Mental Representations of Diagrams, Views about Diagrams, and Problem Solving

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Abstract
This study investigated people’s mental representations of diagrams and whether these relating to views about diagrams and problem solving performance. The participants were 93 undergraduate students who were asked to complete a questionnaire which included free writing on the topic of diagrams, and problem solving. Analysis of the statements and ideas that the students wrote revealed four categories through which diagrams may be mentally represented: uses/purposes, exemplars, personal opinions, and structure. Personal opinions responses were found to negatively correlate with views about the usefulness of diagrams, and with experiences and confidence in using diagrams. In contrast, responses about the uses/purposes of diagrams positively correlated with confidence in using diagrams. Evidence was also found suggesting that, among students studying math, greater knowledge about the uses/purposes of diagrams facilitated better problem solving performance.

Keywords: Mental representation of diagrams; problem solving; articulation; free writing.

Introduction
Diagrams have many different fields of application (see, e.g., Blackwell & Engelhardt’s, 2002, list of academic fields that they identified as having research interest in diagrams), and their use is generally considered as efficacious. In problem solving, for example, Larkin and Simon (1987) explained how diagrammatic representations have distinct advantages over sentential representations because the ways in which diagrams index information can more effectively support useful and efficient computational processes. Hembree (1992) also found that, among the instruction methods he examined in a meta-analysis, training in diagram drawing provided the largest performance improvement in problem solving.

Despite the many reported positive attributes of diagram use, there are numerous problems that have been identified in relation to that use. For example, prior knowledge about diagrams appears necessary for their effective use (see, e.g., Grasmeyer & Cox, 2008; Larkin & Simon, 1987), and student have generally been found to lack spontaneity in using diagrams (see, e.g., Dufour-Janvier, Bednarz, & Belanger, 1987; Uesaka, Manalo, & Ichikawa, 2007). In essence, these suggest that many individuals probably fail to benefit from diagram use: if they lack sufficient knowledge about how to effectively use them, and – even if they did know how to use diagrams – if they nevertheless neglect to make use of them.

Most of the published research on diagrams have focused on their effects and functions (e.g., Ainsworth & Th Loizou, 2003; Cheng, 2002, 2004; Mayer, 2003), with very few studies that have investigated possible ways of understanding and addressing the problems associated with users noted above. The few studies that have considered issues concerning users of diagrams include Uesaka et al. (2007) which found that lack of confidence and perceptions of difficulty in diagram use, and viewing diagrams more as a strategy that teachers use (rather than a strategy that they themselves can use), were deterrents to students’ spontaneous use of diagrams. Uesaka et al.’s findings indicate that how individuals view diagrams influence their use of diagrams – suggesting that understanding the ways in which diagrams can be mentally represented could be key to addressing issues/problems about their use.

There is not a lot in the research literature, however, that deals with how people mentally represent diagrams. Numerous studies have considered mental processes relating to graphical representation: for example, Stern, Aprea, and Ebner (2003) examined the effect of “active” versus “passive” graphical representation (i.e., passive encounter with, as opposed to active construction of, linear graphs) on processing transfer from one subject content area to another. However, such studies have not directly addressed the question of how people might structurally represent diagrams in their minds (e.g., as images and/or propositions, in terms of their functions and/or specific examples?).

Blackwell and Engelhardt (2002) proposed a meta-taxonomy that can be used to analyze and compare existing taxonomic systems of diagrams. Their meta-taxonomy was aimed at facilitating the study of diagrammatic representations, such as assessing the relevance of different representations to specific research questions. One of the taxonomic dimensions they proposed was “cognitive” and, although they did not elaborate on this dimension in any detail, their suggestion of focusing on the nature of the
representation and the ways in which people might differ appears appropriate in any attempt to understand the user perspective in diagrams use.

Perhaps the closest attempt at finding out how people mentally represent diagrams was carried out by Cox and Grawemeyer (2003). They used a card sorting task to assess their participants’ semantic knowledge about a wide range of diagrams (that they referred to as “external representations” or “ERs”), and found through cluster analysis 9 major categories of ERs. Furthermore they found that participants differed in the categories they created according to how well they scored on ER reasoning tasks: the group of participants who scored well tended to create fewer categories that were based on semantic distinctions between ERs, while the group who scored lower created more categories that tended to focus more on superficial aspects of ERs (e.g., what the ERs looked like).

