

# Sources of Difficulty in Imagining Cross Sections of 3D Objects

**Cheryl A. Cohen (c\_cohen@psych.ucsb.edu)**

Department of Psychology, University of California, Santa Barbara  
Santa Barbara, CA 93107

**Mary Hegarty (hegarty@psych.ucsb.edu)**

Department of Psychology, University of California, Santa Barbara  
Santa Barbara, CA 93107

## Abstract

A novel 30-item multiple choice psychometric test was developed to measure individual differences in a spatial visualization task that involves identifying the cross section that results from the intersection of a cutting plane and a geometric solid. An initial study with 59 participants established the internal reliability and external validity of the test. Items in the test varied in complexity of the geometric solid and orientation of the cutting plane. Cutting plane affected performance such that people made more errors when the cutting plane was oblique to the main axes of the object than when it was orthogonal. Effects of cutting plane also interacted with the type of geometric structure. Low-spatial participants had consistently poorer performance. Analyses of their errors suggested that sources of difficulty in this task included failure to shift mental perspective from the current viewpoint to imagine the cutting plane from another orientation and failure to decompose the complex solids into geometric primitives.

**Keywords:** individual differences, spatial ability; strategies

## Introduction

Spatial visualization ability can be defined as the cognitive ability to understand, mentally encode and manipulate three-dimensional visuo-spatial forms. Component processes of spatial visualization include encoding a visuo-spatial stimulus, constructing a visual-spatial image from perceptual input, mentally rotating an image, switching one's view perspective, and comparing a visual stimulus to an image in working memory. (Carroll, 1993; Hegarty & Waller, 2005).

Some spatial visualization tasks involve relating two-dimensional representations and three-dimensional representations. One such task is inferring a cross section, a two-dimensional slice of a three-dimensional object. The ability to infer and interpret the spatial properties of cross sections is essential in many disciplines of science, and in science education. Researchers in biology education have determined that ability to form mental representations of sectional shapes of anatomical structures is a required skill for biology students in both for high school (Russell-Gebbett, 1985) and university (Rochford, 1985) and Lord (1985) developed a successful spatial visualization training program to teach high school biology students how to visualize cross sections of geometric figures. Kali & Orion

(1996) coined the term "visual penetrative ability," to describe ability to predict the internal structure of a geological formation, a required skill in structural geology. The ability to mentally represent cross sections is also intrinsic to success in engineering drawing (Gerson, Sorby, Wysocki & Baartmans, 2001; Hsi, Linn & Bell, 1997). In medicine, the ability to infer and comprehend cross sections is central to learning anatomy and using medical images, such as x-ray and magnetic resonance images (Hegarty, Keehner, Cohen, Montello & Lippa, 2007).

In previous research, we have found that ability to infer and draw a cross-section of an anatomy-like object is correlated with spatial ability (Cohen, 2005; Cohen & Hegarty, in press). Given the well-documented individual differences in spatial visualization skills, and the relevance of that ability to interpreting cross sections, questions arise about the extent and nature of individual differences in this ability. A novel 30-item multiple choice measure (the Santa Barbara Solids Test) was developed to examine sources of difficulty in inferring cross sections. The criterion task was to identify from four answer choices the correct cross section of a 3D geometric figure.

To capture specific aspects of performance that may result from variation in spatial visualization ability, items in the Santa Barbara Solids Test vary along two hypothesized difficulty dimensions: geometric structure and orientation of cutting plane. There are three levels of complexity of geometric structure: Simple, Joined, and Embedded figures. Simple figures are primitive geometric solids: cones, cubes, cylinders, prisms, or pyramids. Joined figures consist of two simple solids attached at their edges. Embedded figures are composed of one Simple solid enmeshed inside another (Figure 1). The use of primitive geometric solids at the lowest level of proposed difficulty is motivated by research which holds that most elementary recognizable three-dimensional forms are primitive solids. (Biederman, 1987; Pani, Jeffries, Shippey & Schwartz, 1997). We assume that it takes more working memory to encode and transform complex objects (joined and embedded) than simple objects. Previous research has shown that spatial working memory is one source of individual differences in spatial visualization ability (Miyake, Rettinger, Friedman, Shah & Hegarty,

2001) so we expected that items involving these complex objects would be more difficult.

