Analysis of the Variability of Three-Dimensional Spatial Relations in Visual Short-Term Memory

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Abstract
In a laboratory experiment, 13 participants reproduced from memory the position of a sphere relative to a second landmark sphere located on the viewing axis of the observer. The relative location of the second sphere varied both laterally and in depth. The stimuli were generated on a stereoscopic display. The paper focuses on the analysis of the structure of the noise in the reproduced object locations, this structure reflecting the mental representation of the stored spatial relations. The results showed that the spatial location of the landmark sphere affects the variability of the reproduced object locations. In particular, the variability in the frontoparallel plane increases with the length of the depth component of the spatial relation. This finding can be interpreted in two ways. First, spatial acuity in perception decreases, or second, participants encode sensory information by transforming it into a mental spherical coordinate system. Both interpretations are discussed.

Introduction
McNamara (2003) proposed that locations are memorized in egocentric and allocentric coordinate systems. Allocentric coordinate systems define locations with respect to objects in the environment. We believe that shifts of attention between several locations in space define the reference axes and planes of local allocentric coordinate systems within which the spatial relations are encoded. This assumption is consistent with the idea that locations are encoded by intrinsic frames of references (Mou & McNamara 2002; Schmidt 2004). These intrinsic reference frames would result naturally from salient landmarks of the scene that attract attention. The structure of the variability in the reproduced locations provides essential information about the nature of the allocentric reference systems and reveals the dimensions in which attributes of the location had been encoded. Only a few reports in the literature have provided a systematic investigation of the dispersion of locations recalled from memory. The most frequently cited work in this field is by Huttenlocher, Hedges, and Duncan S. (1993), who conducted an experiment in which the participants had to reproduce locations within a circle, the observed distribution of which was consistent with encoding the locations relative to the center of the circle in terms of the distance from the center and the polar angle. Furthermore, they found systematic distortions of the reproduced polar angles for locations near the virtual horizontal and vertical lines that divide the circle into quadrants. The participants misplaced the locations toward a central location in each quadrant. Huttenlocher et al. proposed a stochastic model based on hypothesized probability density functions for the recall of the locations from memory. Based on these findings Werner & Diedrichsen (2002) investigated the time course of the memory distortions for the location of a dot in relation to two horizontally aligned landmarks. These works and the work of McNamara (2003) complemented each another, if the recall of locations from memory is described by probability density functions according to the dimensions of the allocentric reference systems.

The aims of the experiment described in this paper are twofold. First to confirm basic parameters of the noise in the mental representation reported in the literature, which we have already used to model phenomena in memorizing object locations in graphical structures (Winkelholz & Schlick 2007a) and for symmetry detection (Winkelholz & Schlick 2007b). Second to gain insight into the structure of the probability distribution of basic three dimensional spatial relations reproduced from memory. Especially, we are interested if subjects encode the stimuli on the basis of values of the perceived attributes or if they transform the perceived attributes into a mental coordinate system.

Experiment
Within the experiment participants reproduced random virtual object locations on three predefined frontoparallel planes. If the object location is represented mentally by a distance and a solid angle relative to a landmark location, then the variability in the lateral coordinates of the
reproduced object locations should increase with their relative distance in depth from the landmark location. If the variability is independent of the reproduced location, the latter’s mental representation might simply be its perceived projection on the screen and a relative distance in depth that is perceived by disparity and the visual angle of the circumference. In general, an increase in the variability of the lateral coordinates might be just the result of visual perception. When the visual system focuses on a location in three-dimensional space through convergence, only the points contained inside Panum’s fusional area near the horopter are fused into a single image. Therefore, outside of Panum’s fusional area oculomotoric sensor information will additionally be used by the visual system to determine the spatial relation. Accommodation should have no effect on spatial acuity, since the stereoscopic stimuli were generated synthetically on a display at a fixed distance from the observer.

