What About Negation in Spatial Reasoning?

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Abstract

How do we reason about spatial descriptions? In recent years a lot of research has been investigated in order to determine factors of complexity in spatial relational reasoning. Several factors like the number of models, the wording of conclusion or relational complexity have been determined. Any of these factors effected reasoning. But the literature focused solemnly on positive premises. Negated expressions like "The fork is not to the right of the cup" had not been investigated. Since in everyday spatial reasoning the role of negation is eminent, we study negated spatial relations from a formal and psychological perspective. Central questions are: How are negated statements represented? If there are various models consistent with the set of premises, which of these is constructed initially? Is there an effect for different reference frames? We conducted three experiments for investigating these research questions. We will show that humans (i) negate a relation by using the opposite relation, (ii) use a cognitive economic principle in constructing mental models, (iii) construct preferred mental models, and (iv) have more difficulties in reasoning with negated relations in comparison to indeterminate positive descriptions.

Keywords: Spatial Reasoning; Cognitive Modeling; Spatial Representation.

Introduction

There is a great body of evidence supporting the mental model theory of spatial reasoning. The key idea of this theory is that reasoners translate the spatial relations in the real or imagined world into a mental model and use this representation to solve given spatial inference problems. To provide an example (cf. Mani & Johnson-Laird 1982):

(I) The plate is to the left of the knife.
The fork is to the left of the knife.
The glass is in front of the knife.
The spoon is in front of the plate.

This describes the following two possible models:

	spoon	glass	|	spoon		glass
fork	plate	knife	|	plate	fork	knife

Assume a child helps his mother to set the table. The child takes the knife and puts it to the left of the plate. But the mother says to the child "The knife does not belong to the left of the plate". Where will the child place the knife? Logically, there are three possibilities: the knife can be placed to the right of, in front of, or behind the plate (not considering that the knife can be placed above or under the plate). After a certain time the child will know how the single components should be placed on the table. This is what we call the background knowledge, but this goes beyond the scope of the paper.

How are such problems with negation processed? Is there a preferred interpretation? The mental model theory (MMT), introduced by Johnson-Laird and Byrne (1991), suggests that people draw conclusions by constructing and inspecting a spatial array that represents the state of affairs described in the premises. It is a three stage process consisting of a comprehension, description, and validation phase. In the *comprehension phase*, reasoners construct a mental model that reflects the information from the premises. If new information is encountered during the reading of the premises it is immediately used in the construction of the model. During the *description phase*, this model is inspected to find new information that is not explicitly given in the premises. Finally, in the *validation phase* alternative models are searched that refute this putative conclusion. However, some questions remain open with regards to how people deal with multi-model problems. For example, which model is constructed first, and does this model construction adhere to certain principles? Why do reasoners neglect some models?

All these questions are not answered by the classical mental model theory. In contrast, the preferred mental model theory (PMMT) has been developed to explain that humans generally tend to construct a preferred mental model (PMM). The PMM is the starting point for deriving at a putative conclusion. In the model variation phase the participants tend to make local and continuous transformations starting from the PMM to search counter-examples (Rauh et al., 2005).

Several predictions of the PMMT about insertion principles as well as transformation strategies in spatial relational reasoning can be shown (Ragni et al., 2006). Assume we have two premises of the form (1) "A is to the left of B" and (2) "A is to the left of C". Humans tend to process these premises sequentially, i.e. first, a model A B is generated and then object C is inserted into the model. There are two possibilities where C can be inserted:

- In between A and B (**f**irst **f**it principle, *ff*-principle) or
- To the right of B (**f**irst **f**ree **f**it principle, *fff*-principle)

PMMs are constructed by using the *fff*-principle. This has been empirically confirmed (Ragni et al., 2006).

