# Visualizing thought: Mapping category and continuum

**Barbara Tversky**
Teachers College, Columbia University, 525 W. 120th Street
New York, NY 10027 USA

**James E. Corter**
Teachers College, Columbia University, 525 W. 120th Street
New York, NY 10027 USA

**Lixiu Yu**
Stevens Institute of Technology, Castle Point on Hudson
Hoboken, NJ 07030 USA

**David L. Mason**
Teachers College, Columbia University, 525 W. 120th Street
New York, NY 10027 USA

**Jeffrey V. Nickerson**
Stevens Institute of Technology, Castle Point on Hudson
Hoboken, NJ 07030 USA

## Abstract

Abstract thought has roots in the spatial world. Abstractions are expressed in the ways things are arranged in the world as well as the ways people talk and gesture. Mappings to the page should be better when they are *congruent*, that is, when the abstract concept matches the spatial one. Congruent mappings can be revealed in people's performance and preferences. Congruence is supported here for visual representations of continuum and category. Congruently mapping a continuous concept, frequency, to a continuous visual variable and mapping a categorical concept, class inclusion, to a categorical visual variable were preferred and lead to better performance than the reverse mappings.

**Keywords:** Diagrammatic reasoning; spatial metaphors; design; networks; information systems

## Introduction

Abstract thought has roots in the spatial world (e. g., Boroditsky, 2002; Lakoff & Johnson, 1980; Shepard, 2001; Talmy, 1983; Tversky, Kugelmass, & Winter, 1991). These abstractions are expressed in the ways people organize space as well as in the ways they speak, gesture, and put things on the page (Tversky, 2011, in press). External visual expressions of thought, from cave paintings to computer bits, go back tens of thousands of years, though expressions of abstract thought have become common only with the widespread use of paper. Visualizations of thought are especially apt for conveying information that is intrinsically spatial, like environments, organisms, and objects, where elements and relations in real space can be mapped onto elements and relations on the page. Yet they are also effective for conveying concepts and relations that are metaphorically spatial, including temporal, social, quantitative, and more, in part because such concepts have "natural" mappings to space (e. g., Landy & Goldstone, 2007; Tversky et al., 1991). These natural mappings seem to come from the ways that we arrange space to suit our needs and the ways that space governs our behavior (Tversky, in press). They are also evident in language, in common expressions and metaphors (e. g., Cooper & Ross, 1975; Lakoff & Johnson, 1980). For example, people, trees, and more grow stronger as they grow taller; taller piles, buildings, and bridges must be stronger than smaller ones. Such associations provide a worldly foundation for the many metaphors associating *up* with *good*, *strength*, and *power*.

By mapping abstract concepts and relations congruently to space and spatial relations, visualizations not only promote comprehension but also inference (cf. Bertin, 1981; Norman, 1993; Zhang, 2000). They allow users to apply highly-practiced skills of spatial reasoning to abstract reasoning (e. g., Tversky, 2001; in press).

Despite natural mappings, representing abstract relations graphically is not always straightforward. Several alternative means of visual expression are often available, and, typically, each of these has several possible interpretations. Many common and useful devices, like dots, lines, boxes, and arrows, are ambiguous, with multiple meanings, not unlike related spatial terms like *link*, *frame*, *field*, and *relationship,* which also have multiple meanings (e. g., Tversky, Zacks, Lee & Heiser, 2002). Arrows, for example, can indicate order, direction, movement, causality, and more (Heiser & Tversky, 2006). Yet, choosing the right representation is essential to fast and clear communication, and to effective reasoning with diagrams.