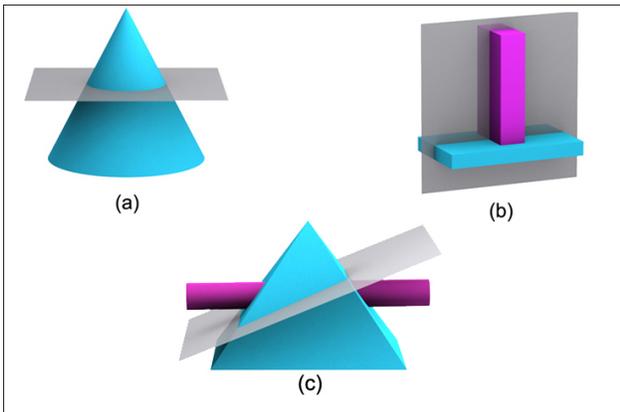


Figure 1. a) Single Orthogonal; b) Joined Orthogonal; and c) Embedded Oblique figures from the Santa Barbara Solids Test

The second proposed difficulty factor is the orientation of the cutting plane of the test figure. Mental transformations of objects with axes oblique to the environmental frame of reference are more difficult to perform than mental transformations of objects whose main axes are orthogonal to the environment. (Rock, 1973; Pani et al., 1977). Thus, the test incorporates two cutting plane orientations: orthogonal (horizontal or vertical) and oblique to the main vertical axis of the test figure.

Figure 1 shows examples of each level of geometric structure and each type of cutting plane. Figure 1a is a Simple figure with an Orthogonal (horizontal) cutting plane. Figure 1b is a Joined figure with an Orthogonal (vertical) cutting plane; Figure 1c is an Embedded figure with Oblique cutting plane.

Each test item shows a criterion figure and four answer choices, as shown in Figure 2. In addition to the correct answer (Figure 2c), each item has three types of distracters. Egocentric distracters (Figure 2d) represent a shape participants might imagine if they failed to change their view perspective relative to the cutting plane of the criterion figure. Combination distracters (Figure 2b) merge two possible sections of the test figure into a hybrid shape. Alternate distracters (Figure 2a) show another possible slice of the test figure.

We predicted that high spatial participants would outperform low spatial participants across all levels of geometric structure and both orientations of cutting plane. Our second prediction was that items cut on oblique axes would be more difficult for all participants than items cut on orthogonal axes. Finally, we predicted that Embedded figures would be more difficult than Joined figures, which would, in turn, be more difficult than Simple figures.

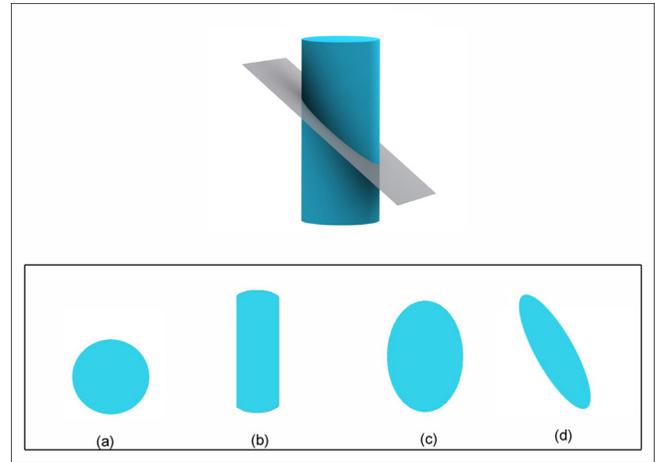


Figure 2: Sample item from the Santa Barbara Solids Test

## Method

### Participants

Sixty participants were recruited from the Subject Pool of the Department of Psychology, University of California, Santa Barbara, and received course credit for their participation. One participant withdrew due to illness.

