Effects of Problem Format in Arithmetic: '3+9' versus 'three + nine' versus 'thrie + nyne'

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Abstract

We explored the impact of operand format (digit, word, pseudo-homophone) on single-digit addition and multiplication. Format manipulations are of theoretical interest because models of arithmetic knowledge differ with respect to predicted format effects. Latencies were shortest when operands were digits, and longest when they were pseudo-homophones. However, there was also an interaction of problem size with format: problem size effects were smaller in the pseudo-homophone condition relative to the digit condition, and were larger in the word format condition relative to the digit condition. We discuss our results with respect to the following question: Can this interaction be attributed (solely) to an encoding phase of processing, or might it (also) arise from a solution phase?

Keywords: math cognition, arithmetic, number, input format, homophone, surface form, encoding, retrieval, computation, problem size

Introduction

In elementary school, children typically learn to do simple arithmetic (e.g., 2+3). Arithmetic fluency is a key predictor of employment and income outcomes (Finnie & Meng, 2006). Thus, cognitive research on arithmetic performance is important for pedagogical as well as theoretical reasons. In the present research, our goal was to learn more about the representations and processes underlying arithmetic performance. In particular, we explored the impact of operand format (i.e., digits: 2+3; words: two + three; pseudo-homophones: tue + thrie) on solution latency and accuracy for simple addition and multiplication problems.

Prior research indicates that response latencies tend to increase by about 30% when operands are in number-word versus digit format (e.g., Campbell, 1994). Practically, Arabic digits are the most conventional and convenient operand format. However, format manipulations are of theoretical interest because different theories about arithmetic representations and processes predict different format effects and interactions. In particular, there has been a debate about whether format costs originate only in an initial encoding phase of processing, or also in the phase in which the answer is accessed. Indeed, it may be that these phases cannot be cleanly individuated but rather are interactive. However, let's start with the simplified assumption that answer production involves an encoding phase followed by a solution phase (in general, the solution phase could involve either direct retrieval or computation).

For problems in regular digit format, a robust effect that is attributed to the solution phase is the problem size effect – response latencies are smaller for problems with small operands (2+3; 2x3) than large operands (9+8, 8x9; reviewed by Zbrodoff & Logan, 2005). For simple arithmetic, self-report data suggests that adults often retrieve answers from memory, but also sometimes use computation procedures (e.g., counting; LeFevre et al., 1996). Even among the subset of problems solved via retrieval (vs. computation), a problem size effect is often still present (LeFevre et al., 1996). Thus, an aggregate problem size effect presumably owes to several contributions: i) among problems that are computed (vs. retrieved), computation takes longer for large (vs. small) problems; ii) among problems that are retrieved, retrieval is faster for small versus large problem, possibly due to differences in practice frequency (e.g., Zbrodoff, 1995), and finally, iii) retrieval, which is faster than computation, is used more frequently for small than large problems (e.g., LeFevre et al, 1996).

That said, in digit format, the majority of arithmetic problems are solved via retrieval. What happens when format changes, say from digit format to word format? Mean latencies are larger in word versus digit format, however the problem size effect is also larger for word versus digit format, that is, there is a format x size interaction (e.g., Campbell, 1994; 1999).

In general, if arithmetic facts in memory are not represented in the same format as the current problem stimulus (e.g., word format), then the current problem stimulus must be encoded (mentally translated) into a form that corresponds to that of the stored arithmetic knowledge.

Encoding Accounts

It seems clear that format (e.g., 3 vs. three) will, at the very least, impact latencies in any encoding phase. Some theories go so far as to suggest that format effects in arithmetic could be attributed *entirely* to encoding (vs. access) processes (e.g., Blankenberger, 2001). For example, McCloskey (1992) suggested that arithmetic knowledge is stored in an abstract format, independent of the input format or input modality (e.g., visual vs. auditory). On this view, to obtain an answer, a person first encodes (translates) the problem stimulus into an abstract representation to access the answer, which in turn is then converted into the required output format. Under this abstract-representation model, manipulations of input format should only affect the encoding phase rather than the answer access phase (which involves the same abstract representations regardless of input/output format).