But how do humans process a premise like "A is not to the left of C"? Do they remain in one dimension (by using the opposite relation only)? Which kind of insertion principle is then used? Kaup and colleagues (2006) focused on the negation in sentences and contradictory predicates. They conducted a verification experiment in which participants had to verify sentences (e.g. the door was not open) and pictures of situations described in the sentences (e.g. closed door, open door). Reaction times were shorter if the sentence and the picture corresponded (affirmation). Since there are only two states possible (the door is open or the door is closed) there is only one opposite state left. Multiple model cases have not been investigated.

Hasson and Glucksberg (2006) examined the difference in understanding affirmative and negated assertions in natural language. The participants had to make decisions on using terms related to either the affirmative or the negative meaning. The results suggest that the affirmative assertion continued to facilitate affirmative-related terms, but the negated assertion did not. In the literature, no work regarding negation in multiple model cases has been reported.

In this paper, we analyze spatial problems with negated relations. The next section contains a formal analysis of negated spatial problems. In the following, we present empirical data in support of our theory. Finally, we discuss the results presented in the paper and give a short overview of some questions that are left open.

Mathematical Background

Johnson-Laird (2001) introduced the principle of truth: "A mental model can represent by default only what is true, but not what is false" (p. 434). The same property holds for mathematical models. It might be worth to analyze the formal structure of the problem: The processing of a premise is a function mapping of a linguistic statement consisting of a propositional statement like "A is to the left of B" to a position in a spatial array. This function is called an interpretation (Ebbinghaus, Flum, & Thomas, 1994). Mathematically it can be described as:

$$I\text{: Premise} \rightarrow \text{Spatial Array}$$

For the example introduced above "A is to the left of B", an interpretation maps Object A to position (0,0) and object B to position (1,0) in a spatial array. But how is a negated statement "A is not left of B" interpreted? Answering this question depends on the used reference frame. Since we are using spatial relations, our reference frame in this case is a relational system consisting of already defined spatial relations. We only take the most parsimonious set of relations, i.e. the relations to the right of, to the left of, over and under. More complex relations then can be defined by using this set of relations and propositional connectives like logical '*and*' and logical '*or*'. Take for instance the ternary relation "B is in between A and C". This relation can be purely represented on a two-dimensional grid by using the four relations. Namely "B is in between A and C" is equivalent to all models of "A is left of B & B is to the left of C *or* A is to the right of B & B is to the right of C *or* A is behind of B & B is behind of C *or* A is behind B & B is behind of C".

How can we define, based on the reference frame and the parsimonious relations, right, left, over, and under the negation of a relation? In our setting the following statement is always true: "A is left of B *or* A is to the right of B *or* A is over of B *or* A is under B", since there are no other possibilities (of course this could be easily extended to three-dimensions). If we receive a premise like "A is not to the left of B", then it follows logically that the rest of the sentence holds, i.e. that "A is to the right of B *or* A is over B *or* A is under B". In several experiments the existence of preferred models has been confirmed (Rauh et al., 2005; Ragni et al., 2006). Compare the following two sets of premises

(II)		(III)	
(II)	A is to the left of B.	(III)	A is to the left of B.
	C is to the right of A.		C is not to the left of A.

If we assume that participants interpret the relation "not" as the opposite relation, then the premise sets II and III have identical models. Therefore, both kinds of problems lead to multiple model cases. There are several options to represent negation:

- Reasoners could insert object A in an arbitrary position to B (*arbitrary interpretation*).
- Reasoners could insert object A to the right of B with/without annotating other possibilities for A (*opposite [annotated] interpretation*).
- Reasoners could insert object A to the left of B and annotating the violence on A (*violating interpretation*).

Other interpretations are possible as well but these are the most reasonable. In the next section we provide data supporting the preferred annotated interpretation.

Empirical Data

We report three experiments on how humans generate and inspect mental models out of given premises when a relation of a premise is negated.