Cox and Grawemeyer’s (2003) findings revealed some important points about how people mentally deal with diagrams: for example, that those who had (presumably) greater knowledge about diagrams were able to perceive meaning-based commonalities between diagrams that may not look alike, while those who had (again, presumably) less knowledge about diagrams may have had to rely on feature-based processes which in turn may have been based on their recollections of diagrams they had previously encountered. Essentially, this suggests that with greater knowledge about diagrams, a person can perceive meaningful relationships between different forms, and categorize accordingly.

It is questionable, however, whether the categories that Cox and Grawemeyer (2003) identified based on their participants’ responses reflect the categories that people normally possess as their mental representations of diagrams (i.e., in normal circumstances, not in response to requests to sort/group diagrams). Would people naturally use such categories in mentally organizing and representing what they know about diagrams? The present authors believe otherwise as the categories that people come up with in response to item sorting tasks would inevitably be influenced by their efforts at incorporating and making sense of items that they either did not know about or had not previously considered as part of the subject in consideration. The categories would also reflect the absence of items they may know about but had not been presented. In other words, the kinds of items presented in the task would unavoidably bias the kinds of categories that are produced.

Furthermore, in the Cox and Grawemeyer (2003) study, the participants’ ability to find meaning (e.g., the semantic distinctions) in the task given does not necessarily mean that those meanings previously guided their mental representation of diagrams: the task itself could have facilitated the development of their insight about those semantic distinctions between different ERs. This therefore means that, despite the valuable contributions of the Cox and Grawemeyer study, the question of how people normally represent diagrams in their minds remains largely unanswered.

One method that has been used to gain insights into people’s cognitive structures about target “objects” is articulation (see, e.g., Scott, 1966): through the descriptions that participants provide of the target object, the structural properties of their cognition can be inferred (i.e., through the definitions, categorizations, connections, and elaborations they articulate). As information provided with the target object can be restricted, potential biases that can inadvertently be communicated to the participant can be reduced. Steps to reduce the potential for such biases are important when attempting to understand how people normally represent certain concepts in their minds.

Free writing, which can be defined as “a procedure in which students are asked to write down whatever they think of and to keep writing without worrying about quality of ideas” (Hayes & Flower, 1986, p. 1106), is one technique that has been used to facilitate participants’ articulation of their beliefs and ideas about target concepts. It has, for example, been used to tap into and understand students’ knowledge about, and associations with, basic scientific concepts they were learning (Curtis & Millar, 1988); and to understand the specific content of marital ideals among newly married couples (Knobloch-Fedders & Knudson, 2009).

In the present study, free writing was utilized as a method to facilitate participants’ articulation of their thoughts and ideas about diagrams – the aim being to explore and try to understand how people might normally represent diagrams in their minds. A further aim of the study was to find out if such mental representations of diagrams are related to participants’ (i) views about the importance of diagrams in teaching and learning situations, (ii) self-assessments of experience and confidence in using diagrams, and (iii) competence in using diagrams in problem solving.

Method

Participants
The participants were 93 undergraduate first-year (freshmen) students in a university of education in Japan (i.e., they were studying to become teachers) who voluntarily participated in completing the questionnaire used in this study. Their mean age was 19.0 years (SD = .53 year); 50 were females, and 43 were males.

Materials and Procedure
The questionnaire administered to students was written in Japanese and comprised of three parts. In part 1, after briefly being informed about “free writing”, participants were first asked to practice free writing for 1 minute on the topic of “friendship”. Following this, they were given 3 minutes to free write on the topic of “diagrams”.

In part 2, participants were asked their opinions about diagrams. First, they were asked to indicate on a 5-point Likert-type scale how important they considered diagrams in teaching and learning. They were then asked to briefly write reasons for their response (however, because of space
categories, analyses concerning the participants’ responses to this question have not been included in this paper). Next they were asked to indicate, again on a 5-point Likert-type scale, how much they usually used diagrams, and how confident they felt in using diagrams.

In part 3, participants were asked to solve three problems. Three minutes were allowed for each of the problems, and participants were explicitly asked to try to construct and use diagrams in their attempts to solve them. The first problem required the comparison of heights, and a pictorial depiction of the heights indicated would have been helpful towards solving it. The second problem required working out the circumference following the arrangement of similar-sized pieces of paper; for this, the construction of a table would have been helpful. The third problem concerned applicant placement at a training and employment agency, and for this problem a decision flow chart would have been helpful. (Again, due to space constraints, analyses relating to appropriateness and quality of the diagrams that participants produced have not been included in this paper.)