Thus, to account for why word problem latencies are longer than digit problem latencies, McCloskey's model implies that converting words into this hypothetical abstract format is more costly that converting digits into abstract format. Note that word (vs. digit) format produces delays in various tasks besides arithmetic (e.g., number comparison, Noel et al., 1997; parity judgments; Campbell et al., 2004).

Another possibility is that arithmetic knowledge is represented exclusively in digit format (rather than an abstract format). Consequently, longer latencies for word-format problems occur because these problems require an encoding step to mentally translate the input from words to digits. Alternately it is possible that all facts are stored phonologically, and that number-word stimuli, digit stimuli, and auditory stimuli all invoke a common phonological access route. In this case problems in both word and digit format would require an encoding step to convert them to phonological format. To account for the shorter response latencies for digit problems (vs. words), this phonological-fact model implies that the digit-to-phonology conversion is more efficient than the word-to-phonology conversion. For example, phonology may be slower for multi-character words versus single character digits. Thus, again, such an account would imply that the (phonological) encoding step is more costly for word inputs than digit inputs.

In summary, models which suggest that arithmetic knowledge is stored exclusively in one format (be that abstract, phonological or digit-orthographic) are compatible with longer RTs for word (vs. digit) problems.

However, could such encoding-based accounts explain the inflated problem size effect for problems in word (vs. digit) format? The inflated problem size effect for number-word problems could be attributed (at least partly) to the influence of word frequency on encoding (i.e., text-to-phonology conversion). Number words, like all known words, are subject to frequency effects: the more frequently a word has been encountered, the more quickly it can be recognized and named. Words for small numbers (e.g., four) appear more frequently in print than words for larger numbers (e.g., nine). Thus, phonological encoding will take longer for large number-word problems (eight + nine) than small number-word problems (three + four). In contrast, there is less variation in frequency (and naming time) across digits than number words. Thus, part of the problem size effect for number-word problems is arguably the artifact of a word-frequency effect (i.e., an encoding locus vs. an access locus; McCloskey et al., 1992; Noel et al., 1997).

Consequently, one motivation for our inclusion of a pseudo-homophone condition (e.g., thrie x ait) was to include a format condition that was amenable to text-to-phonology conversion, but reduced problem-size artifacts associated with the variable frequencies of actual number words. Pseudo-homophones are non-words, and are thus all equally infrequent in the corpus. Thus, the encoding cost of converting pseudo-homophones into phonological form should not vary systematically with operand size. Thus, any persisting problem-size effects and format interactions can be more clearly attributed to access processes rather than encoding processes. To the extent that the inflated problem size effect for word format arises from word-frequency encoding effects, we expected smaller problem size effects for pseudo-homophone format than number-word format.

Thus, above we discussed encoding-based explanations for word format effects – that is, longer latencies and inflated problem size effects for word (vs. digit) problems. Alternately or additionally, format might influence the solution phase. Below we will discuss some access-based explanations for format effects.

Access-based Accounts

At the opposite end of the spectrum from McCloskey's abstract-representation model is the possibility that arithmetic fact knowledge might be stored in multiple (redundant) formats – for example, in Dehane's (1992; Dehaene & Cohen, 1995) model, memory contains both digit-orthographic representations (*visual Arabic Number Form*) and phonological representations (*auditory verbal word frame)* of arithmetic facts. Presumably then, the format of the stimulus problem would influence which type of representation (i.e., which access route) would be employed. For example, a problem presented aloud (auditory input format) would likely trigger the phonological route; whereas a problem presented as visual digits (e.g., 2+3) would likely activate the digit-orthographic route. Because people rarely solve problems or see facts in word format (two + three = five), it is unlikely that memory contains arithmetic representations explicitly encoded in number-word format. Thus, we expect that when people solve word-format problems, the access to the answer is mediated by mentally converting the operands into phonological and/or digit form.