Selecting the right representation for an abstraction does not have to be at the whim of a designer. The Production-Preference-Performance program provides empirical methods for deciding (cf. Kessell & Tversky, 2011; Tversky, Agrawala, et al., 2007). In the 3Ps program, one group of participants *produces* graphic representations for a concept or group of concepts, for example, keeping track of a set of people as they move in time. People's spontaneous graphic productions for representing information reflect their understanding of how that information is structured (e. g., Novick & Hurley, 2001; Zacks & Tversky, 1999). Another group is presented with a set of graphic representations for the same information, for example, a matrix or a graph, and asked which they *prefer*, that is, which is a better or best way to convey the information. In some cases, *interpretation* of the graphic device is added or substituted for preference. A third group is asked to make judgments or inferences from one of several graphic representations, allowing comparison of *performance* under each representation. Comparing these measures can help select the right graphic representation and can also provide insight into the cognition underlying the concepts. Ideally, the mappings that are more successful in performance and preference are more successful because they are more congruent.

Designing direct, comprehensible visual devices to express abstract meanings can be more challenging when those meanings are superimposed on a system structure. Structure, especially spatial structure, has priority for the use of space in a diagram over time and abstract relations (e. g., Kessell & Tversky, 2011; Nickerson, Tversky, Corter, Yu, & Mason, 2010). Some cases are relatively straightforward, for example, superimposing causal relations on the structure of the circulatory system or a bicycle pump or the water cycle by adding arrows indicating the sequence and direction of causality (e. g., Heiser & Tversky, 2006).

Superimposing abstract relations on structural ones is more complicated in other cases. Consider the problem explored here, a network diagram conveying social or computer interrelations. Suppose that we want to show not only the links among the nodes that represent the people or the computers but also how frequently pairs interact or the subgroups that they are part of, issues faced frequently in visualizations, including those of networks (e. g., Tollis et al., 1998). Effectively diagramming frequency and subgrouping are critical in the design of information systems, where balancing efficiency, rooted in frequency, and security, rooted in subgrouping are central issues. Representing frequency and grouping are basic to other network problems, and, more generally, to statistical and information graphics. Frequency is a paradigmatic continuous variable and grouping is a paradigmatic categorical variable.

Spatial organization in the world suggests some possibilities for representing frequency and grouping, possibilities that have been produced in practice. All other things equal, individuals who are closer in space interact more frequently; conversely, when high interaction is desired, individuals—and computers—are placed in close proximity. *Distance* is arguably the most common way to use space to represent abstract relations, where distance in space indicates distance on some abstract dimension. Individuals or components that form a subgroup are often put in the same enclosed space; similarly, individuals in the same enclosed space are more likely to form a subgroup. That is, subgroups are often in the same *container*. Finally, thicker pipes carry more water, and thicker cables carry more wires. Thus, *thickness* of links connecting components is a natural way to represent the intensity of interrelations, especially among components. Each of these real-world and diagrammatic expressions appears in talk as well. We say we've grown apart or *distant*, we talk about band*width*, we say a system or a group *contains* so and so or such and such as members.

Spatial organizations in the world, then, form a basis for abstract thought. They also form a basis for *congruency* of mapping from the conceptual to the spatial world of a diagram (Tversky, Morrison & Betrancourt, 2002). Because frequency is continuous, it is more congruently matched to a continuous spatial variable, such as distance or thickness; similarly, because grouping is a categorical variable, it is more congruently matched to a categorical variable such as containment. Some support for congruence in mapping continuous and categorical concepts comes from prior work on line and bar graphs, where participants understood and produced lines for continuous variables and bars for discrete ones (Zacks & Tversky, 1999), but this was for mapping only those variables, not for superimposing that information on a structure, as in the present studies.

Given that several visual expressions of frequency and inclusion have been produced, as in the previous research, we turn to ask whether one or some conceptual mappings to space are more effective in performance or more compelling in preference. To insure comparability of performance and preference, both frequency and grouping were treated as binary variables: high vs. low frequency and included or not included in a subgroup. Although the more typical ways of regarding these concepts, as categorical or continuous, is expected to affect performance and preference, particularities of the diagrams and the tasks may modulate the predictions derived from congruence.

## Experiment 1: Performance

Will performance with conceptually congruent mappings be better than with less congruent mappings? That is, will people make more inferences about *frequency* when actors interacting more frequently are represented as closer in proximity or connected by thicker lines than when contained in a common frame? Distance and thickness are continuous, thus more congruent with continuous concepts like frequency, whereas frames are categorical, thus more congruent with categorical concepts like grouping. Will people make more accurate inferences involving *grouping* when groups are contained in the same frame or connected by thicker lines than when they are merely in closer proximity?

