

Concrete and Imagined Simulation of Situation Models Enhances Transfer of Solutions to Structurally Different Algebra Word Problems

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Abstract

A number of studies on analogical transfer to algebra word problems have demonstrated that adapting a known solution to nonisomorphic problems of the same type is challenging, and that most instructional aids do not alleviate this difficulty. We designed a non-interactive intervention intended to encourage students to formulate situation models for base and target, and to ground their equations in these representations. One of our experimental groups had to simulate the situation models via manipulating physical objects. The other group had to perform internal simulations. Both conditions outperformed a control group not required to run simulations, yielding comparable advantage. Situation model elaboration proved more effective when targets posed more difficulty of equational assimilation. The implications of these findings for the design of instructional interventions are discussed.

Keywords: transfer; solving problem; algebra word problems; analogy

Introduction

Solving a problem by analogy entails transferring a solution from a known problem (the *base problem*: BP) onto a new problem whose solution is unknown (*target problem*: TP). In learning environments, the solution to a BP serves as scaffolding for the application of a general method until the student has gained the fluidity required to apply this general methods directly (Holyoak, 2005; Koedinger & Nathan, 2004). In a series of studies of near transfer in task-completion problems, Reed, Dempster and Ettinger (1985) investigated to what extent college students can use a worked out example—which could be consulted at any time—to solve TPs that maintain structural differences with it (i.e., when the base equation needs to be adapted to fit the TP). Whereas in the absence of structural differences 70% of the students successfully transferred the base solution, in their presence the rate of success dropped to 12%. In subsequent studies, Reed and colleagues developed a series of instructional aids to help students assimilate these variations into the base equation (e.g., construction of tables for relating quantities to variables, Reed & Ettinger, 1987; provision of explicit rules for incorporating the variations,

Reed & Bolstad, 1991; and instruction on unit cancellation, Reed 2006). In general, the successive instructional aids aimed at alleviating this difficulty proved unsuccessful (see Reed, 1999 for a revision).

A possible account of transfer difficulties

According to Nathan, Kintsch and Young (1992) and Kintsch (1998), solving algebra word problems ideally implies forming a text base of propositions that capture the meaning of the problem story. Together with a set of inferences drawn from general knowledge, this text base allows for the construction of a situation model that represents the situation depicted in the problem. In the situation model, objects and their interactions are represented qualitatively, without consideration of the exact magnitudes stated in the text. After a situation model has been built, students apply several subschemas to construct a quantitative problem model that captures the algebraic structure of the problem, and ultimately leads to an equation that affords calculation of the unknowns. Consider the following problem:

Problem: Fred can paint a wall in 8 hours, while Bob can paint it in 5 hours. Fred starts painting the wall one hour before Bob, and then they keep painting it together.

If painting started at 12, at what time will it be finished?

The situation model should display a painter that starts painting a wall on his own before a second painter joins. This situation model should also picture them painting together, albeit at different rates, until they jointly finish the task. Construction of the problem model is hypothesized to unfold in the following manner: Once the pertinence of the “ $r \times t = w$ ” subschema is acknowledged (r = rate, t = time, w = work), its variables should be substituted first with the values for Bob, and next with the values for Fred, the latter requiring the inference that Freds’ t equals Bobs’ t plus 1 h, thus inviting the “ $t_2 = t_1 + 1 \text{ h}$ ” subschema. Construction of the problem model will also demand the inference that once the task gets completed, the sum of the parts advanced by both painters will equal the whole wall, leading to the

application of the " $T_1 + T_2 = T$ " subschema. Nathan et al. (1992) termed these two key inferences *support relations*.

Nathan et al. (1992), as well as Greeno (1989), consider that novices, as opposite to experts, don't ground equations in situation models, thus failing to notice, for instance, when a problem's solution—albeit mathematically correct—leads to a semantically absurd situation (Paige & Simon, 1966). With these ideas in mind, Nathan et al. (1992) developed ANIMATE, a learning environment that provides a set of subschemas intended to help students build equations for encounter problems. Once an equation has been built, ANIMATE runs a schematic—but quantitatively faithful—simulation of a situation, as determined by such equation. This way the students can check the simulations derived from their problem model against the situation model they had built on their own, and ultimately modify their problem models when these representations diverge. Nathan et al. (1992) compared a training using ANIMATE against two control conditions: one trained in the construction of problem models without situational support and the last one receiving neither kind of support. Students trained with ANIMATE outperformed both control conditions in a series of tasks including formulation of support relations, building equations for problems, fixing wrong equations, and even inventing word problems to fit abstract equations.