First, we questioned which relations between two objects were accepted if a relation was negated or not? Second, we were interested in the generation process of mental models: (i) How is a object inserted into a model if the relation of the object is negated? (ii) Do participants use certain insertion principles if a model contains a negated relation in a given premise which leads to a preferred mental model during the construction process? (iii) Are the preferred mental models with negated problems different from indeterminate positive problems? Finally, we examined the constructed model during the inspection phase that participants had in mind: (i) Which influences have different construction directions, for example, if a model was built from left to right or from right to left, and different term arrangements in two-dimensional models? (ii) Are there differences between indeterminate and negated problems during the inspection phase?

We assumed that the participants interpret the negation of the relation as the logical negation in the same dimension. According to this hypothesis, we expected for the premise "A is not to the left of B" that the participants construct a model in which "A is to the right of B". Another assumption was that models with negated relations are harder to obtain than models without negation. A further assumption was that the complexity of the model that participants have in mind is higher if a relation is negated in comparison to an indeterminate relation.

First Experiment - Acceptance

In this experiment the participants had to accept or reject statements about positive or negated relations between two objects. In the first part of the experiment we tested the acceptance with an underlying grid, since the SRM use distinct positions in a grid for solid objects without overlapping. During the second part of the experiment we tested without an underlying grid as the grid could bias the acceptance due to a clear horizontal and vertical arrangement which is not natural for a mental model.

Participants, Materials, Procedure and Design. Thirty-six students of the University of Freiburg took part in this experiment (with/without grid: n = 20/16, age in years: M = 24.3/24, SD = 2.4/2.8). The participants were presented with pictures of two related objects and a statement. Figure 1 shows examples with (I) and without (II) an underlying grid.

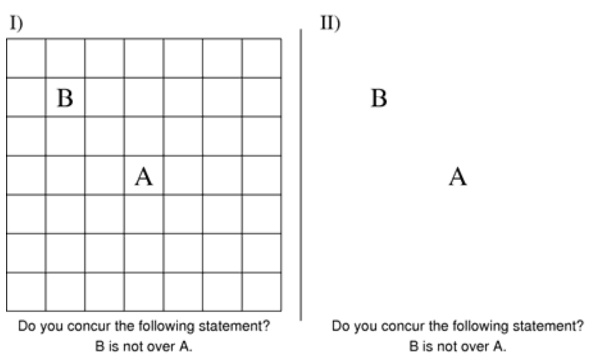

Figure 1: I) with underlying grid, II) without underlying grid. Statement with negated relation of the two objects that have to be accepted or rejected.

The letter **A** had a fixed position in the center while the letter **B** was randomly swapped through all other 48 free cells in the grid. Every possible constellation of A and B was presented with a statement ("B is not over A", "B is not right of A"). We also asked for "B is over A", and "B is right of A" in order to compare the data with positive cases. The last two statements were tested on 16 of the 48 possible cases (see Figure 2II). Response times and accuracy were recorded for each statement.

Results. The participants made a clear decision for affirmative (right/over) and negated (not right/not over) statements. Figure 2 indicates that the distinction whether or not B is over/not over A is clear. In both cases (with or without underlying grid) the results are similar for all four statements (over/not over and right/not right).

I) Not Over — NO

100 3256	100 1767	100 2910	93 2482	100 3302	100 2447	100 2312
93 2528	100 2616	100 3165	100 1871	100 2578	100 3652	93 2589
100 1815	100 2987	100 2628	100 1852	93 2478	100 3412	93 2169
7	19	6	A	13	7	19
0	7	0	14	0	0	7
19	23	6	8	0	7	8
8	0	0	0	7	0	6

I) Not Over — YES

0	0	0	7	0	0	0
7	0	0	0	0	0	7
0	0	0	0	7	0	7
93 2761	81 3823	94 3936	A	87 2652	93 2173	81 4821
100 2997	93 2447	100 2577	86 2260	100 2113	100 2283	93 2462
81 2031	77 2106	94 2534	92 1711	100 2086	93 2029	92 2339
92 1667	100 1640	100 2550	100 2534	93 2299	100 2375	94 3265

II) Over — NO

	0	7	0	0	7	
	0				0	
	93 2265		A		100 2210	
	100 2077				100 1949	
	100 2558	93 2376	85 1856	94 2331	100 2015	

II) Over — YES

	100 1533	93 2207	100 2053	100 1771	93 2028	
	100 1855				100 2116	
	7		A		0	
	0				0	
	0	7	15	6	0	

Figure 2: I) Contains the negated statement "not over". The left square shows the overall decisions in percent for NO and the right square for YES answers (correct answers marked grey). The numbers below contain the reaction times for the correct decisions. II) Shows the affirmative statement "over" (all other information is similar to I).

