children who were best fit by the linear function on the posttest number line estimation task, to "non learners," those children who were best fit by the logarithmic function on the posttest number line estimation task, on the fractional units task.

As predicted, children who learned to provide linear estimates on the number line task were less accurate (*p's* < *.01*) than random responding (modeled in three Monte Carlo simulations), whereas children who continued to generate logarithmic estimates on the number line task were not different from random responding. Learners' overall errors (MAbsErr, *M* = .57, *SD* = .02) were greater than non learners' overall errors (MAbsErr, *M* = .41, *SD* = .01), *F (1, 19)* = 7.499, *d* = 10.12. As predicted, children who continued to generate logarithmic estimates on the number line task provided more accurate estimates of fractional magnitude (*M* = .59, *SD* = .02) than did children who learned to provide linear estimates of integer magnitude (*M* = .43, *SD* = .01), *F* (1, 19) = 7.499, *p* < .05, *d* = 10.12 (see Figure 3).

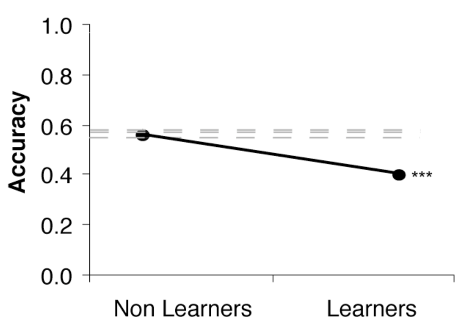

Figure 3: Accuracy for learners and non learners. Dashed lines represent the levels of accuracy obtained in three Monte Carlo simulations. Asterisks represent the number of simulations differing reliably (*p* < .01) from observed responses.

To determine whether the large errors in both the learner and non learner groups came from ignoring fractional context, we regressed the value of the fractional denominator against the median estimate of fractional value (see Figure 4). As expected, the logarithmic function (R^2 = .95) provided a much better fit to non learners' estimates than to all three sets of randomly generated data (log R^2 = .00 - .16), and the logarithmic function also provided a better fit than did the linear function (R^2 = .68). In contrast, the linear function (R^2 = .87) provided a better fit to learners' estimates than to randomly generated data (lin R^2 = .00 -.12), and the linear function provided a slightly better fit than did the logarithmic function (log R^2 = .85).

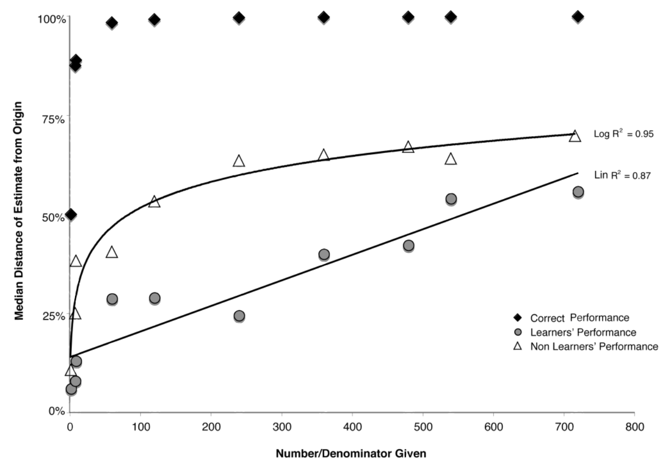

Figure 4: Median estimates for learners and non learners on fractional units task.

General Discussion

We tested the general claim that age-related improvements in estimation are caused by the development of linear representations. Previous support for this hypothesis came from age-differences in performance on tasks in which a linear representation necessarily yields accurate estimates (Opfer & Siegler, in press; Siegler & Opfer, 2003), which has left it unclear whether the development of representations (Siegler & Opfer, 2003) or improvements in general estimation skills could be responsible for estimation improvement (Dowker et al., 1996; Hiebert & Wearne, 1986). In these studies, we sought to provide a particularly strong test for the representational change hypothesis by examining development on an estimation task that favors the logarithmic representation at the expense of the linear one (i.e., estimation of fractional magnitudes). An interesting prediction generated by the representational change hypothesis was that children would outperform adults when making estimates of fractional magnitudes.

The results of the present studies supported this hypothesis. When estimating the value of a salary by estimating its place on a scale expressed in fractional notation (i.e., on a scale where a logarithmic mental scaling of salaries provided a more accurate gauge than a linear mental scaling), children outperformed adults. Clearly this difference did not stem from children having greater mathematical skills than adults. Rather, it appears that children used a logarithmic representation of the denominator's value as a guide for placing their estimates, whereas adults used a linear representation of the denominator's value as a guide for placing their estimates. The clearest evidence for this hypothesis came from a comparison of estimates on our 1/1 – 1/1440 task and Siegler and Opfer's (2003) 0-1000 task: as Figure 2a illustrates, the two sets of estimates are nearly indistinguishable, with the fit of the logarithmic function being very high in both tasks for children (this study, log $R^2 = .99$; S&O, log $R^2 = .95$) and the fit of the linear function being very high in both tasks for adults (this study, lin $R^2 = .94$, S&O, lin $R^2 = 1$).

In contrast, when estimating the value of a salary by estimating its place on a scale expressed in decimal notion (i.e., on a scale where a linear mental scaling provides a more accurate gauge than a logarithmic scaling), adults outperformed children. Adults' superior performance on this decimal units task also suggests that children's superiority on the fractional units task did not stem from children being smarter than adults, paying more attention, taking more time, being more motivated, or having a superior understanding of money, time, rates, or the specific magnitudes assessed. Rather, adults' superior understanding of the decimal system was an advantage on the decimal units task, though a hindrance on the fractional units task. The consequences of training, in Study 2, provided still more direct evidence for the hypothesis that short-term, as well as long-term, changes in estimation accuracy stem from changes in numerical representation rather than independent changes in estimation skills or general mathematical knowledge.

How might we generalize our findings across other fractional magnitude tasks? One interesting class of *k/n* problems (e.g., comparing 1/1 vs. 1/1000) are those where $k > 1$ (e.g., comparing 10/10 vs. 10/1000). Previously, a ratio-bias has been observed (Denes-Raj, Epstein, & Cole, 1995), wherein people judge the occurrence of an event of low probability as less likely when its probability is represented by a ratio of smaller (e.g., 1/100) than of larger numbers (e.g., 10/1000). On these problems, the representational change hypothesis predicts much worse performance from children owing to their linear representation of the 1-100 increment. If correct, this finding would highlight that children's inaccuracy—like adults' inaccuracy—can stem from correct representations being applied inappropriately.

Finally, we should emphasize that we do not interpret these findings as a challenge to the truism that "performance improves with age" (which of course it does), nor to argue that children are better estimators than adults are (normally they are not). Rather, we believe that the finding of children outperforming adults in these tasks is significant for a very different reason: alternative hypotheses are most usefully distinguished when they are tested against events with a low base-rate of probability. We believe that this principle is a particularly important one for making causal inferences in the development of mathematical cognition and in developmental psychology more broadly. Finally, on the basis of this type of event, we believe that we have provided strong evidence for children and adults possessing different representations of numerical magnitude.

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