Increased Availability of Arithmetic Facts Following Working Memory Processing

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Abstract

The current study demonstrated an increase in the availability of simple arithmetic facts following minimal working memory processing. Previous studies by Woltz and Was (2006, 2007) demonstrated faster response times and greater accuracy in processing semantically related but unattended category exemplars following a simple memory load and category-membership identification task. The current study assumed that simple arithmetic facts are stored in long-term memory in a highly interconnected network. Results demonstrated that simple working memory processing led to increased availability of long-term memory elements related to simple arithmetic facts. The results support the theoretical assertion that arithmetic facts are stored in a highly interconnected network. More important to the current line of inquiry, the results support theoretical models of working memory that assume complex cognitive processes rely on increased working memory capacity through activation of memory elements related to but not in the focus of attention.

Keywords: working memory; long-term working memory

Introduction

Recent conceptualizations of working memory (WM) (e.g., Cowan, 1995; Ericsson and Kintsch, 1995; Oberauer, 2002) have attempted to compensate for the fact that the limited capacity of the Baddeley and Hitch (1974; see also Baddeley, 1986) multiple components model of WM is challenged to explain complex cognitive processes that go beyond the model’s stated limits. Baddeley himself attempted to expand the capacity of the multiple components model by adding the episodic buffer to explain the interaction between WM and long-term memory (LTM) (Baddeley, 2000, 2001).

Two models of WM influential in the current research are those proposed by Cowan (1995; 1999) and Ericsson and Kintsch (1995). Cowan’s embedded processes model of WM describes working memory as consisting of 1) information or memory elements in the focus of attention, 2) memory elements not currently in the focus of attention, but in a highly active state, 3) and LTM elements. Attention is considered the portion of activated information or memory elements that is currently being processed. Attention processes are controlled by executive processes as well as involuntary processes controlled by an attention orienting system. Active but unattended information increases the capacity of WM by allowing for the rapid accessibility of these memory elements by the focus of attention.

Similarly, Ericsson and Kintsch (1995) proposed a model of WM that allows for effortless and rapid access of LTM elements. In the model, WM is bifurcated into specific WM components. The first, short-term working memory (ST-WM) is consistent with Baddeley’s WM model. The second, long-term working memory (LT-WM), describes the instant and effortless access to LTM elements through retrieval structures developed with expertise and practice. These retrieval structures are domain-specific but increase WM capacity in a manner similar to that of Cowan’s model.

As previously stated, Baddeley (2000) also introduced a new component to the multiple components model to account for empirical evidence that required the model to expand the limitations of WM capacity. Baddeley defined the episodic buffer as a limited capacity buffer with links between the phonological loop, the visual spatial sketchpad, the central executive, and LTM. The episodic buffer employs a multi-modal code to support the integration of LTM representations with WM components.

Recently, researchers utilizing experimental techniques have attempted to demonstrate the link between processing in WM and the increased availability of LTM memory elements. In particular, Woltz and Was (2006, 2007) demonstrated relatively long-lasting priming effects for related but unattended long-term memory (LTM) elements following minimal working memory processing. Following a series of experiments, Woltz and Was claimed the increased availability of LTM elements (ALTM) was due in part to a strengthening of specific memory procedures (e.g.,
the identification of a category and the comparison of category exemplars).

The basic ALTM task included a memory load of 4 words belonging to two semantic categories (e.g., *oak, poodle, elm, beagle*). Next, an instruction to remember the words from one of the two categories was presented (e.g., *Remember the TREES*), followed by a prompted recall of the two words (e.g., *What was the first word you were supposed to remember?*). Following recall of the relevant memory load items, participants completed a series of category comparison trials. Category comparison trials required participants to determine if two simultaneously presented words represented exemplars from the same or different categories. Analyses revealed that participants were faster and more accurate at completing positive trials of category comparisons of words related to the memory load items than neutral (unprimed) category comparisons. Woltz and Was (2006) attributed the increased availability of LTM elements in part to the strengthening of semantic and lexical memory representations, and in part to the strengthening of specific memory procedures. In other words, the ALTM priming effects could be partially explained by memory for mental operations performed in each trial. This procedural memory account attributes facilitation in primed category comparisons to the strengthening of category-specific membership identification operations.