Reaction time for the negation problems with or without underlying grid for "not over" is significantly longer than for "not right" (with/without grid: t = 7.076/5.589, df = 19/15, $p \leq 0.01$), as well as affirmative problems for "over" in comparison to "right" (with/without grid: t = 3.326/4.062, df = 19/15, $p \leq 0.01$).

In most cases the reaction time is significantly shorter (see Fig. 3) if the statement and the actual state of the relation of A and B is true (with/without grid: "**not over**" t = 0.288/4.124, df = 19/15, p = n.s./p $\leq$ 0.01; "**not right**" t = 1.717/3.186, df = 19/15, p = n.s./p $\leq$ 0.05; "**over**" t = 2.810/3.550, df = 19/15, $p \leq 0.05$/p $\leq$ 0.01; "**right**" t = 4.157/2.422, df = 19/15, $p \leq 0.001$/p $\leq$ 0.05).

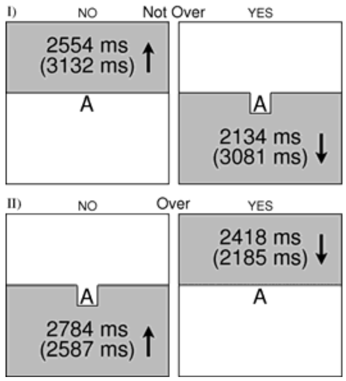

Figure 3: I) Contains the negated statement "not over". The left square shows the overall reaction time for NO and the right square for YES answers (only correct answers without underlying grid). The numbers in parenthesis contain the reaction time with underlying grid. II) Shows the positive statement "over" (all other information is similar to I).

Second Experiment - Simple generating experiment

In this experiment, we investigated how people construct a model if premises contain negated relations between two objects. Additionally, we analyzed if participants construct a preferred mental model (PMM). We assume an increase in the difficulty for the generation of negated problems due to the higher cognitive effort. Additionally, we test the accuracy of the participants with determinate problems, which serve as an exclusion criteria for the ability of the participants to deal with the relational reasoning problems.

Participants, Materials, Procedure and Design. Twenty-three students of the University of Freiburg took part in this experiment (age in years: $M = 25.81$, $SD = 4.45$). It was designed as a pen and paper experiment consisting of sixteen problems (Table 1 and 2) in which the participant had to construct a mental model out of four given premises. This model should then be drawn on a sheet of paper. The models were varied in the dimension (one- and two-dimensional), determination (determinate and indeterminate) and negation (affirmative and negated). Every model was presented twice but had different term names (total of 16). All of the 16 problems were constructed in the same way. Four premises arranged five different objects with the relations left, right, over or under. The relation of the third premise was always negated. Note that models with negation were always indeterminate due to the undetermined position of the object.

In order to guarantee that a model was constructed in working memory only, each problem contained three pages. On the first page the participants were given the first two premises, the following two premises were given on the second page. The third page was empty and the participants were asked to draw only one model even if multiple models could be constructed. Additionally, the participants were instructed not to use any kind of aid (no sketch, etc).

Table 1: Contains four premises for the positive I) and negative II) problems for one-dimensional (a, b) and two-dimensional (c, d) problems for both determinate (a, c) and indeterminate (b, d).