Under this view involving digit and phonological retrieval routes, one co) format via the following assumptions: i) digit input format induces a head-start or bias to access digit-orthographic representations of arithmetic facts; ii) the digit-orthographic access route is more efficient than the phonological route, because people are most frequently exposed to facts in digit (vs. auditory) format; iii) digit input may also be amenable to rapid conversion to phonological form, so the answer may arguably receive activation from both routes.

Thus, number-word format would lead to longer latencies than digit-format for several reasons. First, access to stored information first requires the conversion of number words to phonological (or digit) form. Phonological conversion could be costly because it may involve the serial sub-vocalization of the problem (vs. perhaps processing a digit problem more holistically, in parallel: 2+3). Second, we expect that the phonological access route itself is less efficient (i.e., less practiced) than the orthographic access route – again, because facts are more frequently seen (i.e., digit format) than heard (i.e., auditory format). It is also possible that number-words could be mentally converted to digits, so the digit-orthographic access route could be employed instead of, or in addition to, the phonological route. However, we suspected in general that number-words would lend themselves more naturally to phonological (vs. digit) encoding, and thus to the use of a phonological access route.

Similarly, under this account, one would expect that pseudo-homophone input format would also prompt the use of stored phonological (vs. digit) representations or arithmetic information. Thus, the same access route and arithmetic representation would be in play for words and pseudo-homophones – so one might predict that performance for pseudo homophones might parallel that for words (perhaps plus some extra delay for the phonological encoding of the unfamiliar pseudo-homophones).

However, the above suggestion that different formats may promote the use of different access routes (i.e., different fact representation formats) does not readily explain the inflated problem size effect for word format problems.

In general, the surface familiarity of a problem stimulus may influence which solution strategy is employed – in particular, the less familiar the stimulus, the lower the likelihood that retrieval is even attempted (e.g., Schunn et al., 1997). Thus, one access-based (vs. encoding-based) explanation for the increased problem size effect in word format is that the less familiar word format increases the use of computation versus retrieval. In some studies on addition, participants self-reported more procedure use for problems in word versus digit format (Campbell et al., 2004; Campbell & Penner-Wilger, 2006). Such a strategy shift towards computation would indeed cause an increase in the problem size effect, because even within digit format problems, the problem size effect (i.e., slope) is larger among problems solved via computation then among problems solved via retrieval (LeFevre et al., 1996a; 1996b). Thus, format may interact with problem size because non-digit formats can result in a 'strategy' shift leading, in the case of number-words, to a heavier reliance on computation (vs. retrieval).

Consequently, our pseudo-homophone condition also constitutes a manipulation of apparent familiarity – participants are most familiar with problems in digit format and are completely unfamiliar with problems in pseudo-homophone format. Under this account, one would expect that the highly unfamiliar pseudo-homophone format would promote an even stronger shift from retrieval to computation – producing an even greater inflation in problem size for problems in pseudo-homophone relative to digit format. Conversely, Penner-Wilger and LeFevre (2005) did not find such a shift when participants solved single-digit multiplication problems. Thus, we included both addition and multiplication problems in the present research.

In summary, under an encoding-based (word frequency) account of word format effects, pseudo-homophones should not exhibit an inflated problem size effect relative to digit format, however, under an access-based (strategy shift) account, pseudo-homophones would exhibit an even larger inflation in problem size (over that for word problems). These predictions were tested in the present study. In particular, we manipulated three independent variables: operation (addition, multiplication); format (digit, word, pseudo-homophone) and problem size (small: operands < 5; large: an operand >=5).

Method

Participants.

Undergraduates (N=28) received course credit for their participation.