**Method**

**Participants**

Thirteen volunteers (11 male, 2 female, average age 27), who were recruited from the staff of our institute, took part in the experiment. All participants had normal or corrected-to-normal vision.

**Apparatus and Stimuli**

The experimental task environment was generated on a Windows workstation equipped with a NVIDIA Quadro graphics card. The subjects used a spacemouse and the standard keyboard to provide input information. The spacemouse, a three-dimensional interaction device with six degrees of freedom, contains a controller cap that can be pushed, pulled and twisted in any direction. The subjects used the spacemouse to control the spatial movement of the object during the response stage. The stereoscopic images were rendered at 120 Hz on a 21″ CRT monitor and a resolution of 1280×1024 pixels. The images for the left and right eyes were separated by shutter glasses, which meant that the frame rate per eye was 60 Hz. The scene was rendered using antialiasing (16 times provided by the driver) to increase the visual spatial resolution and thereby enhance perception of the disparity. The monitor screen was located 60 cm in front of the subject. The spheres were displayed using the user-centric projection method that is commonly employed in virtual environments such as caves and workbenches (Cruz-Neira, Sandin & DeFanti 1993). Points in object space are projected onto the screen according to the positions of the user’s eyes. Each eye perceives the points on the surface of a virtual object from the correct solid angle as if the object was actually present. In other words, the disparity of the displayed objects on the screen and the viewing angle of the projected size of the spheres were the same as if real spheres had been placed at these coordinates.

To accomplish this, subjects were advised to sit in an appropriate position so that their head was within the range of the parameters used in the projection model. In the following, the stimulus parameters are reported in virtual coordinates according to this user-centric projection model. All spheres were displayed to appear on one of three virtual planes. The screen is defined to be at z = −60 cm. The first plane was −1.5 cm [z = −58.5 cm] in front of the screen, and the second and third planes were located 1.5 cm [z = −61.5 cm] and 4.5 cm [z = −64.5 cm] behind the screen, respectively. Hence, the disparities shown on the display were −9.5′ for z = −58.5 cm, 9.1′ for z = −61.5 cm, and 26′ for z = −64.5 cm. The diameter of the spheres was 1 cm and the corresponding visual angles of the displayed size were 58.8′, 55.9′ and 53.3′, respectively. The landmark sphere was always displayed on the z axis. The sphere whose location had to be memorized was positioned at two distinct distances from the center axis. The radii of the circles were chosen so that the viewing angle of the distance to the center was constant across different virtual planes. The visual angle, $\alpha_{xy} = \tan^{-1}(\sqrt{x^2 + y^2} / z)$, of the lateral distance was 2.9° for the inner circle and 4.8° for the outer circle. The associated distances of the projected locations on the screen from the center were 3.0 cm for the inner circle and 5.0 cm for the outer circle. This procedure was used to ensure that effects in the xy-component of reproduced spatial relations did not result simply from different distances on the retina, which would make reasoning about the effects more difficult. On the other hand, this procedure makes it more difficult to analyze the effects in the displayed, virtual, three-dimensional space. However, variation in the radii of the test stimuli in virtual space was quite small and only resulted in additional noise that was identical for each factor level of $\Delta z$ and therefore did not affect the main effects. In the virtual space, the range of the radii was [2.8 cm, 3.1 cm] for the inner circle and [4.9 cm, 5.4 cm] for the outer circle.