## Method

**Participants.** 399 volunteers from Amazon's Mechanical Turk website participated, distributed fairly evenly across 6 conditions. The average age was 30, with a range from 18-63. 56% were male, 45% were native English speakers, and 48% had a college degree. Collecting data on a website increases the range of responders, making the data more representative of a general population, but decreases control, which may add variance to the results, decreasing the chances of finding significance for a fixed sample size.

**Design**. There were six groups. Each participant saw one of three visualizations (Figure 1) and answered one of the two questions below (*Frequency* or *Grouping):*

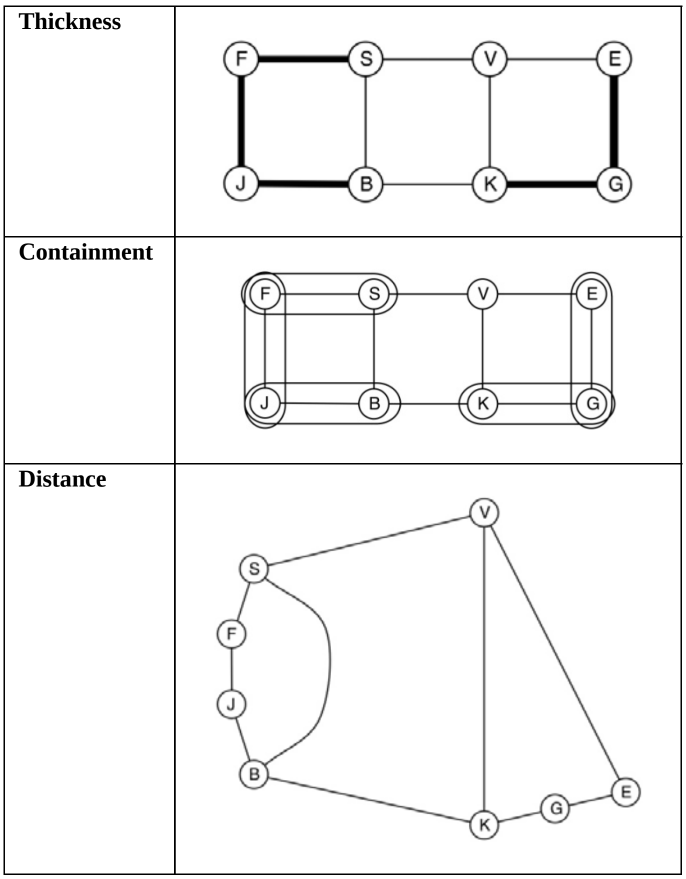


Figure 1: The diagrammatic prompts

*Frequency.* The diagram below represents computers that can all communicate with each other. S and F communicate with each other six times a second. F and J communicate with each other six times a second. J and B communicate with each other six times a second. E and G communicate with each other six times a second. K and G communicate with each other six times a second. All the other communication links shown indicate that the nodes communicate with each other at the rate of one time a second.

S needs to transmit a message to E. Along which pathway will the message arrive first? (Please list all nodes along the pathway.)

*Grouping.* The diagram below represents computers that can all communicate with each other. S and F are part of the same system. F and J are part of the same system. J and B are part of the same system. E and G are part of the same system. K and G are part of the same system. Links within a system are six times as secure as other links.

S needs to transmit a message to E. Which pathway is the most secure? (Please list all nodes along the pathway.)

In pilot experiments using simpler diagrams (such as those in the preference experiment described below), performance was at ceiling. With the more complex diagrams used here, accuracy was about 50%, a level that allowed detection of differences across diagrams and inferences, but makes direct comparison to the preference results more difficult.