Materials

The stimuli were simple addition and multiplication problems with operands between 1 and 9. The operands were presented in three formats:

i) digits: **2 x 5**;
ii) words**: two x five**;
iii) pseudo-homophones: **tue x fyve**

Pseudo-homophones are non-words (English) that are pronounced the same as the corresponding number words. The complete set of pseudo-homophones used in the present experiment was: wun, tue, thrie, fowr, five, siks, sevin, ait, and nyne. For each operation, participants solved the nine tie problems (e.g., 2+2, 2x2); and 36 non-tie problems in each of the three formats. Operand order (2+3 vs. 3+2) was counterbalanced so that if a participant saw 2+3 in the addition block, the other operand order was used in the multiplication block (3x2). Within each operation (block) half the problems had the larger operand first, and half had the larger operand second. Following Campbell (1994), small problems were classified as those with both operands less than 5.

Procedure

Problems were blocked by operation (multiplication vs. addition), however, format varied within each block. Each problem was presented horizontally on the screen after a central fixation prompt (prompt duration 500 ms). The problem remained visible until the participant made a verbal response, triggering a microphone. The experimenter then recorded the participant's response (or an error code in the rare cases in which the microphone did not register the response, or accidentally triggered prior to the response). No accuracy or latency feedback was provided.

Results

A few trials (<5%) were excluded from the analyses because the participant's response did not trigger the microphone, or another sound prematurely triggered it. Repeated measures ANOVAs were conducted for the dependent measures of accuracy and latency. Analyses of response times are based on trials in which correct responses were received. The independent variables were operation (addition, multiplication); format (digit, word, pseudo-homophone) and size (small: operands < 5; large: an operand >=5). If the format variable failed Mauchly's test of sphericity, the Greenhouse-Geisser adjustment was used to assess significance. The Bonferroni adjustment was used for all pairwise comparisons.

Accuracy. Accuracy was higher on addition problems than multiplication problems, $F(1,26) = 4.38$, $p < .046$, and for small (both operands <=5) versus large problems, $F(1,26) = 26.94$, $p < .001$. Problem-size interacted with operation, $F(1,26) = 10.60$, $p < .003$, such that for small problems accuracy was comparable across operations, but for large problems, accuracy was lower for multiplication than addition. There was also a main effect of format on accuracy, $F(1,26) = 13.99$, $p < .001$. In particular, participants were less accurate on words than digits ($p < .034$), and less accurate on pseudo-homophones than on either words ($p < .030$) or digits ($p < .030$). There was no interaction of format and operation (addition vs. multiplication), $F(1.5,38.1) = .88$, $p = .393$. However, as shown in Figure 1, there was a marginal interaction of problem size and format, $F(1,26) = 3.12$, $p < .053$ – For small problems, accuracy was comparable for word and digit formats, whereas for large problems accuracy was highest for digit format and comparable for word and pseudo-homophone format. .

Latency. Ties and non-ties were analyzed separately because ties may be subject to distinctive processing (e.g., they typically have attenuated problem size effects and encoding may be different when the same operand is repeated) .

Latency (Non-Ties): Addition problems were solved more quickly than multiplication problems, $F(1,26) = 12.23$, $p = .002$; and small problems were solved more quickly than large problems, $F(1,26) = 103.67$, $p < .001$. Size interacted with operation, $F(1,26) = 4.77$, $p < .039$ such that thee problem size effect was larger for multiplication than addition. In terms of format, RT was smallest in the digit condition and largest in the pseudo-homophone condition, $F(1,42.38) = 228.747$, $p < .001$. There was also an interaction of size and format, $F(2,52) = 6.62$, $p = .003$, such that the problem size effect was smallest in the pseudo-homophone condition and largest in the word condition. As was the case in the accuracy analysis, format did not interact significantly with operation (addition vs. multiplication). Mean response latencies and problem size effects are presented in Table 1 (collapsed across operation).