Problem	***PMM/alternative models***
(a) A is to the left of B. B is to the left of C. *I) C is to the left of D.* *II) C is not to the right of D.* D is to the left of E.	(1) **A B C D E** For negated problems: "C is placed right of D" is impossible
(b) A is to the left of B. B is to the left of C. *I) D is not to the left of B.* *II) D is to the right of B.* D is to the left of E.	(1) **A B C D E** (2) A B D E C (3) A B D C E For negated problems: "D is placed left of B" is impossible
(c) A is over B. B is to the left of C. *I) C is not to the right of D.* *II) C is to the left of D.* D is under E.	(1) **A E** **B C D** For negated problems: "C is placed right of D" is impossible
(d) A is over B. B is to the left of C. *I) D is not to the left of C.* *II) D is to the right of C.* D is under E.	(1) **A E** **B C D** (2) A E B D C For negated problems: "D is placed left of C" is impossible

Results. The correct answers (see Tab. 3) indicate, that one-dimensional problems are significantly more often correct than two-dimensional problems (Wilcoxon-Test: Z = 3.109, p = 0.002).

Table 3: Shows the correct responses (in percent) for one- and two-dimensional, affirmative and negative, as well as determinate and indeterminate problems.
Aff. = affirmation, Neg. = negation

	1-dim.		2-dim.	
	Aff.	Neg.	Aff.	Neg.
Det.	87	78	76	52
Indet.	85	78	67	57

Furthermore, there is a significant difference between affirmative and negated problems (Wilcoxon-Test: Z = 2.618, p = 0.009). However, there is no significant difference between determinate and indeterminate problems.

An additional question was how participants understand negated problems. If one direction is negated, then all other possible directions are allowed. There was a stable preference for the opposite direction in negated problems. Table 4 shows that except for indeterminate two-dimensional problems the use of the opposite direction was significantly more frequent. A further question was the preference for a model. For both dimensions we found a significant difference from zero for the PMM. When we analyzed only the indeterminate problems for affirmative versus negated problems, we did not find any significant differences.

Table 4: Shows the preference for the opposite direction in percent for one- and two-dimensional, as well as determinate and indeterminate negated problems. The numbers divided with colons denote the number of correct answers for the opposite direction in comparison to all correct answers. The last row indicates the proportion of preferred models (fff) in comparison to the other principle (ff). Note that the models for determinate negated problems in this task do not provide the discrimination between preferred and alternative models.

$^{*}p \leq 0.05$, $^{**}p \leq 0.01$, $^{***}p \leq 0.001$

	Correct answers	Negation	
		1-dim.	2-dim.
Det.	Opposite	81%***	83%**
	Opposite : all	29 : 36	20 : 24
Indet.	Opposite	75%*	65%
	Opposite : all	27 : 36	17 : 26
	fff / ff	24*** / 3	14*** / 3

Third Experiment - Model inspection and negation

In the last experiment we analyze the inspection phase for indeterminate and negated problems. We assume that if participants were asked to inspect and validate recently constructed mental models the difficulty for negated problems increase due to the higher cognitive effort (determinate < indeterminate < negated problems).

Participants, Materials, Procedure and Design. Sixteen students of the University of Freiburg (age in years: M = 24.3, SD = 2.4) took part in this experiment. Two participants were excluded due to the low accuracy rate (≤ 50%) in determinate problems. We conducted a computer experiment in order to measure reaction time and accuracy as well as reading time for given premises. The experiment contained 24 problems, 12 one-dimensional and 12 two-dimensional (see Tab. 5).

Table 5: Shows the material for the problems. The cursive part indicates the three different types of the models: negated (3a), indeterminate (3b), and determinate (3c). Half of the problems had the relation "under" (I) in the fourth premise, the other "over" (II). Every type of problem was presented twice.