### Materials

**Spatial ability tests.** Participants completed the Vandenberg Mental Rotation Test (Vandenberg & Kuse, 1978) (MRT) and a modified version of Guay's (1976) Visualization of Views Test (Eliot & Smith, 1983). In the Vandenberg Mental Rotation Test, participants view a depiction of a three-dimensional target figure and four test figures. Their task is to determine which test figures are rotations of the target figure, as quickly and accurately as possible. The maximum score is 80. The Visualization of Views Test (VV) measures the participant's ability to visualize an unfamiliar three-dimensional object from an imagined perspective. The criterion figure is a line drawing of a transparent cube with a small block floating in its center (Figure 3). The participant must identify the corner of the cube from which a given alternate view of the small block would be visible. The maximum score is 24.

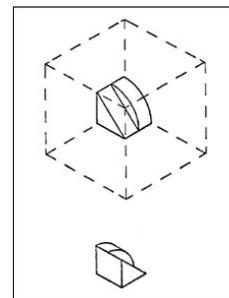


Figure 3. A sample item from the Visualization of Views Test

**Santa Barbara Solids Test.** The Santa Barbara Solids Test consists of 30 multiple choice items. Three levels of geometric structure and two types of cutting planes are distributed evenly across the 30 items: one-third of the figures are Simple solids, one-third are Joined solids, and one-third are Embedded solids. Due to experimenter error, the correct answer for one embedded oblique item was omitted, so that the score for that category was based on the remaining 4 embedded oblique items. Joined and Embedded test figures are composed of two solids of different colors. (Figure 1); answer choices for Joined and Embedded items are similarly colored. Fifteen of the test items are bisected by an Orthogonal cutting plane, and fifteen by an Oblique cutting plane. Test figures and answer choices were created in 3D Studio Max, PhotoShop, and Illustrator software. The test was printed in color on 8 ½ x 11 paper, with two test items per page.

## Procedure

Participants were tested in four groups of 15. The experimenter asked the participants to read the instructions silently as she read them aloud to the group.

## Test instructions

Four pages of written instructions, including a sample problem, precede the 30 test items. The instructions define the term ‘cross section,’ and illustrate the types of geometric structure and cutting plane orientations in the test. The instructions also direct participants to imagine how the cross section will appear from a mental viewing perspective that is perpendicular to the cutting plane of each test item. This direction is illustrated by a cartoon figure that adjusts the orientation of his head so that his gaze is perpendicular to horizontal, vertical, and oblique cutting planes. Furthermore, the experimenter demonstrated the process of changing view perspective with a Styrofoam egg-shaped prop that was sliced to reveal a cross section by tilting her head so that it was perpendicular to the cross-sectional surface of the prop. The experimenter then directed participants to complete the sample problem, verified the correct answer, and instructed participants to proceed at their own pace on the untimed test. After all participants had finished the test, the experimenter administered the Vandenberg Mental Rotation (Vandenberg & Kuse, 1978) and the Visualization of Views Test (Eliot & Macfarlane Smith, 1983).

## Results

**Internal consistency.** Cronbach’s Alpha, an estimate of internal consistency, was computed for the 29 items included in the analysis. The coefficient alpha was .86, indicating satisfactory reliability.

**Descriptive statistics.** Mean score on the Vandenberg Mental Rotation Test was 30.37 (SD=18.69), and on the Visualization of Views Test was 9.00 (SD=6.30). Performance (proportion of items correct) on the Santa Barbara Solids Test is shown in Figure 3. Mean performance (proportion correct) across all 29 figures of the Santa Barbara Solids Test was .54 (SD=.22). Highest means were on Embedded Orthogonal items (M=.75, SD=.25), and lowest performance was on Embedded Oblique items (M=.36, SD=.25). Oblique problems were consistently more difficult than Orthogonal problems, across all three levels of geometric structure.