Results

Categories that Emerged from the Free Writing Task

The participants’ responses to the free writing task about diagrams were analyzed initially by looking through these responses and identifying themes or categories of ideas that they conveyed. Understandably, because it was a free writing task and participants were asked to write continuously for the 3-minute duration irrespective of the relevance of the ideas that came to their minds, a large proportion of what they produced appeared unrelated to the topic of diagrams (e.g., single word statements like “compass” and “PC”, phrases like “book that has a catchphrase of ‘easy to understand’”, and sentences like “It keeps appropriate distance.”). Apart from these unrelated statements, however, the participants’ responses appeared to fall into four broad categories: statements or ideas concerning (a) the uses of diagrams, (b) specific examples of diagrams, (c) personal opinions about diagrams, and (d) the structure of diagrams.

These categories were therefore used to sort and tabulate the participants’ responses. The responses were sorted in terms of single, complete ‘units of ideas’: these could be single words that conveyed a complete idea and appeared intended by the participant to be so (e.g., by being separated from other ideas spatially or through the use of punctuations), complete phrases, sentences, and – in a few cases – diagrams that participants drew.

Table 1 shows the five categories (including the “unrelated” category), the number of ideas or statements that participants wrote in these categories, and the number and percentage of participants who wrote ideas or statements that belonged in these categories.

Under the category of “Uses or purposes” were included statements or ideas that pertained to general or specific uses, purposes, or functions of diagrams. Examples of general statements/ideas of this kind were: “it can help to summarize” and “promotes understanding”. Examples of more specific references to uses, purposes, or functions included: “diagrams are used to represent problems more concretely, as a result people can visualize better and find a hint for finding the solution more easily” and “although mathematics is separate from daily life, many people can reach common understanding by using diagrams”.

Table 1: Responses to the free writing task.

<table>
<thead>
<tr>
<th>Category</th>
<th>No. of ideas/statements</th>
<th>No./percentage of participants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uses or purposes</td>
<td>178</td>
<td>69 (74%)</td>
</tr>
<tr>
<td>Specific examples</td>
<td>163</td>
<td>52 (56%)</td>
</tr>
<tr>
<td>Personal opinions</td>
<td>80</td>
<td>46 (49%)</td>
</tr>
<tr>
<td>Structure</td>
<td>17</td>
<td>12 (13%)</td>
</tr>
<tr>
<td>Unrelated</td>
<td>470</td>
<td>81 (87%)</td>
</tr>
</tbody>
</table>

The category of “Specific examples” was used when participants simply listed, described, or drew specific kinds or forms of diagrams. Examples included: “graph”, “bar chart”, “pie chart”, and “table”.

Included in the “Personal opinions” category were participants’ ideas or statements that pertained or related to experiences they had had with diagrams. Examples of statements and ideas placed in this category were: “troublesome … complicated”, “many are difficult to understand”, and “I get irritated when I cannot draw them well”. Almost all were negative.

Under the “Structure” category were included participants’ references to the general or specific ways in which diagrams structure, organize, or present data/information. Examples of the general ways participants mentioned included: “a diagram is a visual representation”, and “a different approach from one that uses language”. An example of a more specific reference to the way in which diagrams structure data/information included: “something represented line-by-line”.

To check the reliability of coding the participants’ responses under these categories, another person independently carried out coding on 10% of the participants’ responses (10% being the minimum acceptable subsample size recommended by Lombard, Snyder-Duch, & Campanella Bracken, 2008, for such purposes). The inter-rater agreement (Cohen’s kappa coefficient) was found to be .63, which was considered as being substantially concordant.

Relationship of Categories to Views About Diagrams

As previously noted, in part 2 of the questionnaire, Question 1 (“Importance”) asked participants how important they considered diagrams in teaching and learning situations, Question 2 (“Experience”) asked how much experience they had had in using diagrams, and Question 3 (“Confidence”)
asked how confident they felt in using diagrams. The participants were asked to respond on 5-point Likert-type scales where 1 was most negative (e.g., not important) and 5 was most positive (e.g., very important).

For the “Importance” question, the mean response was 4.38 ($SD = .59$), indicating that the participants generally viewed diagrams as being very important in teaching and learning situations. For the “Experience” question, the mean response was 3.44 ($SD = 1.04$), indicating that most of the participants had experiences of occasionally using diagrams. And for the “Confidence” question, the mean response was 2.59 ($SD = .84$), indicating that the participants were generally tending toward being doubtful about their ability to use diagrams well.

To find out whether there were any possible relationships between (i) the categories of statements and ideas that participants produced in free writing about diagrams and (ii) their views about diagrams as gauged in part 2 of the questionnaire, correlations between these were examined.