The main objective of the present study was to investigate whether a stimulation to ground equations in situational models—a key feature in the ANIMATE environment, but absent in most interventions developed by Reed and cols.—would also aid transfer to TPs with structural variations. In the present experiment, which followed a traditional near transfer paradigm, both the learning context and the superficial content of the problems were kept constant.

The second objective of our experiment was to determine whether a prompting to construct situation models was more effective when carried out with physical objects as compared to an internal simulation. Across domains as diverse as text comprehension (Glenberg, Gutierrez, Levin, Japuntich & Kaschak, 2004), memory for instructions (Engelkamp, 1999) and metaphor comprehension (Wilson & Gibbs, 2007), psysical and internal simulations yielded comparable beneficial effects. Following the lack of difference observed in the above studies, we predicted that concrete and imagined simulations of situation models would equally promote transfer to target problems introducing structural variations.

The third objective in this study was to investigate if the type of situation model stimulation interacts with the degree of difficulty posed by a given variation. In the following section we present the BP and the three TPs used in this study, and flesh out a theoretical analysis of their relative difficulty of equational assimilation. We hypothesized that a stimulation to control the problem model from situation model would be more beneficial in those cases in which the structural differences introduced by a TP posed greater difficulties of equational assimilation.

Interaction between situation model simulation and the type of variations introduced by the TPs.

Below we present a BP and three TPs, each introducing a different type of structural variation:

BP: Peter can paint a wall in 10 hours, while John can paint that same wall in 15 hours. If they start painting the wall together at 12, at what time will it be finished?

TP with speed variation (TP_{speed}): Ned can paint a wall in 8 hours, while Louis takes twice as long to paint the same wall. If they start painting the wall together at 12, at what time will it be finished?

TP with work variation (TP_{work}): Bob can paint a wall in 20 hours, while Mark can paint that same wall in 12 hours. One third of the wall has been painted by other painters. If Bob and Mark start painting the remainder at 12, at what time will it be finished?

TP with time variation (TP_{time}): Fred can paint a wall in 8 hours, while Bob can paint that same wall in 12 hours. They mostly paint it together but, overall, Bob paints one more hour than Fred. If painting started at 12, at what time will it be finished?

The following model represents the BP, as well as their structural variations. In the model (see Figure 1) knowledge is represented as a propositional web of nodes (concepts) and predicates (attributes and relations). Concepts appear in rectangles, operations and relations between concepts appear in circles, while numerical values—or the procedures needed to obtain them—are represented with ovals. Vertical lines indicate permitted substitutions. While the highest level corresponds to the upper principle " $w_1 + w_2 = w_{total}$ ", the intermediate level corresponds to the variables r and t which, when multiplied, conform the left terms of such principle. The lower level refers to the known and unknown quantities and to the procedures needed to obtain them.

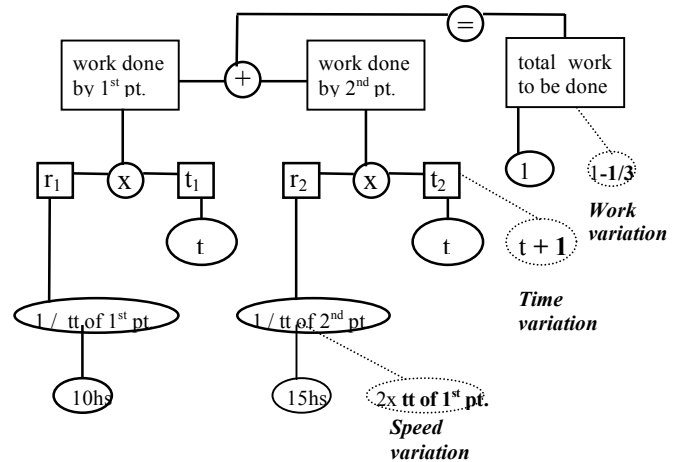


Figure 1. Representation of the equational structure of task-completion problems, according to the model proposed by Reed (1987). *Note.* tt: total time; pt: painter.

