The Role of Comparison in Mathematics Learning

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Abstract

To better understand how comparison can be effectively used in mathematics instruction, we reviewed research in psychology and education, with the aim of identifying types of comparison that take place in mathematics learning, and considering the effects of comparison on procedural and conceptual understanding. We identified three types of comparison that are commonly utilized in mathematics instruction and learning: (1) problem-to-problem comparisons, (2) step-to-step comparisons, and (3) item-to-abstraction comparisons. Of these three types, only the effects of problem-to-problem comparisons on learning have been well documented. This paper therefore highlights the need for further research to elucidate the unique contributions of different types of comparison in mathematics learning.

Keywords: comparison; mathematics; learning; instruction

Students often have difficulty learning both mathematical procedures and their conceptual underpinnings (e.g., Kamii & Dominick, 1997; Pesek & Kirshner, 2000). In order to improve students' procedural and conceptual understanding, it is important to understand the cognitive processes involved in mathematics learning.

In an effort to support student learning, teachers often make connections between problems or concepts by comparing them, but they sometimes fail to provide students with the cognitive support needed to help students benefit from these comparisons (Richland, Zur, & Holyoak, 2007). How can comparison be used to effectively promote mathematics learning? Several techniques utilized in mathematics education involve comparison, but the effects of different types of comparison are not well understood.

This paper reviews research in psychology and education in order to (1) identify what types of comparison take place in mathematics learning, and (2) consider the effects of comparison on procedural and conceptual understanding of mathematics.

Three Types of Comparison

We identified three types of comparison that are commonly utilized in mathematics instruction and learning: (1) problem-to-problem comparison, (2) step-to-step comparison, and (3) item-to-abstraction comparison. Later sections of this paper define each type of comparison and document its effects on learning.

Effects on Procedural and Conceptual Learning

The effects of comparison on mathematical learning could be measured in myriad ways. In this paper, we focus on two critical aspects of mathematical knowledge: procedural and conceptual knowledge (Hiebert, 1986). *Procedural knowledge* refers to the ability to execute action sequences for solving problems (including the ability to adapt procedures for new problems) (Rittle-Johnson & Alibali, 1999) and *conceptual knowledge* refers to explicit or implicit understanding of principles that govern a domain and of interrelations among aspects of mathematical knowledge (Rittle-Johnson & Alibali, 1999; Tennyson & Cocchiarella, 1986).

Examining the effects of comparison on procedural and conceptual understanding is a primary goal of this paper. However, in reviewing this literature, it quickly became clear that few studies have sought to determine the *unique* effects of comparison on procedural and conceptual knowledge. Many studies have investigated how comparison affects procedural knowledge, but few studies have addressed the effects of comparison on conceptual knowledge.

In the domain of math, gains in procedural and conceptual knowledge are often difficult to assess separately. The two forms of knowledge are tightly linked, with procedural knowledge affecting conceptual knowledge and conceptual knowledge informing procedures (Gelman & Gallistel, 1978; Siegler & Crowley, 1994; Rittle-Johnson & Alibali, 1999). Many studies measure only procedural gains, which seems to imply that procedural knowledge is the most important measure of learning. This review will highlight the need for studies investigating the effects of comparison on conceptual knowledge, and will emphasize the unique and interrelated importance of both types of knowledge.

Inclusion Criteria

The studies included in this review were limited to those pertaining to students' mathematical learning found in the psychology and education literatures. The keywords "math*", "student", and "learn*" were used in combination to search the databases PsycINFO, ERIC, and Web of Knowledge for relevant empirical articles and book chapters. Although this may not be a complete sample of studies, we have tried to include a representative sample of relevant research articles from the analogical problem solving, contrasting cases, and self-explanation literatures. To be included in the final set of studies reviewed, each study had to include (1) some type of comparison in a lesson or intervention, and (2) at least one measure of procedural or conceptual learning.

Studies were coded according to the types of comparison used. Many of the studies were not specifically designed to assess effects of comparison on learning, so the studies were categorized by which types of comparison must have taken place given the procedure or instructions in the study.

We found that the three different types of comparison sometimes occurred in combination, and sometimes on their own. To discern the separate effects of each type of comparison on conceptual and procedural learning, we focused on studies where only one type of comparison was implemented. We use studies involving combinations of comparison types for illustrative purposes in cases where the research for a particular type of comparison is sparse. Studies reviewed are marked with an asterisk in the Reference section.

Problem-to-Problem Comparisons

Problem-to-problem comparisons involve comparing the structure of one problem to that of another, or comparing the solution strategy used for one problem to that used for another. Opportunities to engage in P-P comparisons may arise from either direct instruction to compare problems or indirect practice with multiple example problems.

When students are presented with a new math problem, recruiting a relevant earlier example is often useful for recognizing what features of the current problem are important or what solution is necessary to solve the problem. Theories of perceptual learning support the notion that opportunities to compare problems highlight features of a problem to which students previously may not have been sensitive (Gagne & Gibson, 1947; Gick & Paterson, 1992). Comparing problems helps students notice similar features as well as distinctive ones, resulting in well-differentiated problem representations (Schwartz & Bransford, 1998).