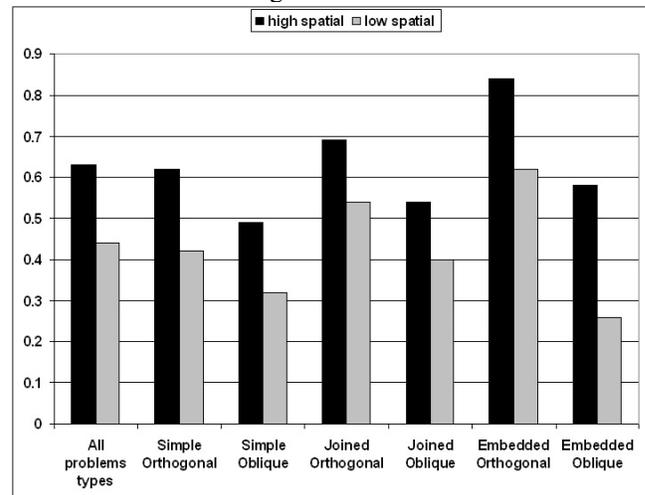


Figure 4. Mean proportion of items correct on Santa Barbara Solids Test, by type of problem.

**Effects of Object Complexity and Orientation of Cutting Plane.** A within subjects, repeated measures ANOVA was conducted to determine the contribution geometric structure and orientation of cutting plane to performance on the Santa Barbara Solids Test. There was a significant main effect of orientation of cutting plane,  $F(1, 5) = 81.68, p < .001$ , and a significant interaction between cutting plane and structure  $F(2, 57) = 21.12, p < .000$ . Across three types of structure, performance was higher on orthogonal than on oblique cutting planes. Within Orthogonal items, performance was highest on Embedded, followed by Joined and Simple figures. Within Oblique items, performance was highest on Joined, followed by Simple and Embedded figures (See Figure 3).

**Effects of Spatial Ability.** The Vandenberg Mental Rotation Test and the Visualization of Views Test were highly correlated with each other ( $r = .47^{**}$ ) and were combined into a single spatial score, which was computed by averaging the z-scores in the two measures. As can be seen in Table 1, the spatial score has substantial correlations with total score on the Santa Barbara Solids test and on each sub-category of items, providing evidence for its validity as a test of spatial visualization ability.

Table 1: Correlations of Spatial Ability Score with different categories of items on the Santa Barbara Solids

Type of Test Figure	Spatial Score
All figures (29 items)	.50**
Simple figures (10 items)	.38**
Joined figures (10 items)	.50**
Embedded figures (9 items)	.31*
Orthogonal figures (15 items)	.42**
Oblique figures (14 items)	.56**

\*  $p < .05$  (two-tailed)  
 \*\*  $p < .01$  (two-tailed)

Using the upper and lower thirds of the distribution of MRT scores<sup>1</sup> as the bounds for high and low spatial ability, we conducted an additional analysis of variance with spatial ability, orientation of cutting plane and object complexity as factors. The main effects of spatial ability and cutting plane were significant in this analysis, but spatial ability did not interact significantly with either cutting plane or object structure.

**Patterns of errors.** We analyzed the frequency of the four answer choices (correct, egocentric, combination, and alternate) across the 29 test item. Overall, participants chose the correct answer more than half the time (mean proportion = .54, SD = .21). The most frequently chosen distracter was the egocentric answer, which was chosen almost one-quarter of the time (Mean = .23, SD = .17). Combination (Mean = .12, SD = .13) and alternate (Mean = .10, SD = .13) answers were chosen less frequently and approximately equally.