Latency (Ties). Ties exhibited the same pattern as non-ties in that there were the three main effects of operation, $F(1,26) = 18.26$, $p < .001$, problem size, $F(1,26) = 35.47$, $p < .001$, and format, $F(2,52) = 76.04$, $p < .001$. Again there was in interaction of problem size and format, $F(2,52) = 5.50$, $p = .007$, such that the problem size effect was smallest in the pseudo-homophone condition and largest in the word condition. No other interactions reached significance.

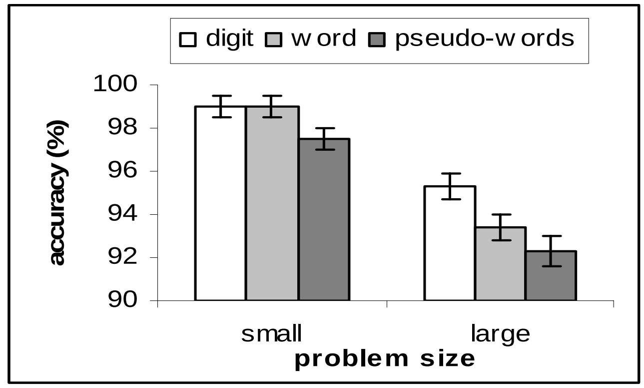

Figure 1. Accuracy (collapsed across operation).

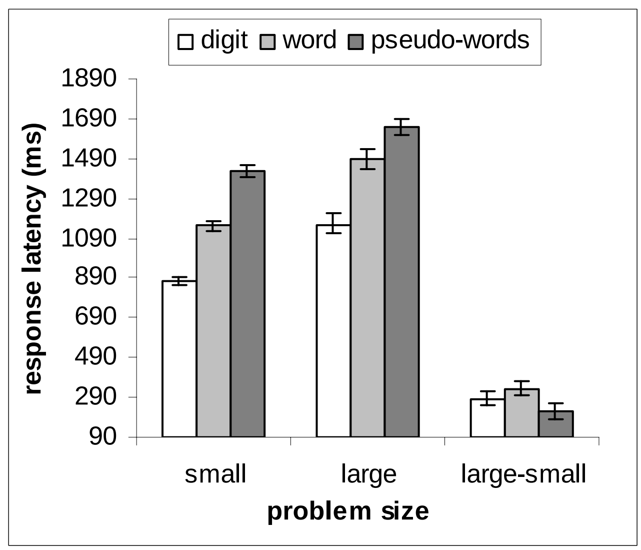

Figure 2. Latency (ms) collapsed across operation

Table 1. Response times (ms) collapsed across operation

	Ties			Non-ties		
Format	Small	Large	L-S	Small	Large	L-S
Digit	836	936	100	875	1160	285
Word	929	1096	167	1153	1486	333
Pseudo	1054	1130	76	1426	1647	221

Discussion

The present study explored the impact of operand format (digit, word, pseudo-homophone) on single digit addition and multiplication. Latencies were shortest when operands were digits, were longer for number word operands, and longest for pseudo-homophone operands. We did not find an interaction of format and operation (addition vs. multiplication; cf. Campbell, 1994). However, in accord with Campbell (1994; 1999) we did find an interaction of format with problem size – that is, there was a bigger problem size effect in word format compared to digit format. Note that our stimuli included an extra format condition – pseudo-homophones – that was not present in the prior research. Consequently, our account of the format by size interaction(s) is somewhat distinct and more general.

Overall, however, our pattern of results seems incompatible with Campbell et al.'s (2004) access-based account in which the increase in the problem size effect (in word format) arises because low format familiarity produces a shift from retrieval to computation. If low familiarity with a problem's surface form induces a bias towards a computation strategy, then computation (vs. retrieval) should be most likely for problems in pseudo-homophone format. Consequently, under a familiarity account, the problem-size effect should be largest in the pseudo-homophone condition. Contrary to this prediction, however, the problem size effect was actually smaller in the pseudo-homophone than in either the word or digit condition[1]. Thus, decreases in format familiarity per se are not cleanly correlated with increases in problem-size effects (nor, by inference, with increases in the use of computation vs. retrieval). Recall also LeFevre and Penner-Wilger (2005) found no increase in procedure use with word format for multiplication.