## Results

Figures 2 and 3 show the proportion of correct responses to the optimal-path inference questions posed in the frequency and grouping problems. In a log-linear analysis, the three-way association among prompt condition (frequency versus grouping), diagram type (container, distance, line weight), and correctness was significant, $\chi^2(2)=12.16$, $p=.002$, meaning that the effects of the diagram types on correctness differed for frequency and grouping scenarios. Mapping frequency to distance or thickness led to superior performance (Figure 2) compared with mapping frequency to containment, $z = -2.99$, $p=.003$. Distance and thickness mappings did not differ, $z = 0.83$, $p=.407$.

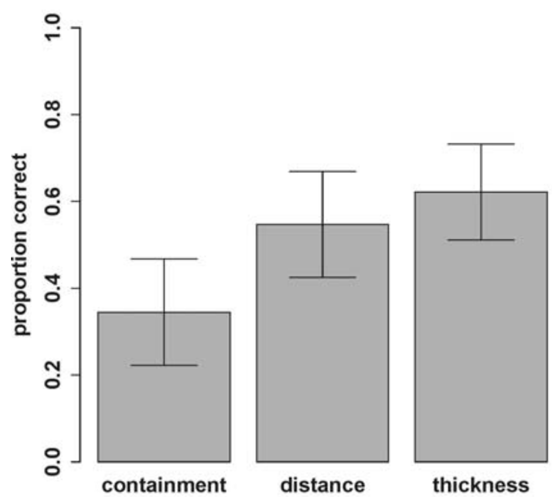

Figure 2: Mean proportions correct for each diagram type for the Frequency problem. Error bars represent the 95% confidence interval for the mean.

In contrast, mapping grouping to containment or thickness led to superior performance (Figure 3) compared with mapping grouping to distance, z=3.30, p=.001. Thickness and containment did not differ, z=1.243, p=.214.

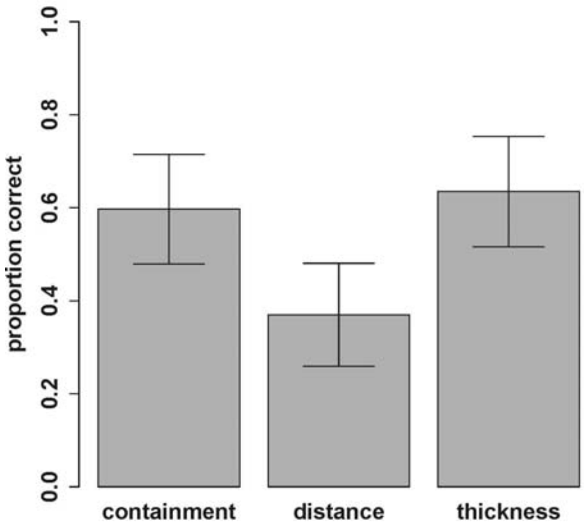


Figure 3: Mean proportions correct for each diagram type for the Grouping problem. Error bars represent the 95% confidence interval for the mean.

## Discussion

Congruence of conceptual mapping can account for the general pattern of results. Participants were more accurate making inferences about frequency when it was mapped to spatial distance or thickness than when it was mapped to containment. Frequency is a continuous conceptual dimension and both distance and thickness are continuous spatial dimensions. Thus the conceptual and visual are congruent.

Inferences about grouping, a categorical relationship, led to a different pattern: participants were more accurate judging groupings of computers when grouping was mapped to containment than to distance. This, too, is a congruent mapping, of a categorical concept, inclusion, to a categorical visual device, a frame. Interestingly, thickness of connection was as good as containment for grouping judgments. In the specific diagrams used in the experiment, thickness had only two levels, so that it could easily be mapped to inclusion, but distance had many levels, hence was more confusing for assessing a categorical concept like inclusion.

# Experiment 2: Preference

Congruence of concept to space accounted for the general pattern of *performance* (i.e., inferences). Here, we examine *preferences* to see if they, too, are congruent. Will people judge the use of boxes to enclose computers belonging to the same system a more natural way to think about grouping and inclusion than putting them close spatially? Will people think putting computer systems that communicate frequently close in space a more natural way to think about frequency than enclosing them?