Across levels, dotted lines are used to denote the variations introduced by *TPwork*, *TPtime* and *TPspeed*, whose intrinsic difficulties of equational assimilation we now turn to analyse. In the following analysis, we will assume that the difficulty posed by a given variation is determined by the degree of comprehension of the equational structure that its assimilation demands.

The speed variation. In *TPspeed*, calculation of the speed of one of the painters only requires knowing the time it would take him to finish the task on his own, which comes from doubling the time taken by the other painter. Given that no knowledge of the equational structure is involved in this variation, we predicted that a simulation of situation models wouldn't result in an increased transfer performance.

The work variation. In *TPwork*, there are two different ways of assimilating this variation: 1) $r_1 \times t + r_2 \times t = 1 - 1/3$ and 2) $r_1 \times t + r_2 \times t + 1/3 = 1$. Either alternative only presupposes comprehension of the upper level of the equational structure (i.e., $w_1 + w_2 = w_{total}$). We thus predicted that the simulation of situation models would result in a moderate increase in transfer performance.

The time variation. In *TPtime* there are three ways to assimilate the structural variation in the problem's equation: 1) $r_1 \times t + r_2 \times t + r_2 \times 1 \text{ h} = 1$; 2) $r_1 \times t + r_2 \times t = 1 - r_2 \times 1 \text{ h}$ and 3) $r_1 \times t + r_2 \times (t + 1 \text{ h}) = 1$. To assimilate this variation the student will need to understand that the extra hour spent by one of the painters should result, at the upper level of the equational structure, in a third chunk of painted wall (e.g., " $w_1 + w_2 + w_3 = 1$ "). To derive this extra term it is necessary to multiply r_2 by 1 h, which also demands comprehension of the intermediate level (i.e., " $r_2 \times t_2 = w_2$ "). Given that all three alternatives demand comprehension of the upper and intermediate levels of the equational structure, we predicted that the simulation of situation models would result in higher beneficial transfer than to simulations in *TPwork*.

We ran a complementary study to verify that adaptation was more difficult for *TPtime* than for *TPwork* and more difficult for *TPwork* than for *TPspeed*. An independent group of eighteen 12th graders at Estación Limay High School (the same population as in the experiment reported here) received the BP and its solution (see Figure 2 below). They were asked to solve each of the TPs, based on the BP and its solution. They had to speak aloud their thoughts during the process, and their responses were tape-recorded. Data analysis was limited to trials in which students showed comprehension of the BP and its solution procedure, the TP and the differences between them (to assess to what extent a variation poses a challenge to the analogical subprocess of adaptation, it is necessary to control that the previous analogical subprocesses of representation building and mapping had been successfully performed). Transfer performance was 70% for *TPspeed*, 53% for *TPwork* and 17% for *TPtime*, thus confirming the theory-driven prediction about the transfer difficulty of the TPs.

Experiment

The present experiment compared transfer performance across three instructional conditions. While the Concrete Simulation Group (cSG) had to simulate situation models via manipulating physical objects, the Imagined Simulation Group (iSG) was asked to run mental simulations of situation models. The group with no Simulation (nSG) was not asked to run simulations of the situation models during any phase of the experiment. All groups passed through three distinct phases: pretest, instruction and transfer. The pretest was included to identify participants that could come up with a solution to the BP. During the instructional phase, participants were given an instruction on how to solve it. During the transfer phase all groups had to solve a TP identical to the BP—included to serve as reference of maximal transfer performance—and the three problems presented in the previous section. Both simulation groups were encouraged to: 1) simulate the situation model of the BP, 2) produce a quantitative situation model of the BP and connect it with equation presented for that problem, 3) simulate the situation model of each of the three TPs, and 4) compare all TPs with the BP in terms of their situation models. For every simulation that these groups had to perform, the nSG had to carry out a non-simulative task that was equivalent to it in terms of the reconsideration of problem information it produced.

Method

Participants and design. Sixty 12th year students at Estación Limay School in Rio Negro, Argentina, volunteered to participate in the experiment. They were randomly assigned to each of the three groups (20 to the cSG, 20 to the iSG and 20 to the nSG). In this 3 x 3 design, the independent variable type of simulation (concrete, imagined and no simulation) received between-subjects manipulation. The independent variable difficulty of equational assimilation (low, medium and high) received within-subjects manipulation.