This interpretation supports models of WM that include activated LTM elements as the contents of WM. Also, and perhaps more importantly, the data and interpretation support models such as the Ericsson and Kintsch (1995) model. For example, the interpretation of Woltz and Was (2006) that the strengthening of the category specific membership identification procedures might represent LT-WM. Specifically, practice at the membership identification procedure for specific categories could possibly lead to direct access to the categories practiced (identified in the memory load).

Previous investigations utilizing the ALTM tasks to measure the effects of WM processing on the increased availability of LTM elements through the strengthening of specific memory procedures have focused on word category comparisons (Liu and Fu, 2007; Woltz & Was, 2006, 2007). The current study was conducted to replicate the previous findings using a version of the ALTM task that did not rely on word meaning category comparisons as the measure of increased availability and word category exemplars as the experimental stimuli. In the current study, simple arithmetic facts served as the stimuli.

Several researchers have proposed that arithmetic facts are stored in a dense network of interconnections. Following a series of six experiments, Galfano, Rusconi, and Umiltà (2003) proposed that multiplication facts in LTM are likely arranged in a highly interconnected network which supports the spread of activation. Aschraft (1995) also argued that numbers in LTM are stored in an associative network.

LeFevre, Bisanz, and Mrkonjic (1988) demonstrated that the presentation of two numbers (e.g., 4 5) subsequently activated the sum of the two numbers (9). There is clearly some agreement that numbers and arithmetic facts are stored in an interconnected network. Just as apparent is the disagreement as to the nature of the connections. Germane to the current study, however, LeFevre et al. argued that their findings were due to automatic spread of activation from the presented nodes to the sum of those nodes in an associative network. Similarly, Niedeggen and Rosler (1999) demonstrated that spread of activation was limited to numbers that are table related to one of the operands in a multiplication problem.

The spread of activation hypothesis does not fully account for the ALTM effects in the previous research. However, models of the categorization of number facts are a necessary part of the theoretical account of these effects because the ALTM effects are proposed to include the strengthening of semantic memory representations.

Based on the findings regarding the mental representations of math facts, the current study operates under the theoretical assumption that simple arithmetic facts, defined as addition, subtraction multiplication, and division problems with a single digit result (e.g., 3 x 3 = 9, 40 / 8 = 5), are stored in LTM memory in a complex set of connections. It was hypothesized that this structure would allow for the similar outcomes as previous studies utilizing the word category ALTM task, using arithmetic facts as the stimuli. If arithmetic stimuli generate similar effects, this would be of two-fold importance. First, it would support cognitive models that propose that simple arithmetic facts are stored in an interconnected associative network. Second, and more importantly to recent models of WM, it would indicate that the strengthening of category identification operations is not limited to semantically and associatively related words but is also applicable to simple math facts.

**Method**

**Participants:** Fifty-eight undergraduate students at a large Midwestern University were given course credit for their participation. Forty-four of the participants were female. Age ranged from 19 to 43 years of age with a mean of 20.3.

**Apparatus** Participants performed the experimental task on IBM compatible microcomputers with SVGA monitors and standard keyboards. The experiment was programmed with E-Prime software (Schneider et al., 2002).

**Experimental Task.** The experimental task consisted of four components: memory load presentation, a selection instruction, arithmetic fact comparison frames, and a memory load recall (see Figure 1). There were 24 trials that each contained the four components in a consistent order.

Experimental stimuli consisted of arithmetic facts derived from the number families of numbers 1 through 9. We defined a number family as all of the addition, subtraction, and multiplication problems of two numbers less than 10.
that result in a given number, as well as division problems that contained one 2-digit number and a single digit number. An example is the number family for the number 6. Family members include 4+2, 9-3, 2x3, and 30/5. In all, there are 18 family members for the number 6.

The number families were organized in 24 sets, with each set containing three categories. The number family triplets were chosen such that there was minimal conceptual overlap between number families. For each participant, one number family from each set was assigned to be the focused number family in the memory load, one was assigned to be the ignored number family in the memory load, and the remaining number represented a number family not found in the memory load. Six versions of the experiment were created that represented a complete counterbalancing of triplet number family assignment to priming condition (focused, ignored, unprimed).