## Method

A total of 377 volunteers from Amazon's Mechanical Turk website participated. 182 participated in the frequency condition (see below), the others in the grouping condition. The average age was 30, with a range from 18-69. 52% were male, 61% were native English speakers, and 70% had a college degree.

In this experiment participants made preference judgments. Participants compared the three diagrams in Figure 4 in one of the two judgment tasks described below, *Frequency* or *Grouping*.

> *Frequency*. The diagrams below represent computers that can all communicate with each other. K communicates frequently with Z. H communicates frequently T. Neither K nor Z communicates frequently with either H or T.
>
> Choose the diagram that best expresses the description. Choose the second best diagram. Choose the third best diagram.
>
> *Grouping*. These diagrams represent computers that can all communicate with each other. K and Z are part of the same system. H and T are part of the same system.
>
> Choose the diagram that best expresses the description. Choose the second best diagram. Choose the third best diagram.

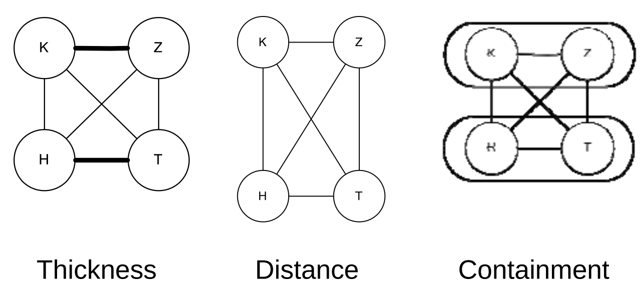


Figure 4: The diagrams presented in the experiment.

## Results

Figures 5 and 6 show the proportions of participants choosing each diagram type as best in the Frequency and Grouping conditions. In a log-linear analysis, the proportions of "best" choices of the three diagram types differed between the two conditions, $\chi2(2) = 83.676$, $p<.001$. In the Frequency condition, line thickness was most often chosen as the best representation, by 48% of participants. Containment was chosen as best by 34% of participants, and distance was chosen as best by only 18%. In the Grouping condition, containment was chosen as the best representation by 70% of participants. Line thickness was chosen as best by only 22% of participants, and distance by only 9%.

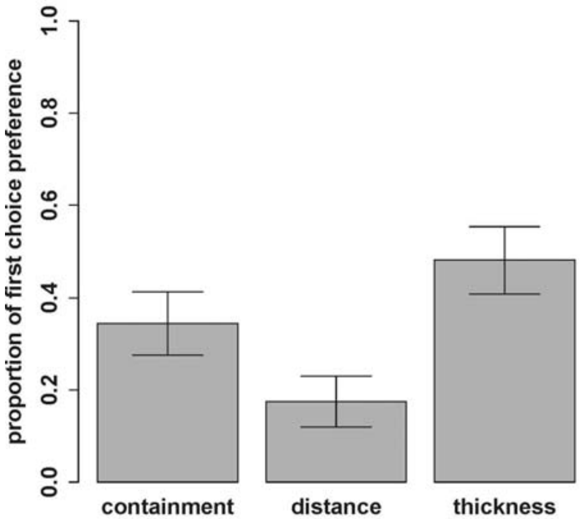


Figure 5: The proportion of participants in the Frequency condition who chose each diagram type as best.

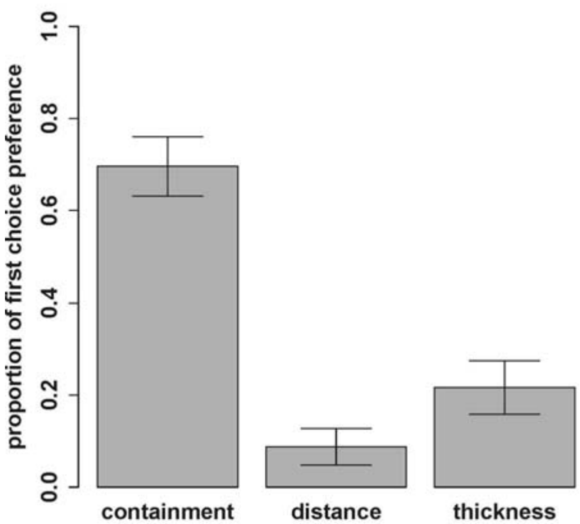


Figure 6: The proportion of participants in the Grouping condition who chose each diagram type as best.