The correlations between the participants’ use or otherwise of the categories, and their responses to the Likert-type scales used in part 2 of the questionnaire, are shown in Table 2.

Table 2: Correlations between participants’ use of the categories and their opinions about diagrams.

<table>
<thead>
<tr>
<th>Category Used</th>
<th>Importance</th>
<th>Experience</th>
<th>Confid.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uses or purposes</td>
<td>.043</td>
<td>.109</td>
<td>.212*</td>
</tr>
<tr>
<td>Specific examples</td>
<td>.127</td>
<td>.107</td>
<td>.020</td>
</tr>
<tr>
<td>Personal opinions</td>
<td>-.195</td>
<td>-.193</td>
<td>-.160</td>
</tr>
<tr>
<td>Structure</td>
<td>-.138</td>
<td>.022</td>
<td>-.004</td>
</tr>
<tr>
<td>Unrelated</td>
<td>-.081</td>
<td>.090</td>
<td>-.112</td>
</tr>
</tbody>
</table>

* $p < .05$

The significant correlation found here suggests that participants who wrote statements/ideas about the uses of diagrams also possessed greater confidence in their ability to use diagrams.

In contrast, the significant negative correlations between “Personal opinions” and “Importance”, “Experience”, and “Confidence” suggest that those who wrote more about their personal opinions about diagrams tended to have lesser appreciation of the value of diagrams in teaching and learning situations, and less experience and lower confidence in diagrams use.

Relationship of Categories to Problem Solving Performance

The mean score for the three problems administered to participants in part 3 of the questionnaire was 2.32 ($SD = .80$). Seventy-three percent correctly solved Problem 1 (heights comparison), 76% correctly solved Problem 2 (paper arrangement circumference), and 83% correctly solved Problem 3 (employment agency placement). In general therefore, the problems appeared quite easy for most of the participants to solve and ceiling effects may have been encountered.

No significant and/or meaningful correlations were found between (i) the categories of statements and ideas that participants produced in free writing about diagrams (both whether or not they used the categories, and the number of statements they made in each of the categories) and (ii) the scores they obtained in their attempts to solve the problems given in part 3 of the questionnaire.

However, when participants were differentiated on the basis of their “math involvement” (i.e., whether or not they were in a math course, or were seeking a math teacher’s license), a significant correlation was found between math involvement and problem solving performance ($r = .28$, $p < .05$). This suggested the possibility that the relationship between math involvement and problem solving performance was mediated by participants’ knowledge about the uses/purposes of diagrams; thus, a mediation effect analysis (Baron & Kenny, 1986) was undertaken. This revealed that when regression analysis on problem solving performance was carried out only with the math involvement variable, the standardized coefficient was significant ($\beta = .208$, $p < .05$). However, when the same analysis was done with math involvement and participants’ use (or otherwise) of the uses/purposes category as independent variables, the standardized coefficient of math involvement diminished and became non-significant ($\beta = .198$, n.s.). This finding suggests that the better problem solving performance of participants with math involvement was likely due to their greater knowledge about the uses/purposes of diagrams.

Discussion

How Do People Mentally Represent Diagrams?

Through the statements and ideas that participants in the present study articulated via the free writing task, it can be inferred that they viewed or mentally represented diagrams
In terms of their uses, specific examples of them, personal opinions/experiences of them, and their structure. Almost three-quarters of the participants (74%) wrote something about the uses, purposes or functions of diagrams. This high proportion is probably understandable considering that diagrams are tools or strategies that can serve particular purposes: thus the majority of people are likely to mentally represent them in terms of, or in relation to, those purposes they are aware of.

Just over half of the participants (56%) also provided specific examples of diagrams, either on their own or in relation to other categories of statements/ideas they articulated (i.e., uses, personal opinions, structure). This suggests that many people also mentally represent diagrams in terms of, or in relation to, specific kinds of diagrams they know – or exemplars.

Almost half of the participants (49%) wrote statements and ideas that appeared to fall into a category of being about their personal experiences and opinions about diagrams. Again, it probably makes sense that many people would do this when one considers that people make sense of the world around them through their personal experiences and the resulting opinions that they form. Thus, where tools/strategies are concerned, these are likely to be represented in terms of notions like “helpful” or “difficult to use” depending on their experiences of using these.