Materials and Procedure. At pretest, participants were given 5 min to solve the BP. During the instructional phase, participants were handed an instruction similar to the one typically used by Reed and colleagues (see Figure 2 below) and were given 5 min to study it. Next they answered a series of oral and written questions that were intended—in the case of the simulation groups—to link equational terms to their respective situation model representations. During the transfer phase participants received the TPs in counterbalanced order, and were asked to apply the learned equation with the modifications they deemed necessary. Instruction material could be consulted during TPs solution. Even though they were not required to solve the unknown values, they had to state how such values—if calculated—should yield an answer to the problems (e.g., for *TPtime*, “we should solve the equation for t , then add 1 hour to that value and finally add the resulting time to 12 hours”). We'll first describe the procedure of the cSG and then present the

differences between cSG and the other groups. Upon completion of the pretest, participants were presented with a white rectangle vertically attached to its base and with two toy painters. They were asked to read aloud the BP at a very slow pace. After each sentence was completed, the experimenter asked them to simulate its content with the given materials. Finally, participants were asked to use both painters to represent the complete situation, from beginning to end. During the instructional phase participants were given 5 min to study the solution to the BP (see Figure 2 below):

Solution: this problem is a work problem in which two people work together to complete a task. The amount of task completed by each person is found multiplying his rate of work by the amount of time he works, as follows:

$$\text{Rate of work} \times \text{Time of work} = \text{part of work done}$$

Because Peter takes 10 h to paint the wall, he finishes 1/10 of the wall in 1 h. In t h he finishes $1/10 \times t$. John finishes 1/15 of the wall in 1 h. In t h he paints $1/15 \times t$. The following table summarizes this information:

Worker	Rate of work (part of task/h)	Time of work (h)	Work done (part of task)
Peter	1/10	t	$1/10 \times t$
John	1/15	t	$1/15 \times t$

If the task is finished, the sum of the fractional part finished by Peter and the fractional part finished by John must equal 1; $(1/10) \times t + (1/15) \times t = 1$

Solving for t yields the following: $(1/10 + 1/15) \times t = 1$ then $(3/30 + 2/30) \times t = 1$; Finally, $t = 30/5 \text{ h} = 6\text{h}$.
Answer: if they started at 12, then they finish at 6.

Figure 2. Instructions for solving task completion problems (adapted from Reed et al., 1985)

After receiving instruction on how to solve the BP, participants had to produce a quantitative situation model of the BP. They were given 20 small boxes, each one containing several magnetized stripes that represented decimal fractions of the wall (e.g., 1/2s, 1/3s... until 1/20s). They were asked to take advantage of these tools and of the known solution to BP in order to represent the situation unfolding on an hour-by-hour *fashion*. After completing this simulation, participants received a sheet of paper on which they had to answer several questions intended to promote a connection between the equation and its corresponding quantitative situation model. First, they were asked to show both in the wall and in the equation the painted part that corresponded to each painter (e.g., “1/10 x 6 h” for the first painter), telling the exact amount of wall completed by each painter. Second, they had to indicate the part of the equation in which the total work was represented (i.e., the “1” in the right term). During the transfer phase participants were asked to simulate a TP in the same way as was done with

the BP. Next, they were asked to simulate the BP once again, representing any difference they deemed relevant between the BP and such TP. Participants were given 6 min to solve each TP. This sequence was repeated for each of the three TPs. All simulations were videotaped.

Situation model simulations in the iSG were identical to that of the cSG, except for the fact that were carried out internally. Even though participants were presented with the set of materials used by the cSG—and were also encouraged to use them for representing the problems—these materials were removed from their sight before the mental simulations were run. They were asked to close their eyes before each simulation.

Concerning the nSG, the procedure was similar to that of the simulation groups except for the fact that each simulative task was replaced by a non-simulative task that was equivalent to the former in terms of the reconsideration of problem information it produced. For instance, when simulation groups had to run the qualitative simulation of the problems, the nSG had to reread the problem text; or when the simulation groups had to construct the quantitative situation models, the nSG had to indicate how many times each painter painted his hourly portion of the wall, as well as indicate the times at which they began and finished the task. Neither group received feedback while performing either simulative or non-simulative tasks. Interventions were exclusively oriented to foster comprehension of the base equation in terms of its quantitative situation model and to encourage simulation and comparison of base and target situation models.