### Discussion

The importance of the congruence of conceptual content to the visual representation is supported by participants' first-choice preferences. To represent grouping, a categorical concept, most participants chose containment, a categorical visual variable. To represent frequency, a continuous concept, the most common choice was thickness, a continuous visual variable.

## General Discussion

Spatial thinking is all around us, in the world, in talk, in gesture, in diagrams. Communication through diagrams can be fast and efficient, and is increasingly common. Mapping spatial entities and spatial relations in the world to spatial elements and relations on the page is fairly straightforward (which is not to say that it is always done well).

But successfully mapping abstract concepts and relations to marks and spatial relations to the page, a metaphoric use of diagrams, can be more complicated. The best mappings use space in ways that are congruent with the abstract concept. For example, since people think of greater height, health, power, and wealth as going upwards, mapping those concepts upwards on a graph is bound to lead to better and faster comprehension and inference. Congruent mappings can be revealed indirectly in language and gesture, and more directly in experiments eliciting production, performance, and preference (e.g., Kessell & Tversky, 2011, Tversky, 2011, in press; Tversky et al., 1991).

Here we explored congruent mappings for a continuous concept, frequency, and a categorical concept, grouping. These representations were superimposed on a graphic structure, a network, rather than appearing in isolation. Interconnected computer systems are usually visualized as nodes linked in a network, much like social networks. It is often desirable to superimpose other information on the networks, notably frequency of interaction of nodes and subgroups of nodes. Several ways to superimpose this information on a network have been proposed in the literature, including distance or lengths of link, thickness of link, and frames or containers (e.g., Bertin, 1981; Harel, 1987).

Since frequency is a continuous concept, mapping it to a continuous spatial variable, either distance or thickness, should be congruent. Since inclusion is a categorical concept, mapping it to frames should be congruent. Congruency predictions were borne out both in performance, making inferences from the diagrams, and in preference. Inferences involving frequency were more accurate when frequency was mapped to the continuous representations, distance and thickness, compared with when it was mapped to the categorical aspect of containment. The opposite held for making inferences involving inclusion or group membership, where containment (but also thickness) led to superior performance. Thickness was actually used as a binary variable in the present graphs, encouraging a categorical interpretation.

For preference judgments, containment was preferred to represent grouping; this was predicted because both relationships are categorical. For representing frequency, the continuous aspect of line thickness was most often chosen as the best representation, as predicted. Distance, though, was a distant third choice, perhaps because in the set of diagrams used in this experiment, the actual distances used in the diagrams were quite small.

Both preference and performance suggest that congruent mappings of concepts to space are effective. Although the specific mappings differed between the experiments, probably because the conditions were not strictly comparable, the overall conclusion holds. Mapping continuous concepts to continuous uses of space and mapping categorical concepts to categorical uses of space were preferred and led to superior performance. These conclusions corroborate earlier work showing congruence of the use of bars for categorical concepts and lines for trends (Zacks & Tversky, 1999). These results also strengthen the case for the general program, of selecting among graphic means to represent abstract concepts and relations by assessing people's productions, preferences, and performance. The findings have broad implications for designing diagrams, for information systems as well as for statistical, scientific, and information graphics in the popular and technical media. The reasoning and the techniques provide a model for empirical methods to reveal design principles. In turn, the findings have implications for the many arenas of life where understanding diagrams is crucial, including navigation in the world, research in science and engineering, and learning in and out of classrooms, Spatial thinking is pervasive and powerful; visualizations can successfully express a range of abstract concepts, as long as the mappings are congruent.

## Acknowledgments

Portions of this research were supported by grants from National Science Foundation IIS-0725223, IIS-0855995, and REC-0440103, the Stanford Regional Visualization and Analysis Center, and Office of Naval Research NOOO14-PP-1-O649, N000140110717, and N000140210534.