Finally, a small proportion of participants (13%) referred to the structure of diagrams, suggesting that such structure formed at least part of their mental representation of diagrams. However it should be noted here that the statements/ideas that participants wrote in relation to structure were fairly general and superficial – mostly just expressing that diagrams represent information in visual or pictorial format. They did not refer to more complex and specific structural qualities/portrayals of diagrams like arrays, sequences, notations, and so on.

Only the four categories of uses/purposes, specific examples, personal opinions, and structure were identified in the written data collected in the present study. However, it is possible that other groups of participants would evidence other categories (different from the four identified here) depending on their knowledge about and experiences in the use of diagrams. It would be important to investigate this in future research.

In the present study, only the participants’ responses in terms of their use of the categories identified, and the number of distinct statements/ideas they wrote that belonged to each of those categories, were coded, analyzed, and reported. However, there are a number of other dimensions of the data that, at the time of writing this paper, the authors had not yet examined. These dimensions include the ‘quality’ of the statements and ideas that participants articulated: for example, participants wrote both fairly superficial as well as more meaningful uses/purposes of diagrams that were not differentiated. Also, most of the statements that participants wrote relating to their personal opinions about diagrams were “negative”; very few could be considered “positive”. Another potentially important dimension is the connectedness of the statements and ideas – both within and between categories. Although outside the scope of the present paper, it would clearly be useful to examine in more detail the possible effects or relationships that may stem from these other dimensions of the data.

**Contributions to Cognitive Research**

As noted in the introduction to this paper, no prior research had looked into how people might naturally represent diagrams in their minds. The present findings suggest that such mental representations involve categories of uses or purposes of diagrams for the majority of people. Furthermore, approximately 50% of people would have mental representations that incorporate their personal opinions about diagrams and/or specific examples or exemplars of diagrams. A small minority may also have mental representations relating to the structure of diagrams.

Although at first glance the mental representations suggested by these findings may appear completely different from those identified by Cox and Grawemeyer (2003) through their card sorting task, there are possible connections and congruence between these representations. Firstly, the 9 categories identified by Cox and Grawemeyer pertained to the structure of diagrams – both semantic and superficial. Although only a small proportion, some of the participants in the present study did articulate structure-related statements and ideas about diagrams. Categorizing diagrams according to their structure may be a natural response in a task like the one used in the Cox and Grawemeyer study (where structure may appear as the most salient feature of the stimuli presented). However, diagrammatic structure may also be a natural way of mentally representing diagrams for some people: perhaps for those with limited knowledge/experience about diagrams, superficial structures may be the only salient basis for mental representation. Likewise, for those who have greater than average knowledge/experience about diagrams, the semantic structures of diagrams may in fact be a natural way of mentally representing and organizing diagrams.

Secondly, in the same way that the Cox and Grawemeyer (2003) study identified semantic and superficial distinctions in participants’ responses according to their possession of greater or lesser knowledge about diagrams, it is possible that the same semantic-superficial dimension underpins the participants’ responses across the different categories identified in the present study. Thus, for example, the participants’ responses in the personal opinions category may well differentiate those with greater from those with lesser knowledge and skills about diagrams according to whether the opinions expressed are superficial in nature (e.g., basic references to ease or difficulty) or more meaningful (e.g., pertaining to what they have learnt about themselves or about diagrams). One possibility is that the mental representation of diagrams lies along two dimensions – one dimension being the kinds of categories identified in the present study, and the other being
meaningfulness-superficiality. Future research will need to examine this, and whether other strategies/tools may also be represented mentally in a similar manner.

**Contributions to Educational Research**

There is evidence in the findings of the present study to suggest that mental representations of diagrams could influence students’ views about diagrams as well as their problem solving performance. That responses in the personal opinions category negatively correlated with participants’ views about the usefulness of diagrams, and their experiences and confidence in using diagrams, is understandable in light of the fact that the majority of statements/ideas written in the personal opinions category were negative. Many of the participants’ more positive personal opinions about diagrams were probably expressed as statements/ideas about their uses – thus falling into the uses/purposes category instead.

The finding that responses in the uses/purposes category not only correlated with confidence in using diagrams, but also appeared to mediate the problem solving performance of those studying math, is likely due to two simple explanations (cf. Uesaka et al., 2007). First, greater knowledge about the uses/purposes of a strategy/tool should promote greater confidence in the use of that strategy/tool. Second, greater knowledge about the uses/purposes of diagrams should enable more appropriate use of them in problem solving situations, which in turn should assist toward better problem solving performance. Further research into the mechanisms of these relationships would be helpful toward the development of their applications in classroom instruction.

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**References**