### Discussion

In summary, we developed a novel psychometric test that measures ability to infer the shape of a cross section of a three-dimensional figure. Two of our three predictions were supported. First, test items with oblique cutting planes were more difficult than items with orthogonal cutting planes, which is consistent with previous research indicating that imagining rotations of objects around orthogonal axes is easier than imagining rotations around oblique axes (Pani et al., 1996; 1997). Second, high-spatial participants outperformed low-spatial participants across all categories of items on the test. There were significant positive correlations between the Santa Barbara Solids Test and the two psychometric spatial ability tests, suggesting that our test measures similar cognitive abilities, namely spatial visualization.

Our third prediction regarding the relative difficulty of the three types of figures (Simple, Joined, and Embedded) was not supported by our analysis. We predicted that Embedded and Joined figures would be more difficult than Simple

figures because they would require more working memory to transform. In fact, for low-spatial participants, mean performance was highest on Joined figures, followed by Simple and Embedded figures. There was a similar trend for high-spatial participants, who showed highest performance on Joined figures, followed by Embedded and Simple figures.

Analysis of the test materials suggests a possible explanation for the lack of support for our third prediction. Answer choices for Embedded and Joined problems show two colored geometric forms in specific spatial relationships to each other. Answer choices for Simple test items, on the other hand, are unitary, monochromatic geometric shapes, lacking internal detail and visible external relationships with other shapes (see Figure 2). A participant could use the additional visuo-spatial information inherent in Joined and Embedded answer choices to eliminate incorrect answers, a strategy that is not available on the Simple test items. An examination of Figure 5, an Embedded problem with an oblique cutting plane, illustrates this point. A participant could rule out answers (5c) and (5d) by reasoning that an oblique cross section of a cylinder is an oval, not a circle. The participant could eliminate answer (5b) by reasoning that the internal structure should be centered across the width of the external structure, rather than placed closer to the right edge of the rectangle.

This explanation is consistent with evidence that problem-solvers can use a variety of strategies on the same spatial task. For example, Schultz (1991) identified three strategies that participants used on a battery of spatial visualization tasks, namely mental rotation, perspective change, and the analytic strategy of identifying key features of the test object and subsequently noting that feature's presence, absence, or change. Participants could use the third strategy to their advantage in Joined and Embedded problems by comparing the relations between features of the two simple shapes that made up the complex solid. This strategy was not available for the simple shapes.

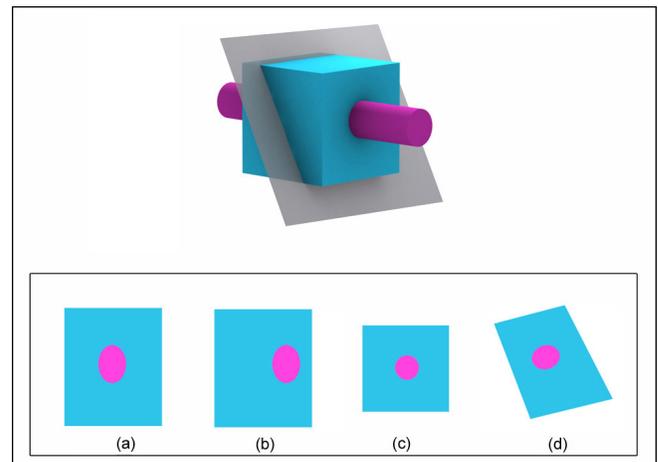


Figure 5. An Embedded problem from the Santa Barbara Solids Test

<sup>1</sup> Low spatial ability was defined as  $MRT \leq 20$ . High spatial ability was defined as  $MRT \geq 38$ .

By examining the frequency of answer choices in each problem, we identified two specific cognitive processes that caused particular difficulty for participants. First, participants may have failed to change from an egocentric representation of the stimulus figure to the 'head-on' perspective defined in the instructions. Second, participants may have failed to use an analytic strategy to choose their answers.