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1 The x, y, and z axes form a right-handed coordinate system.
Procedure
In each experiment the subject’s task was to reproduce the location of one sphere relative to a second sphere. All participants performed training sessions to familiarize themselves with the stereoscopic information display and the spacemouse. Each experiment used a 3x3x2 within-participants design. The first factor was the virtual frontoparallel plane on which the landmark sphere was located. The second and third factors indicated the virtual frontoparallel plane and the eccentricity of the location that had to be memorized, respectively. The polar angle of the location on the circle in question was randomized by a uniform distribution. All object configurations were tested in a randomized order. At the beginning of each trial, both spheres were displayed for one second, followed by a blank screen shown for two seconds. Finally, the landmark sphere was displayed at its previous location and a second sphere was shown at the default location, the origin. This second sphere had to be moved to the memorized location using the spacemouse. When the subject was confident that the second sphere was located at its remembered location he/she confirmed the location by pressing the spacebar on the keyboard. After a blank screen had been displayed for a short time, a new trial containing new locations for the spheres followed. For movement of the sphere, the translation of the controller cap was modeled as a three-dimensional Cartesian vector. Because the controller cap can be moved along all dimensions simultaneously, this Cartesian vector can point in any direction and has no preferred movement along a particular axis. The sphere moved in the virtual display space in the direction of this vector with a speed proportional to the vector norm.

Dependent Variables
In the following, the triplet \((x_0, y_0, z_0)\) represents the coordinates of the landmark location, \((x, y, z)\) are the coordinates of the location that had to be memorized, and \((x', y', z')\) are the coordinates of the location that was reproduced by a subject. In this study, the relative distances of the locations to the landmark location are of major interest: \(\theta = (x - x_0, y - y_0, z - z_0)^T\), \(\theta' = (x' - x_0, y' - y_0, z' - z_0)^T\).

To test the hypothesis that relative depth is encoded independently of relative lateral location, we first investigated the response errors, \(\tilde{e} = \theta' - \theta\), in Cartesian coordinates. The reliability of the memorized location is reflected in the variability of the responses. By itself, the error vector reflects systematic distortions in the mental representation. Without a systematic component of distortion in the mental representation, the mean error equals zero. The variability of the errors is identical to the variability of the responses. We used the average absolute deviation to measure variability and the median to measure central tendency.

Results and Discussion
All trials on which the distance between the reproduced location and the correct location was larger than the distance between the correct location and the landmark location were considered as outliers. Since the exclusion of outliers resulted in empty cells for two of the participants, their data were excluded from further analysis. There were 6.9% outliers in the remaining group of 11 participants.

Cartesian coordinate system
For each factor level, the mean response error, \(\tilde{e}\), was determined. Using these means, the absolute deviations of each component, \(x\) and \(y\), of the response error were calculated. The absolute deviation was analyzed using a repeated measures ANOVA with \(|\Delta z|\) as the within-subject factor. The ANOVA results showed that the absolute deviation varied systematically with \(|\Delta z|\) (\(F(2,20) = 5.88, p < .01\), \(\eta^2 = .37\)), its value being smaller for \(|\Delta z| = 0\) cm (\(Mean = .36\) cm, \(SEM = .04\) cm) than for \(|\Delta z| = 3\) cm (\(Mean = .44\) cm, \(SEM = .03\) cm) and \(\Delta z = 6\) cm (\(Mean = .47\) cm, \(SEM = .06\) cm). No significant difference was found between \(|\Delta z| = 3\) cm and \(|\Delta z| = 6\) cm. The analysis revealed neither a main effect of the component type (\(F(1,10) = .60\), \(p = .44\)) nor a main effect of the component type (\(F(1,10) = .81\), \(p = .39\)).
levels are shown in Fig. 3. Since it was assumed that the reference axis points in the same direction as the spatial relation, the zenith angle only varied from 0° to 90°. Based on the grouping of these two values, factor levels were defined for the zenith angle and the lengths of the tested spatial relation. The defined factor levels are shown in Fig. 3.