An encoding-based account is, however, compatible with the finding that problem size effects are smaller in pseudo-homophone format than in word format. Following Dehaene (1992), we assume that arithmetic knowledge can be stored/accessed in phonological format. Further, to access this knowledge, we assume that both word and pseudo-homophone problem formats typically involve a mental conversion from text to phonology. For word-format problems this conversion to phonology causes problem-size contributions associated with word frequency effects. In contrast, the pseudo-homophone format should produce systematic frequency-related problem-size contributions in this encoding stage. Thus, it is possible that the inflated problem size effect associated with word format is predominantly due to encoding (vs. access).

An encoding-based account can also address why mean RTs are larger for problems in pseudo-homophone format relative to word format. For regular number words which are in our mental lexicon, text-to-phonology conversion can be holistic (i.e., a stored association mapping the whole word to its sound), whereas for non-words, like our pseudo-homophones, participants have to sound them out serially, which takes more time. In summary, an encoding-based account seems able to address the differences between the word and pseudo-homophone formats (i.e., the latter has larger latencies but smaller problem size effects).

[1] However, if the pseudo-homophone format somehow inflated procedure use for small problems more than large problems then it is possible such a procedure shift could result in no overall inflation of the problem-size effect.

That said, we have not yet contrasted digit format with pseudo-homophone format. Our pseudo-homophone pattern evidenced a similar slow down for small and large problems relative to digit format. In fact, the The psuedo-homophone format does not contribute any additional problem-size effect beyond that present for digits. Why would the problem-size effect was slightly smaller for problems in pseudo-homophone format than for digit format. Because the phonological encoding of a pseudo-homophone problem is arguably a serial process (sounding out the first operand and then the second), the answer-access phase may begin before the phonological encoding of the second operand is complete – in particular, once encoded, the first operand may prime the phonological representations of relevant facts involving that operand. Thus, by the time the second operand is encoded, a relatively small amount of time is needed to individuate the needed fact from among the facts already primed by the first operand. Inter-fact retrieval interference (which contributes to the problem size effect; Zbrodoff, 1995) is reduced because the first operand constrains the solution context. In contrast, we assume that digit problems are processed and accessed more holistically (both operands in parallel) so the context is initially unconstrained, and latency (problem size) is sensitive to the problem as a whole (i.e., practice frequency). That said, the digit-orthographic route is still far more efficient than the phonological route (which involves the serial encoding of pseudo-homophones). In summary, a phonological access route (i.e., for pseudo-homophones) could explain the reduced problem-size effect relative to digit-format because the serial nature of phonological encoding may allow the some of the access phase (fact-priming based on the first operand) to proceed in parallel with the encoding of the second operand.

In support of our suggestion that a phonological route might result in reduced problem size effects, researchers have reported smaller problem size effects for multiplication problems presented in auditory format versus visual digit format (e.g., LeFevre et al., 2001; Metcalfe & Campbell, 2008). However, context effects due to processing a problem serially should also then have been present for word format problems, but word format lead to an inflated problem size effect.

We have suggested that word format may encourage the use of a phonological access route (vs. digit-orthographic access route). Further, we have suggested that digit-format problems may be processed holistically (i.e., both operands simultaneously) whereas the phonological encoding of a problem is serial (first operand then second operand). This distinction correctly predicts that word-format problems will be processed more slowly than digit-format problems. However, we would expect the serial nature of phonological encoding to also attenuate problem size effects in the word-format condition relative to the digit condition. Contrary to this expectation, problem size effects are larger in the word condition than the digit condition. Thus, either serial processing does not confer much of an access (problem-size) advantage, or word-frequency effects might exacerbate the problem size effect to the point of drowning out any serial processing advantage.

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