Results

As all participants failed to solve BP at pretest, no participants were excluded from the data analysis. Solutions to TPs were scored as correct only when: 1) the participant correctly incorporated in the corresponding equation all the data directly or indirectly available in the problem text, such that the equation could be solved for the unknown value, and 2) the participant successfully stated how the solution of the equation should yield an answer to the problem.

Across TPs, transfer to problems with structural variations averaged 68%, which clearly surpasses the performance typically observed by Reed and colleagues with this kind of problems. Due to high rate of success in TPspeed (95%) our subsequent analysis of the advantages of the simulation of situation models on analogical transfer will be limited to performance in TPwork and TPtime. To assess the effects of the difficulty of equational assimilation (TPwork: medium, TPtime: high) and of the type of situation model simulation (cSG, iSG and nSG) on analogical transfer, we computed a 2 x 3 mixed analysis of variance (ANOVA) with repeated measures on the difficulty of equational assimilation. Main effects were observed for both the difficulty of equational assimilation $F(1, 60) = 31.933, MSE = .104, p < .0001$ and the type of situation model simulation, $F(2, 60) = 10.476, MSE = .222, p < .0001$. An interaction between both factors

was also found to be significant, $F(2, 60) = 3.433$, $MSE = .104$, $p < .05$.

Paired comparisons revealed that the cSG (83% of correct answers) outperformed the nSG (38%), $p < .001$ (Bonferroni adjustments). The iSG (75% correct) also outperformed the nSG, $p < .001$. These data thus confirmed our hypothesis that promoting situation model simulation aids transfer of a base solution to structurally different target problems. Bonferroni comparisons also revealed that performance in both simulation groups did not differ reliably, $p > .05$, thus confirming our prediction that situation model simulation via physical manipulation would not differ from internal simulation in facilitating transfer to target problems.

Our third hypothesis was that the advantage of situation model simulation in promoting transfer would be higher for problems that posed a greater challenge of equational assimilation. We performed two separate 2×2 ANOVAS with repeated measures on the factor difficulty of equational assimilation: one pitting nSG against iSG, and the other pitting nSG against cSG.

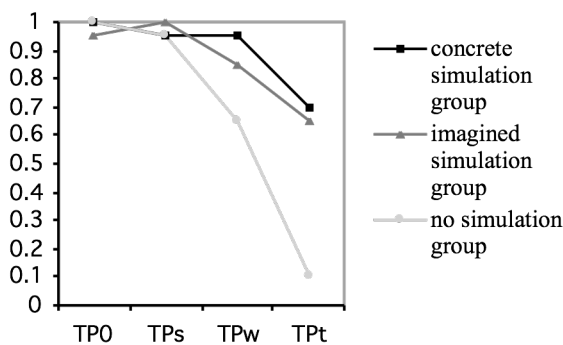


Figure 2. Proportion of correct solutions for the three simulation conditions of the experiment on each TP. Note: *0*: identical; *s*: speed; *w*: work; *t*: time.

The 2×2 ANOVA with type of situation model simulation (no simulation and imagined simulation) as between-subjects factor and difficulty of equational assimilation (medium and high) as within-subjects factor revealed a significant interaction between these factors, $F(1, 40) = 5.712$, $MSE = .107$, $p < .05$. Bonferroni adjustments showed that the iSG reliably outperformed the nSG in TP_{time} (65% vs. 10%), $p < .001$ but not in TP_{work} (85% vs. 65%), $p > .05$. Contrasts for paired samples revealed that performance in TP_{work} was higher than in TP_{time} both within iSG, $t(19) = 2.179$, $p < .05$, and within nSG, $t(19) = 4.819$, $p < .0001$. In a similar vein, the 2×2 ANOVA with type of simulation (no simulation and concrete simulation) as between-subjects factor and difficulty of equational assimilation (medium and high) as within-subjects factor revealed a strong tendency towards interaction between these factors, $F(1, 40) = 3.931$, $MSE = .114$, $p = .055$. Bonferroni comparisons showed that the cSG reliably outperformed the nSG both in TP_{time} (70% vs. 10%), $p < .001$, and in TP_{work} (95% vs. 65%), $p < .05$.