A two-way repeated-measures ANOVA showed significant effects for the angular component ($F(1,10) = 15.30$, $p < .005$, $\eta_p^2 = .54$) and the zenith angle of the tested spatial relation ($F(3,30) = 6.46$, $p < .01$, $\eta_p^2 = .39$). The interaction of these two factors was not significant ($F(3,30) = 2.13$, $p = .12$, $\eta_p^2 = .18$). The increase in the absolute deviation of the reproduced azimuth angle was smaller ($\theta_{90^\circ}$: $Mean = 5.14^\circ$, $SEM = .85$; $\theta_{25^\circ}$: $Mean = 7.12^\circ$, $SEM = .97$) than the increase in absolute deviation of the reproduced zenith angle ($\theta_{90^\circ}$: $Mean = 9.15^\circ$, $SEM = .79$; $\theta_{25^\circ}$: $Mean = 15.53^\circ$, $SEM = 1.96$). The strong dependence of the absolute deviations in the reproduced zenith angles on the tested zenith angle contradicts the predictions of a pure spherical geometry for the mental representation. Therefore, as a next step the absolute deviation of the reproduced length of the spatial relation was analyzed. For each defined factor group, the tested lengths have a given absolute deviation, which need to be considered in the analysis. For a spherical geometry it must be expected that the absolute deviations of the reproduced lengths increase linearly. In contrast, the analysis showed a disordered picture for the reproduced lengths, the mean of the reproduced length being smaller for $d_s$ than for $d_d$ (Fig 4b). A one-way repeated-measures ANOVA revealed no significant difference for these two groups ($F(1,10) = 2.93$, $p = .12$, $\eta_p^2 = .23$). Therefore, the mean of the reproduced length for $d_s$ ($Mean = 5.45$ cm, $SEM = .19$ cm) was at least equal to or possibly smaller than that for $d_d$ ($Mean = 5.73$ cm, $SEM = .13$ cm). However, groups $d_d$ and $d_s$ also differed in zenith angle for the tested spatial relation. In $d_d$, the mean zenith angle was $42^\circ$, whereas for $d_s$ the mean zenith angle was $90^\circ$. For $d_s$, the spatial relation had no depth component, and the absolute deviations did not increase with the length of the tested spatial relation.

The absolute deviation for the tested length for $d_s$ was significantly smaller than that for $d_d$ ($F(1,10) = 13.6$, $p < .005$, $\eta_p^2 = .58$). Again, both groups also differed in zenith angle ($d_d$: $\theta = 20^\circ$, $d_s$: $\theta = 42^\circ$), and consequently by the fraction of the depth component. These findings suggest an independent analysis of the depth and lateral components of the length of a spatial relation. Therefore, the data are grouped by $|\Delta z|$ and the length of the $xy$-component of the tested spatial relations. The absolute deviations increased with the length of the related length. A one-way repeated-measures ANOVA showed that this effect was significant for the $z$-component ($F(2,20) = 35.1$, $p < .001$, $\eta_p^2 = .78$) and the $xy$-component ($F(1,10) = 20.3$, $p < .001$, $\eta_p^2 = .67$). Since the absolute deviations from the given spatial relations also increased itself for the $xy$-component, an additional two-way repeated-measures ANOVA was performed on pooled data from the tested spatial relations and the reproduced spatial relations. This analysis, which included reproduced vs. original spatial relations as an additional factor, revealed a significant interaction between reproduced vs. original spatial relation and length ($F(1,10) = 6.59$, $p = .028$, $\eta_p^2 = .39$). This interaction indicated that an additional increase in the absolute deviation results from the

$p = .46$) nor an interaction effect of component type and $|\Delta z|$ ($F(2,20) = 1.00$, $p = .38$). The absolute deviation of $e_{xy}$ parameterized with $\alpha_{xy}$, is plotted in Fig. 2b. The absolute deviation varied systematically with $\alpha_{xy}$ ($F(1,10) = 19.5$, $p < .001$, $\eta_p^2 = .66$), and the interaction of $|\Delta z|$ and $\alpha_{xy}$ was not significant ($p > .5$). The absolute deviation was smaller for $\alpha_{xy} = 2.9^\circ$ ($Mean = .37$ cm, $SEM = .04$ cm) than for $\alpha_{xy} = 4.8^\circ$ ($Mean = .48$ cm, $SEM = .04$ cm).