Each trial began with the statement Get ready to memorize answers to equations, which was displayed for 4 sec followed by a blank screen for 1 sec. This was followed by four problems presented on the display one at a time. Each problem set was preceded by an asterisk displayed for 750 msec in the location of the problems (center of screen) and then a blank screen for 1 sec. Each problem was displayed for 1500 msec followed by a blank screen for 500 msec. Order of the memory load problems was constrained such that the exemplars from a number family could not be contiguous (i.e., number families were alternated). Which number family came first was randomized.

There was a 2 sec delay after the final memory load item. This was followed by an instruction frame that directed the participant to remember the result of two of the four problems in the memory load. Although the instruction was to remember the result of the problems, it was always presented using the problems rather than the number result to be remembered (e.g., Remember the answer to the equations 3 x 3, 45 / 5). Participants could take as long as needed to identify and rehearse the result of the two problems in the memory load. They were instructed to press the space bar when ready to proceed.

The press of the space bar was followed by a 1 sec delay followed by the instruction, Get ready to COMPARE Equations... Rest your fingers on the L and D keys. This instruction was presented for 3 sec followed by a 2 sec blank screen to allow participants to prepare for the comparison frames. Each comparison frame began with two asterisks presented for 500 msec, one on top of the other in the location that the two problems would appear. The asterisks were followed by a blank screen for 750 msec, and then the two stimulus problems. The stimuli remained on the screen until the participant responded by pressing either the L or D key. A 1 sec interval separated the response and the asterisks preceding the subsequent comparison. During the entire set of comparison frames, the lower left portion of the display contained the reminder D=Different, and the lower right portion of the display contained L=Like. As described earlier, participants were instructed to decide if the two problems in each comparison resulted in the same answer (L response) or different answers (D response).

There was a total of eight category comparison frames in each trial. The first two frames were warm-ups that contained problems unrelated to the number families of the memory load and the unprimed category in the stimulus set. The remaining six frames were in random order for each participant. They consisted of two frames each, one positive match and one negative match, for the three categories of the stimulus set (focused, ignored, unprimed).

Following the category comparison frames, participants were required to report the number they were asked to remember, by entering the number on the key board and pressing enter.

Feedback was provided for the entire trial. Participants were informed of their accuracy for the recall frames and their average accuracy and response time for the problem comparison frames. Prior to the next trial, participants were reminded that they should try to obtain perfect accuracy on the recall frames and try to respond as quickly as possible without making errors on the problem comparison frames. The feedback and goal reminder frames were self-paced.
Procedure. Participants performed the experimental task in a single 1 hr session. They performed the experiment in groups of 1-4 subjects, with each participant seated in a computer carrel separated by sound-deadening panels. Every attempt was made to generate equal numbers of participants in each of the six counterbalanced versions of the experiment. However, due to unforeseen technical problems and the elimination of the outliers as described below, an unequal number of participants completed the six versions. Table 1 displays the number of participants in each version as well as the percent in each version.

Table 1. Number and percentage of participants completing each counterbalanced version of the experimental task

<table>
<thead>
<tr>
<th>Task Version</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>14.8</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
<td>20.4</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>18.5</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>14.8</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>14.8</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
<td>16.7</td>
</tr>
<tr>
<td>Total</td>
<td>54</td>
<td>100</td>
</tr>
</tbody>
</table>

Results

A total of four participants' data were eliminated from the analyses. Two of the participants were eliminated because their overall accuracy for the problem comparison trials was 50% indicating not better-than-chance performance. Two other participants were eliminated because overall mean reaction times were beyond three standard deviations above the mean. Elimination of these four participants left 54 participants for the final analyses.

Recall of the focus number family appeared to be at a substantially high value ($M = .98$, $Minh = .93$, $SD = .17$). This result would imply that overall participants were capable of making at least one of the calculations required for the focus number family and maintaining that number in WM while completing the problem comparisons. However, there was a good deal of skewness (-3.30; std. error of skewness = .325), indicating that although the majority of participants successfully completed the recall task, there were a number who had particular difficulty, while others were well below the average. Response time was reasonable but did display a somewhat large variance as well ($M=2786.04$ msec, $SD = 979.87$).