As one example, in a classroom study by Rittle-Johnson and Star (2007), 7th-grade students were asked to study two examples of a problem solved with different solution strategies, either studying the examples separately, or studying them simultaneously while comparing and contrasting them. Students who compared the two solution strategies were more successful at solving transfer problems and were more likely to explore alternative solution strategies than students who viewed the strategies separately. Actively comparing two problems' solution strategies allowed students to gain a better understanding of the structure of the problems and the strategies used to solve them.

Effects on Procedural and Conceptual Learning

Among the 52 studies of comparison in mathematics learning that we reviewed, 28 assessed the effects of problem-to-problem comparisons on learning on their own (i.e., not in combination with other types of comparison) (see Table 2). Of these 28 studies, 24 included procedural measures of learning. Nineteen (79%) of these 24 studies found positive effects, and the remaining 5 found no effects of comparison on procedural learning. Nineteen studies measured the effects of problem-to-problem comparisons on conceptual learning; of these, 16 (84%) found positive effects of conceptual learning and 3 did not.

Table 1 displays the procedural measures used in problem-to-problem comparisons to show that the pattern of effects for procedural learning holds across various procedural measures, including near and far transfer problems. Problems were coded as far transfer if they were labeled as such in the original study or if they were novel problems that required considerable adaptation of known procedures to solve. All other problems (e.g., equivalent problems and isomorphs) were coded as near transfer.

Table 1: Effects for procedural measures used in P-P comparison studies

Procedural Measures	Number of Studies	Effects	
		Yes	No
Near transfer	24	19	5
Far transfer	9	9	0
Response time	2	2	0
Procedure Recall	1	0	1
Procedural Flexibility	3	3	0

The majority of studies reviewed do report gains in procedural knowledge (e.g., Novick & Holyoak, 1991; Reed, 1989; Ross & Kennedy, 1990). For example, one study found that students who solved several problems before solving a target problem successfully transferred their procedural knowledge during transfer tasks (Bernardo, 2001).

However, a few studies have reported no greater procedural gains for students who contrasted problems versus those who did not (e.g., Hattikudur & Alibali, 2010; Reed, 1987; VanderStoep & Seifert, 1993). There may be certain conditions under which comparison is more or less likely to promote procedural learning; however, more research on this issue is needed.

The effects of problem-to-problem comparisons on conceptual knowledge in math are also generally positive, with the majority of relevant studies reporting gains in conceptual knowledge. Hattikudur and Alibali (2010), for example, found that students received a lesson contrasting the equal sign with inequality symbols showed greater gains in conceptual understanding than those who received a lesson about the equal sign alone.

Table 2: Evidence for the Three Types of Comparison in the Literature and their Effects on Learning

Type of Comparison	Number of Articles	Procedural Measures	Procedural Effects	Conceptual Measures	Conceptual Effects
Problem to Problem	28	24	19 yes, 5 no	19	16 yes, 3 no
Step to Step	1	1	0 yes, 1 no	0	N/A
Item to Abstraction	1	1	1 yes, 0 no	1	0 yes, 1 no
P-P and S-S	10	9	8 yes, 1 no	4	3 yes, 1 no
P-P and I-A	10	9	8 yes, 1 no	7	4 yes, 3 no
S-S and I-A	1	1	1 yes, 0 no	0	N/A
P-P, S-S, and I-A	1	1	1 yes, 0 no	0	N/A

However, Rittle-Johnson and Star (2007) found opposite results, in that students who contrasted multiple solution strategies showed no greater gains in conceptual understanding than those who encountered the solution strategies sequentially. However, it is worth emphasizing that, of the 28 studies that included solely problem-to-problem comparisons, only 19 included conceptual measures. More research that investigates the effects of comparison on conceptual learning is needed.

Step-to-Step Comparisons

Comparison can also occur within a single problem, as is the case with step-to-step comparisons in which one step of a problem is compared to the next step in the problem. Comparison is critical to forming inferences that explain the links between successive steps in an example. For example, in order to understand successive steps in a worked-out example of a linear equation, the reader must compare one step (e.g., $3x + 5 = 20$) with the next step (e.g., $3x = 15$), in order to deduce that both sides of the equation were subtracted by 5. In a study conducted by Chi and colleagues (1989), students studying example physics problems provided self-explanations that compared consecutive example statements to each other, giving meaning to each new quantitative expression. These bridging inferences promote deeper reasoning about the example and allow the learner to form a more cohesive global representation of the problem (McNamara et al., 2006). Comparison of one step in an example to the next is a critical feature of self-explanations, allowing readers to make sense of the procedure used in an example.