The fact that the egocentric distracter was the most common error, and that it was chosen on nearly one-quarter of the trials, suggests that changing view perspective relative to the orientation of each test figure was a frequent problem for participants. This interpretation is consistent with the spatial visualization literature, which holds that low-spatial individuals have difficulty transforming spatial representations through mental rotation or change in view perspective. It is supported by the significant correlations we observed between the Santa Barbara Solids Test and the Vandenberg Mental Rotation, and Visualization of Views Tests, which measure mental rotation ability, and perspective-taking ability, respectively. It is possible that egocentric errors resulted from participants' failure to understand a basic requirement of the test, that is, to imagine what the cross section would look like from a perspective perpendicular to the plane of the cross section. However, given the explicitness of the test instructions, including the experimenter's demonstration of changing view perspective to accommodate changes in cutting plane orientation, this explanation is unlikely and it is more likely that low-spatial participants were simply unable to change their view perspective.

The second proposed explanation for the pattern of errors was that participants may have failed to use an analytic strategy to choose their answer. Whereas the use of a feature matching strategy (cf. Schultz, 1991). may account for the improved performance on Embedded vs. Simple figures, the trends in Figure 3 indicate that this might have been used more by high-spatial than low-spatial ability participants. In addition, we observed a particularly diagnostic error for some of the complex items, in which the cross section of one of the simple solids making up the complex solid was not correct. For example, in an item showing an oblique section of a cylinder making up part of a complex object, a participant might chose an answer choice showing a circle rather than an ellipse as the cross section of the cylinder. This type of error indicates that the participant failed to use a possible strategy of decomposing the complex object into its primitives and choosing only answer in which the cross sections of the primitive objects were correct.

Finally, it is possible that differences between high- and low-spatial individuals on this test reflect differences in availability of stored memories about the results of sectioning objects in addition to differences in ability to transform spatial representations or in strategies. Thus high-spatial individuals might be more fluent in forming spatial memories of cross sections or in accessing these memories when required for a specific task. Mental images can be produced from recently acquired visual percepts, representations previously stored in long-term memory, and from verbal descriptions. Images can be also be generated by combining separately stored parts that have been held in long-term or short-term memory (Kosslyn, 1980; Kosslyn, Brunn, Cave, & Wallach 1984). It is possible that high-spatial participants have had more extensive experience with spatial forms or pay more attention to spatial forms on in everyday activities (e.g., noting the shape of cross sections when slicing vegetables), so that they have more relevant stored memories to draw on in a task like ours. Thus, a participant who had previously stored mental images of orthogonal or oblique cross sections of any of the geometric forms used in our test might have an advantage over a participant with limited experience forming and manipulating images of simple geometric solids. This possibility is consistent with recent research suggesting that training in spatial tasks leads to gains that are quite specific to the experience given, suggesting that what is acquired is specific memories rather than a more general ability to transform spatial forms (Pani, Chariker, Dawson, & Johnson, 2005).

**Future research.** The pattern of errors we observed in this study leads to hypotheses about the nature of the spatial visualization deficiencies among low-spatial individuals. One hypothesis is that, compared to high-spatial individuals, low-spatial individuals have difficulty changing their view orientation during spatial visualization tasks. Another hypothesis is that low-spatial individuals do not adopt spatial strategies such as feature matching and task decomposition. A third hypothesis is that, compared to high-spatial individuals, low spatial participants have less experience encoding, manipulating and retrieving spatial images.

We are using these hypotheses to design and test training interventions with the goal of improving the spatial visualization performance of low-spatial individuals. Future experiments will use the Santa Barbara Solids Test as a measure of performance before and after training to compare the benefits of different forms of spatial visualization training.

This research was supported by grant 0313237 from the National Science Foundation. The authors would like to thank Kevin Beer and Jerome Tietz for technical support.