**Spherical coordinate system**

To analyze the variability of the responses using a spherical coordinate system, both the length and the zenith angle were calculated for all spatial relations that had been analyzed. Since it was assumed that the reference axis points in the same direction as the spatial relation, the zenith angle only varied from 0° to 90°. Based on the grouping of these two values, factor levels were defined for the zenith angle and the lengths of the tested spatial relation. The defined factor levels are shown in Fig. 3.

![Fig. 3: Factor levels of the stimuli used for the analysis.](image)

The factor levels with different lengths for a single zenith angle are of special interest. This is the case for $\theta_{25^\circ} \approx 42^\circ$ and $\theta_{90^\circ} = 90^\circ$. Therefore, if the response errors are examined in spherical coordinates, the absolute deviations of the angles should be identical for different lengths of the spatial relation. To verify this, the absolute deviations of zenith and azimuth angle for each response were calculated. A three-way repeated-measures ANOVA was conducted using Euclidean length and zenith angle of the tested spatial relation and the angular component of the reproduced spatial relation as within-subject factors. There was no significant effect of the length of the tested spatial relation on the absolute response deviation ($F(1,10) = 1.52$, $p = .240$). Therefore, the absolute deviation of the reproduced spherical angles was also calculated for $\theta_{25^\circ}$ and $\theta_{90^\circ}$. In Fig. 4a, the absolute deviations of the reproduced angles are plotted for all zenith angles under study. The absolute deviations of the reproduced zenith angles increased for smaller zenith angles of the tested spatial relation, whereas this dependence seemed to be weaker for the reproduced azimuth angle.
mental representation. Notably, the absolute deviations of the z-component appeared to increase linearly with $|\Delta z|$ for the tested spatial relation but not with the reproduced length. Furthermore, the z-component shrinks in memory. The strengths of the growth and shrinkage depended on $|\Delta z|$. A more detailed analysis is out of the scope of this paper.

**General Discussion**

The results of the experiment showed that the variability of a location reproduced from visual spatial memory is influenced by the relative distance in depth to a landmark. With increasing distance in depth, not only did the variability of the reproduced depth component of the distance increase, but the variability of the reproduced lateral location also increased. The effect of landmarks on locations reproduced from memory generally indicates that participants include spatial relations between the location and the landmark in the encoded location. The structure of the variability of the reproduced locations provides insight into the mental representation. For an analysis a detailed model should describe the actual information processing steps that transform sensory information into a cognitive representation and then into a reproduction. Such a model can be greatly simplified if noise contained in the mental representation is much greater than the noise contained in visual sensory information. In this case, the noise from sensory information can be neglected. For two dimensional stimuli, visual acuity was much higher than the variability of the reproduced locations. For example, the visual acuity at an eccentricity of 5° is about 3°. Under the assumption that the landmark location can be assessed with a resolution of 1′, the lateral direction of a location relative to the landmark location should be discriminated by $2 \times \tan(5°/(4°/2)) \approx 0.08°$, which is much lower than the usually obtained variability of directions reproduced from memory. Similar arguments apply for the reproduced lateral distance to the landmark location. The lateral noise parameter $\sigma_\theta$ and $\sigma_{\alpha_x}$, determined from the data can be compared to values reported in literature. Huttenlocher et al. (1991) reported $\sigma_\theta = 10^\circ$, which is somewhat higher than the value of $\sigma_\theta = 6.3^\circ$ found in our data. In contrast, we found $\sigma_{\alpha_x} = 0.11^2$ for the standard deviation to reproduce radial distance, which is larger than $\sigma_{\alpha_x} = 0.025$, the value reported by Huttenlocher et al. (1991). This difference may be caused by the fact that participants in the experiments of Huttenlocher et al. had to estimate locations within a circle. To do so, initially they had to estimate the center of the surrounding circle as the landmark location, which adds noise to the direction, whereas the radial component could be estimated more efficiently by using more than one landmark located on the circumference of the circle. For three dimensional stimuli, the assessment of sensory acuity is much more complex than it is for two dimensional stimuli. There are several sources of sensory information that can be exploited by the visual system to deduce information about depth: disparity, accommodation, and vergence. To the best of our knowledge, the quantity representing the effect of an increase in disparity on lateral spatial resolution has not been described in the depth perception literature. In contrast, the dimensions of Panum’s fusional area have been well studied (Kenneth & Ogle 1952). Additional studies have focused on the dependence of the stereo acuity on eccentricity (Rawlings & Shipley 1969) and the effect of object size on stereoscopic spatial depth acuity (Schlesinger & Yeshurun 1998) A decrease in spatial acuity in the lateral dimensions due to increasing disparity is to be expected, because double images are perceived outside Panum’s fusional area. However, we believe that the additional noise from disparity is less than the increase in noise that was found in the data. Furthermore, because the stimuli had horizontal disparity, this noise should only affect the horizontal component of the