Accuracy for positive match and negative match comparison trial conditions was relatively high, $M = .918$, $SD = .07$ and $M = .915$, $SD = .07$ respectively. Mean latency (reaction time) in msec for problem comparison trials was slightly longer for positive match trials, ($M =3334$, $SD = 1203$), than negative match trials ($M =3292$, $SD = 1142$). However, the difference was not significant, $t(53) = .77$, $p = .47$. The Pearson correlation between mean accuracy and reaction time over all comparison trials revealed a slight speed accuracy trade-off in the current study, $[r = .36, p < .05]$. On average, as participants' reaction time increased so did their accuracy.

In previous studies using the ALTM task design, Woltz and Was (2006, 2007) found priming effects for both accuracy and latency, and combined the measures to create a speed score. However, others utilizing the ALTM measures found priming effects only in latency (e.g., Liu & Fu, 2007). Therefore, in the current study latency and accuracy were analyzed separately for priming effects. Mean error rates and latencies for the three categories of the stimulus sets are presented in Table 2.

Table 2. Mean Proportion Correct (in Percentages) and Response Latencies (in msec) for Problem Comparisons by Match Type and Trial Condition (With Standard Deviations)

<table>
<thead>
<tr>
<th></th>
<th>Positive Match</th>
<th>Negative Match</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M$</td>
<td>$SD$</td>
</tr>
<tr>
<td>Comparison Type</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unprimed</td>
<td>91.3</td>
<td>8.6</td>
</tr>
<tr>
<td>Ignored category</td>
<td>92.4</td>
<td>8.9</td>
</tr>
<tr>
<td>Focus category</td>
<td>91.8</td>
<td>8.6</td>
</tr>
</tbody>
</table>

Hypothesized priming effects for the ignored and focused category stimuli were analyzed using a repeated measures ANOVA with two orthogonal contrasts. Recall accuracy was included as a covariate. The inclusion of recall accuracy was warranted by the negative skew of the distribution of recall accuracy amongst participants. We assumed that if participants were inaccurately calculating the result of the arithmetic problems, it was likely they had activated the incorrect number family. Alpha was set at .05 for all analyses. The first contrast compared the average of the ignored and focused comparisons with the unprimed comparisons. This first contrast was a test of overall priming effects for having processed the number families in WM. The second contrast compared the focused and ignored number family comparisons in order to test whether the degree of attention dedicated to number family exemplars in WM affected the degree of residual ALTM.

We first discuss the analysis of reaction time. Statistical analysis of the problem comparison trials was completed separately for the positively and negatively matched trials. Previous research and theory support the conclusion that priming is minimal or non-existent for negative match comparison trials (e.g., Woltz, 1990, Woltz and Was, 2006).
As expected, there was no overall priming effect for negative match comparisons when controlling for recall accuracy.

Figure 2 presents the reaction time data for category comparisons. For positive match comparisons, controlling for recall accuracy participants responded in less time to focus and ignore comparisons than to unprimed comparisons \([F (1, 52) = 7.13, MS_e = 429,307.74, \text{ partial } \eta^2 = .12, p < .01]\). This result demonstrates a priming effect for number families that were previously encountered compared to problems related to number families not previously processed. It was expected that the focused category problem comparisons would be responded to faster than the ignored category. However, the difference between ignored and focused problem comparisons occurred opposite to the hypothesized direction; although, it did not reach significance \([F (1, 52) = 1.66, MS_e = 67040.53, \text{ partial } \eta^2 = .03, p > .05]\). Finally, because the relationship between ignored and focused category comparison was not significant, yet in the opposite of the expected direction, an extra comparison was made to determine if the focused problem comparisons were significantly faster than the unprimed comparisons. Participants’ reaction time was longer for the unprimed compared to focused category comparison trials \([F (1,52) = 4.17, MS_e = 268162.14, \text{ partial } \eta^2 = .07, p < .05]\).

![Figure 2](image)

**Figure 2.** Category comparison reaction time. Error bars reflect standard errors

In the analysis of accuracy data, there was no overall priming effect when controlling for recall accuracy, for negative match trials.