## References

- Biederman, I. (1987). Recognition-by-Components: A Theory of Human Image Understanding. *Psychological Review*, 94(2), 115-147.
- Carroll, J.B. (1993). Human cognitive abilities: A survey of factor-analytic studies. Cambridge University Press Cambridge; New York.
- Cohen, C. (2005). The influence of spatial ability on the use of a dynamic interactive animation in a spatial problem solving task. In T. Barkowsky, C. Freksa, M. Hegarty & R. Lowe (Eds.). *Reasoning with mental and external diagrams. Computational modeling and spatial assistance*. Menlo Park, CA: American Association for Artificial Intelligence Press.
- Cohen, C.A. and Hegarty, M. (In press). Individual differences in use of an external visualization to perform an internal visualization task. *Applied Cognitive Psychology*.
- Eliot, J. & Smith, I. M. (1983). *An international directory of spatial tests*. Windsor Berks: Nfer-Nelson.
- Gerson, H., Sorby, S., Wysocki, A., & Baartmans, B. (2001). The development and assessment of multimedia software for improving 3-D spatial visualization skills. *Computer Applications in Engineering Education*, 9 (2) 105-113.
- Hegarty, M. & Waller, D. (2005) Individual differences in spatial abilities. In P. Shah, and A. Miyake (Eds.), *The Cambridge Handbook of Visuospatial Thinking*. (121-167). New York: Cambridge University Press.
- Hegarty, M., Keehner, M., Cohen, C., Montello, D. R., & Lippa, Y. (2007). The role of spatial cognition in medicine: Applications for selecting and training professionals. In G. Allen (Ed.) *Applied Spatial Cognition*. Mahwah, NJ: Lawrence Erlbaum Associates
- Hsi, S., Linn, M.C., & Bell, J.E. (1997). The role of spatial reasoning in engineering and the design of spatial instruction. *Journal of Engineering Education*. 151-158.
- Kosslyn, S. (1980). *Image and Mind*. Harvard University Press. Cambridge, MA.
- Kosslyn, S., Brunn, J., Cave, K., & Wallach, R. (1984). Individual differences in mental imagery ability: A computational analysis. *Cognition*, 18, 195-243.
- Lord, T. (1985). Enhancing the visuo-spatial aptitude of students. *Journal of Research in Science Teaching* 22, 395-405.
- Miyake, A., Rettinger, D. A., Friedman, N. P., Shah, P & Hegarty, M. (2001). Visuospatial working memory, executive functioning and spatial abilities. How are they related? *Journal of Experimental Psychology: General*, 130, 621-640.
- Pani, J.R., Chariker, J. H., Dawson, T. E. & Johnson, N. (2005). Acquiring new spatial intuitions: Learning to reason about rotations. *Cognitive Psychology*, 51 285-333.
- Pani, JR., Jeffries, J.A., Shippey, G.T., and Schwartz, K.J. (1996). Imagining projective transformations: Aligned orientations in spatial organization. *Cognitive Psychology*, 31(2), 125-167.
- Pani, JR., Zhou, H., Friend, S. M (1997). Perceiving and imagining Plato's solids: The generalized cylinder in spatial organization of 3D structures. *Visual Cognition*, 4(3), 225-264.
- Rochford, K. (1985). Spatial learning disabilities and underachievement among university anatomy students. *Medical Education*, 19, 13-26.
- Rock, I. (1973) *Orientation and form*. New York. Academic Press.
- Russell-Gebbett, J. (1985). Skills and strategies: Pupils' approaches to three-dimensional problems in biology. *Journal of Biological Education*, 19(4), 293-298.
- Schultz, K. (1991). The contribution of solution strategy to spatial performance. *Canadian Journal of Psychology*, 45(4), 474-491.
- Schultz (1991). The contribution of solution strategy to spatial performance. *Canadian Journal of Psychology* 45(4), 474-491.
- Vandenberg, S. & Kuse, A. (1978). Mental rotations: Group test of three-dimensional spatial visualization. *Perceptual and Motor Skills*, 47, 599-604.