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2 To be compliant with Weber-Fechner-Law the standard deviation of reproduced eccentricity scales linear with the eccentricity of the actual memorized visual angle $\alpha_{xy}$, is given by: $\sigma_{\alpha_x} = \sigma_{\alpha} \alpha_{xy}$
lateral location and not the noise in the vertical component, which surprisingly increased by similar amounts. Nevertheless, the analysis of the absolute deviation of the reproduced distances as a function of the distances examined in the experiment showed that the depth component of distance was crucial, since the variability was much greater in depth than it was in the lateral dimensions. This is consistent with the findings of Norman et al. (1996), who observed that participants are highly sensitive to small differences in the length of lines presented in the frontoparallel plane, while the sensitivity decreases by an order of magnitude when the line segments are presented at random orientations in depth. In case of a mental representation of the spatial relation in a spherical coordinate system a model should include this noise in the perception of depth, while the zenith angle is deduced from this noisy depth component. The dependence of noise in the depth component on eccentricity, where landmark location and the to-be-reproduced location are in the same frontoparallel plane ($\theta = 90^\circ$), was similar to values reported in the literature. It is known that stereoscopic acuity is a decreasing function of eccentricity. Rawlings and Shipley (1969) reported a stereo acuity of 21″ at the point of focus and 155″ at an eccentricity of 4°. If 25% is assumed to be the threshold of the just-noticeable difference, an interpolation of the data reported in this paper will predict a stereo acuity of 221″ at an eccentricity of 4°. On the one hand, this finding does not deliver a new argument that spatial relations are mentally represented in a spherical coordinate system, since the additional noise might simply be the result of the subject’s carelessness when adjusting the stimulus to the remembered location. Yet on the other hand, this finding does not contradict the argument that the noise contained in the mental representation results from noisy perception. A model assuming a mental representation in a spherical coordinate system would explain both effects—the increase in depth variability with eccentricity and the increase of lateral variability with relative distance in depth—using only one noise parameter for the zenith angle $\sigma_\theta$, whereas a model considering independent dimensions for the depth and the lateral location needed two parameters: one noise parameter for the lateral projected distance in dependence on the depth component ($\sigma_\phi(\Delta z)$), and a second noise parameter for the noise in the depth component in dependence on the eccentricity of the spatial relation ($\sigma_\theta(\alpha_{xy})$). In future research the mathematical modeling of human performance variability using probability density functions would clarify the underlying assumptions regarding dependencies between spatial attributes. The resulting parameterized models could be used to describe the recollection of locations from memory. The distortions at categorical boundaries emerged naturally at the boundaries of the probability density functions. Furthermore, the results of this study should be generalized. In the current experiment, the viewing axis was a natural choice for the polar axis of the spherical coordinate system, since there was only one landmark sphere present. If there are two landmark spheres, we suggest that the line connecting the two spheres serve as the polar axis of the mental representation.

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