		Problem	***Expected PMM***
(a)	1	A is over B.	(I) A E B C D
	2	B is to the left of C.	
	3a	*D is not to the left of B.*	
	3b	*D is to the right of B.*	(II) A B C D E
	3c	*C is to the left of D.*	
	4	D is under E. (I) / D is over E. (II)	
(b)	1	A is over B.	(I) E A D C B
	2	B is to the right of C.	
	3a	*D is not to the right of B.*	
	3b	*D is to the left of B.*	(II) A D C B E
	3c	*C is to the right of D.*	
	4	D is under E. (I) / D is over E. (II)	

All four premises were given on the computer screen at once. After pressing a key the premises disappeared and a statement was presented. One object of the statement was taken from the third premise and the other object from another premise. This guaranteed that the participant had to prove the model containing the negated relation and had to infer an implicit relation between two objects. The relation in the statement was always missing so that the participant had to fill in the correct answer, or, in case of indetermination, a relation that seemed the most possible. There were four possible relations for an answer: left, right, over and under.

Results. We found no differences in the overall premise reading times between determinate, indeterminate, and negated problems. The different term arrangements (introduced through the fourth premise with "under" or "over") and dimension were not different likewise.

Again, we found a strong preference for PMM (indeterminate/negated: alternative mental model, AMM = 24%; PMM = 76%; Binomial-Test $p \leq 0.001$). The accuracy of the answers decreases significantly from determinate to indeterminate to negated problems (Page-L Test N = 14, k = 3, L = 178, $p \leq 0.05$).

General Discussion

Without negated relations relational reasoning seems to be inherently incomplete. But how do humans interpret and reason with negated relations? A formal analysis showed that there are at least three possible interpretations. First of all, we are able to show that a negated expression like "A is not to the left of B" is interpreted by the opposite relation "to the right of". Even, if the participants had the chance to use correctly "over" or "under" they tend to maintain the dimension (horizontal if the negated relation is horizontal etc.). Especially the independence of interpreting relations by using/not using a grid is remarkable. It is a justification for using grid structures as a help in both designing experiments and modeling relational reasoning computationally (SRM, Ragni et al., 2005) by using a grid. We found that the reaction time for the negation problems with or without underlying grid for "not over" was significantly longer than for "not right". We assume that this depends on the dimension the participants had to reason about. It seems that it is more common for reasoners to handle horizontal tasks. Finally, negation is a form of reasoning with indeterminacy, but reasoning is more complex and leads to more errors than reasoning with determinate descriptions. The accuracy of the answers decreased from indeterminate positive relations to negated relations. Problems with negated premises offers a greater variety of consistent models, but as well as in the classical case (Ragni et al., 2006) humans tend to construct preferred mental models to reduce complexity. Predictions about how the negated relation is interpreted, as well as the construction of a preferred mental model could be confirmed by all experiments. In other words, there are definitively preferred mental models in reasoning with negated assertions, and in indeterminate cases the participants constructed the preferred models by using the fff-principle (Ragni, et al., 2006).

Some previous research has covered the linguistic processing and comprehension. Kaup and colleagues (2006) showed that the processing of matching sentences and pictures are easier if the sentence and the picture correspond. Hasson and Glucksberg (2005) examined the question if negated information entails affirmation. They were able to show that negated metaphors are most likely represented as affirmation. However, in spatial reasoning multiple model cases are possible. Therefore, the negation of a spatial relation is not necessarily the opposite relation. Additionally, the information about other possible models has to be stored. In this case it seems reasonable to adapt an approach of Vandierendonck, Dierckx, and De Vooght (2004) for positive indeterminate model cases: to represent the alternatives by annotations at the object that is related with negation in the initial premises.

Our investigation can be contrasted to the work of Gapp (1995) who investigated the question what kind of configuration of two objects could still be described by a relation. Participants had to rate configurations on a scale. As a result he showed that participants accepted for instance the relation "to the right of" in-between the angles ± 22.5°. Contrary, we did not offer a continuous scale, because humans have to build a mental model and therefore they use a unique interpretation.

Negation plays an important role in representation and reasoning. Since negation makes reasoning difficult, even more than ambiguous descriptions, humans tend to construct preferred mental models. Future work should cover aspects of how participants find or neglect counter-examples and how three-dimensional reference frames are being processed.

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