Effects on Procedural and Conceptual Learning

We identified 13 studies involving step-to-step comparisons in mathematics learning, but only one focused solely on the effects of S-S comparisons (i.e., not in combination with other types of comparison). This study (Novick & Tversky, 1987) included procedural measures, but revealed no effects of comparison on procedural knowledge. It did not include any conceptual measures. In this study, students who studied and compared the steps in a procedure were not able to transfer this knowledge to a new problem. Instead, they were so focused on the sequence of steps in the learned problem that they were not able to solve the new problems using a different sequence of steps. This lack of gain in procedural knowledge when comparing steps is supported by other studies that include step-to-step comparisons in combination with other types of comparison (Didierjean & Cauzinille-Marmeche, 1997; GroBe & Renkl, 2003). Future research should more directly examine whether step-to-step comparisons are actually detrimental to transfer.

Because no study using step-to-step comparisons included conceptual measures, we cannot assess the unique contribution of this type of comparison on conceptual learning.

Item-to-Abstraction Comparisons

The third type of comparison we identified involves comparing an item--either a step or a problem as a whole--to an abstraction. An *abstraction* is some generalizable information that stands apart from any concrete or specific aspects of a problem, such as a domain principle, concept, or schema for solving a problem.

In item-to-abstraction comparisons, students relate steps in an example, or a problem as a whole, to domain concepts and principles. For example, in one probability learning study, students providing “principle-based explanations” linked each step in the problem to a principle such as “It gets multiplied, because the events are independent from each other” referring to the multiplication principle in probability (Renkl, 1997). Comparisons to a principle or concept promote a deeper understanding of the links between abstract concepts and problem-solving procedures. As students make these comparisons, new information is integrated with prior knowledge--a critical aspect in the learning process (Kintsch & Kintsch, 1995). Through comparison, students connect their understanding of the steps in a procedure to domain concepts, and in doing so, expand their prior knowledge of those concepts and the domain.

One example of a study that facilitated learning through item-to-abstraction comparisons involved students solving probability problems (Atkinson, Renkl, & Merrill, 2003). College undergraduates were asked to work through the problems step by step on a computer screen. As students worked through the steps, they were prompted to identify which principles of probability were relevant to each step. As students linked and compared the steps to principles, they were able to better understand the problems as revealed through both near and far transfer of problem-solving skills.

Effects on Procedural and Conceptual Learning

We identified 13 studies involving item-to-abstraction comparisons in mathematics learning, but only one of these (Atkinson et al., 2003) focused solely on the effects of this type of comparison (i.e., not in combination with other types of comparison). This study did include measures of procedural knowledge, and found positive effects on procedural learning. This study also included measures of conceptual knowledge, but did not find that I-A comparison led to conceptual learning.

This study measured procedural learning with transfer problems, and found that item-to-abstraction comparisons lead to gains in procedural knowledge. Students transferred their procedural knowledge to novel probability problems better when the worked-out problems they studied included information about relevant probability principles (Atkinson et al., 2003).

Although this study did include a measure of conceptual knowledge--asking students to produce a principle that is relevant to the problem--no effect of item-to-abstraction comparisons on conceptual learning was found. Although students studied worked-out problems that invited them to compare problem steps with probability principles, when asked to solve novel transfer problems students were not able to identify which principle was most relevant. Students in this study who engaged in item-to-abstraction comparisons were not able to transfer their conceptual knowledge.

Discussion

This paper reviewed research on three types of comparison in mathematics learning: problem-to-problem, step-to-step, and item-to-abstraction comparisons. Of these three types, only the effects of problem-to-problem comparisons on learning have been well documented. Problem-to-problem comparisons, in which students compare one problem to another problem, lead to both procedural and conceptual gains in learning.

The effects of step-to-step and item-to abstraction comparisons are much less well understood. Very few of the studies reviewed investigated the unique contributions of these types of comparisons. As a result, little is known about whether or not they promote learning of procedural or conceptual knowledge in mathematics. Future studies should directly investigate these types of comparisons.

The lack of empirical support for step-to-step comparisons in this review bears further discussion. Although only one study measured the unique effects of step-to-step comparisons on procedural and conceptual learning, it revealed no benefits for step-to-step comparisons. It may be the case that step-to-step comparisons are not useful in the domain of mathematics, because these comparisons narrow a student's focus to the specifics of one particular procedure. When students are given another similar problem, they may have difficulty adjusting this procedure to a new problem. Domains for which the specifics of one process (e.g., the circulatory system) need not transfer to another process (e.g., digestion) may improve with better understanding of the specific steps in the process. However, in situations where transfer is necessary from one problem to another (as is often the case in mathematics), step-to-step comparisons may in fact be detrimental.

More generally, future research is needed to better understand the benefits and potential drawbacks of comparison in math learning. Research specifically geared to assess the unique contributions of each type of comparison in math is needed to better understand what types of comparison are most useful in math learning. With a better understanding of the contributions of each type of comparison, mathematics teachers and curriculum designers will acquire the information they need to implement effective instruction utilizing comparisons in ways that will best lead to learning.

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