For positive match comparisons, controlling for recall accuracy, participants responded with greater accuracy to focused and ignored comparisons than to unprimed comparisons. Although this difference was not significant, it did approach significance \([F (1, 52) = 3.74, MS_e = .006, \text{ partial } \eta^2 = .07, p = .59]\). Participants responded with greater accuracy to ignored problem comparisons than to unprimed problem comparison \([F (1, 52) = 4.46, MS_e = .011, \text{ partial } \eta^2 = .08, p < .05]\). The difference between ignored and focused category comparisons was not significant.

**Discussion**

Results of the statistical analysis support the hypothesis that ALTM effects would occur in a similar pattern when using simple arithmetic facts, as effects in previous studies using word categories and exemplars as the experimental stimulus. These findings support theoretical models of cognitive arithmetic that conceptualize simple mathematical facts as being stored in a highly interconnected network. Are intention was not to describe the nature of the relationships in the network, but instead to demonstrate that specific memory procedures might be strengthened in order to increase access to the network.

The findings are similar to previous findings using the ALTM task. We believe that the current data further support the conclusion that minimal processing of information in WM not only leads to priming that can be partially attributed to a spread of activation, but may also lead to the strengthening of specific memory procedures. Recall that the problem category comparisons did not include any of the problems from the memory load, but instead utilized problems that resulted in the same number. Therefore, perceptual or repetition priming is not the source of the increased access to the number families. In the current study, identifying the result of at least one of the focused category problems in the memory may activate the number family (operationalized as simple arithmetic problems resulting in the same single digit number) associated with that result. And, the processes of identifying whether or not two problems have the same result may strengthen the specific memory procedure of identifying the result of a problem. This is similar to the explanation of Woltz and Was (2006) that the results of the ALTM task could be due to the strengthening of the category-specific memory procedures.

There are some caveats, that must be considered when discussing the current findings. First, the magnitude of priming effects was not of the same magnitude as those discussed by Woltz and Was (2006). Secondly, as discussed previously, accuracy and latency were not combined in the current study. This was in part because of the large variance in the accuracy of problem comparison trials. Overall percent correct for problem comparison trials was 90%. However, the standard deviation was 10%. Although at first glance this may not seem like a large spread, it does indicate that a substantial proportion of participants found this task to be challenging. In fact, 33% of the participants scored below 90% correct over all comparison types.

There are also alternative explanations for the reduced reaction time for ignored and focused problem comparison as compared to unprimed problem comparisons. It is possible that a simple spread of activation account will
explain the current findings. By our own definition of a number family, committing problems to memory and calculating results might activate an entire number family. However, as in previous studies, there were a minimum of two intervening comparison trials before a primed comparison trial was encountered. However, even though traditional priming effects have been demonstrated to be very short lived (a few seconds at the longest), more persistent priming effects have been demonstrated following complex operations and multiple priming events (Becker, Moscovitch, Behrmann, & Joordens, 1997; Hughes & Whittlesea, 2003; Joordens & Becker, 1007). It is possible that the calculation of simple problems requires more complex processing than simple category or number family identification leading to more persistent priming effects.

The finding of focused problem comparisons not differing from ignored comparison is contrary to the findings of Woltz and Was (2006). This might indicate that participants were calculating the result of the ignored problems as well. Woltz and Was (2006, 2007; see also Liu and Fu, 2007) found that even minimal processing led to increased availability, but that the degree of processing related to the magnitude of the priming effect. Participants calculating the result of the ignored problem might explain this particular finding of the current study.

Finally, the number of subjects in each of the counterbalanced versions of the task was not equal. This could possibly have led to a stimulus effect. In other words, it is possible that the outcomes were biased by a set (or sets) of stimuli that were more likely to produce the hypothesized effects. We feel this is unlikely, but certainly needs to be considered.

In conclusion, it is our contention that the results of the current study support models of WM that call for an increased availability of LTM memory elements to support complex cognitive processes. Many of these models propose that quick and accurate access to LTM is supported by task and perhaps item specific memory procedures that are acquired through experience and practice. The arithmetic version of the ALTM task supports these hypotheses.

References