Contrasts for paired samples showed that performance on TP_{work} was higher than on TP_{time} both within cSG, $t(19) = 2.517$, $p < .05$, and within nSG, $t(19) = 4.819$, $p < .0001$. The observed interactions support the thesis that the advantage of situation model elaboration is higher for problems implying greater difficulty of equational assimilation. Analysis of the simulations carried out by the cSG revealed that in 95% of the trials participants correctly simulated the qualitative situation model of the TPs as well as their relevant differences with that of the BP. The quantitative simulation of the BP and its solution was carried out successfully in 90% of the trials.

Discussion

Authors like Greeno (1989), Nathan et al. (1992) and others have proposed that novices—as opposed to experts—fail at formulating correct solutions to algebra word problems partly because they neither elaborate situation models for problems, nor do they relate them to their equations. These authors tend to agree with the idea that, in most cases, this deficit is due to a lack of disposition originated by current instruction rather than to the lack of cognitive capabilities. We considered that if these assertions are right, then a simple stimulation to elaborate situation models would also aid transfer to problems demanding assimilation of structural differences—a task that proved immune to several interventions developed by Reed and colleagues. On the one hand, the situation models generated by the cSG showed that students can easily build them for the base and target problems and compare them in order to pinpoint their differences. On the other hand, the fact that both simulation groups outperformed a control group that did not receive any kind of prompting to simulate situation models demonstrated that analogical transfer to nonisomorphic target problems can be enhanced by encouraging elaboration of situation models. Importantly, the fact that performance in iSG was comparable to performance in cSG further suggests that transfer performance can be boosted by rather austere interventions, not even requiring the manipulation of concrete materials. It should be noted, however, that the mathematical background of our students was probably superior to that of the population typically evaluated by Reed and colleagues. It seems likely that a strong mathematical background is required to take advantage of a situational scaffolding such as the one provided in the present study. It should be noted that the situation models of the problems used in this study were rather simple. Perhaps with problems requiring more complex situation models (e.g., mixture problems) students will need assistance along the process of elaborating them.

A second objective of our experiment was to test the hypothesis that a physical simulation wouldn't differ from an internal simulation at promoting analogical transfer. Such prediction was derived from previous results obtained by Glenberg et al. (2004) on text comprehension, Engelkamp (1998) on memory for actions and Wilson and Gibbs (2007) on metaphor comprehension. The fact that transfer performance in the concrete simulation group was not reliably superior to performance in the imagined simulation

group thus confirmed such prediction. However, the observed lack of difference shouldn't be readily generalized to problems with more complex situation models. It seems reasonable that with problems with situation models more prone to overloading working memory, a concrete simulation might turn out to be more beneficial.

The influence of the type of variation introduced by target problems has received little attention within the study on analogical problem solving, specifically within studies dealing with mathematical and statistical problems. Data from our independent group showed that transfer to target problems of a certain type could be more or less difficult, depending on the depth to which a given variation demands comprehension of the equational structure. Concerning the implications of these results for the design of instructional materials (e.g., textbooks), more attention should be drawn to the proper selection of problems to be included in their expository and practice sections. The variations of a worked out example should be selected so as to demand the analysis of different parts and aspects of the equational structure, thus promoting a flexible use of the structure being learned. However, more studies are needed to further understand qualitative differences among problem variations, as a determinant of success in transfer to nonidentical problems.

Concerning the implications of these results for the broader study of analogical problem solving, the data obtained here run contrary to the thesis that the analogical subprocess of *adaptation* is intrinsically difficult. Like in Minervino, Trench and de la Fuente (2005), data from our independent group showed that adaptation can be relatively easy for certain problems—sometimes as easy as repeating the base procedure—and that performance at adaptation is not always inferior to performance at mapping and inferencing.

Our third hypothesis proposed that the advantages of a stimulation to elaborate situation models would depend on the difficulty of assimilating a given variation in the base equation. Interactions between the difficulty of equational assimilation and the presence of situation model simulations were observed when pitting the no simulation condition against the imagined and the concrete simulation ones. Our simulations thus proved more beneficial for problems with higher difficulty of equational assimilation.

In future studies it would be interesting to investigate the effect of the idealized/concrete character of the objects employed during the simulations. As in Goldstone and Son (2005), it is likely that a progression from concrete to idealized simulation materials will also prove beneficial for transfer to algebra word problems with variations in content.

Acknowledgments

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