## References

Bertin, J. (1981). *Graphics and graphic-information-processing*. N. Y.: Walter de Gruyter.

Boroditsky, L. (2000). Metaphoric structuring: Understanding time through spatial metaphors. *Cognition* (75:1), 1-28.

Cooper, W. E. & Ross, J. R. (1975). World Order. (1975).In R. E. Grossman, L. J. San, and T. J. Vances (Eds.) *Papers from the parasession on functionalism*. Pp. 63-111. Chicago: Chicago Linguistic Society.

Harel, D. (1987). Statecharts: A visual formalism for complex systems. *Science of Computer Programming,* 8, 231-274.

Kessell, A. M. & Tversky, B. (2011).Visualizing space, time, and agents: Production, performance, and preference. *Cognitive Processing*, 12, 43-52. DOI: 10.1007/s10339-010-0379-3

Lakoff, G. & Johnson, M. (1980). *Metaphors we live by*. Chicago: University of Chicago Press.

Landy, D., & Goldstone, R. L. (2007). How abstract is symbolic thought? *Journal of Experimental Psychology: Learning, Memory, & Cognition, 33,* 720-733.

Nickerson, J. V., Tversky, B., Corter, J.E., Yu, L. & Mason, D. (2010). Thinking with networks, *Proceedings of the 32nd Annual Conference of the Cognitive Science Society.*

Norman, D. A. (1993). *Things that make us smart*. Reading, MA: Addison-Wesley.

Novick, L. R., & Hurley, S. M. (2001). To matrix, network, or hierarchy: That is the question. *Cognitive Psychology 42,* 158–216.

Shepard, R. N. (2001). Perceptual-cognitive universals as reflections of the world. *Behavioral and Brain Sciences, 24*, 581-601.

Talmy, L. (1983). How language structures space. In H. L. Pick, Jr. & L. P. Acredolo (Eds.), *Spatial orientation: Theory, research and application*. Pp. 225-282. N. Y.: Plenum.

Tollis, I. G., Di Battista, G., Eades, P., & Tamasssia, R. (1998). *Graph drawing: Algorithms for the visualization of graphs*. NY: Prentice Hall.

Tversky, B. (2001). Spatial schemas in depictions. In M. Gattis (Ed.), *Spatial schemas and abstract thought* Pp. 79–111. Cambridge: MIT Press.

Tversky, B. (2011). Tools for thought. In B. Benedetti and V. Cook (Editors), *Language and bilingual cognition.* Pp. 131-139. NY: Psychology Press.

Tversky, B. (In press). Visualizing thought. *Topics in Cognitive Science.*

Tversky, B., Agrawala, M., Heiser, J., Lee, P. U., Hanrahan, P., Phan, D., Stolte, C., & Daniel, M.-P. (2007). Cognitive design principles for generating visualizations. In G. Allen (Editor). *Applied spatial cognition: From research to cognitive technology. P*p. 53-73. Mahwah, NJ: Erlbaum.

Tversky, B., Kugelmass, S., & Winter, A. (1991). Cross-cultural and developmental trends in graphic productions. *Cognitive Psychology, 23*, 515-557.

Tversky, B., Morrison, J. B. & Betrancourt, M (2002). Animation: Can it facilitate? *International Journal of Human Computer Studies. International Journal of Human Computer Studies*, 57, 247-262.

Tversky, B, Zacks, J., Lee, P. U., & Heiser, J. (2000). Lines, blobs, crosses, and arrows: Diagrammatic communication with schematic figures. In M. Anderson, P. Cheng, and V. Haarslev (Editors). *Theory and application of diagrams*. Pp. 221-230. Berlin: Springer.

Zacks, J., & Tversky, B. (1999). Bars and lines: A study of graphic communication. *Memory and Cognition, 27*, 1073-1079.

Zhang, J. (2000). External representations in complex information processing tasks. In A. Kent (Ed.), *Encyclopedia of library and information science* (Vol. 68, pp. 164-180). New York: Marcel Dekker, Inc.