The Whole Number Bias in Fraction Magnitude Comparisons with Adults

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Abstract

The current study examines the extent to which the whole number bias, especially whole number ordering, can interfere with adult understandings of fractions. Using the framework theory approach to conceptual change as outlined by Vosniadou (2007; Vosniadou, Vamvakoussi & Skopeliti, 2008), this study supports the idea that initial concepts formed in childhood can have lasting effects into adulthood. Twenty-eight CMU undergraduates participated in a fraction magnitude comparison task. Half of the fraction comparisons were designed with the larger fraction consistent with whole number ordering; the other half was inconsistent with this ordering. Comparisons in the consistent condition had the larger magnitude fraction have larger whole number parts than the opposing fraction. Comparisons in the inconsistent condition were the opposite. Participants were more accurate and faster to respond to comparisons in the consistent condition, supporting the hypothesis that an initial concept of number as natural number constrains operations with fractions even in adults.

Keywords: conceptual change; mathematical development; fractions; whole number bias

Introduction

In recent years conceptual change approaches to learning have been applied to mathematics in order to examine how mathematical concepts develop (Vosniadou & Verschaffel, 2004; Gelman & Williams, 1998). The discussion of fractions from a conceptual change approach can shed light on why so many students have trouble developing their conceptual understanding of fractions (National Council of Mathematics, 2007; Mazzocco & Delvin, 2008). Particularly, the framework theory approach has been shown to have strong explanatory power for the phenomena found within fractions misconceptions (Stafylidou & Vosniadou, 2004; Christou & Vosniadou, in press; Vamvakoussi & Vosniadou, 2010).

The framework theory approach to conceptual change draws attention to the intuitive theories that children develop based on their experiences with their environment and prior knowledge. Vosniadou & Verschaffel (2004) argue that before they are exposed to rational number, students have formed an initial concept of number, which is based on the act of counting and resembles the mathematical concept of natural number (see also Ni & Zhou, 2005). This initial number concept is a complex knowledge system encompassing a number of background assumptions and beliefs that underlie students' expectations about what counts as a number and how it is supposed to behave (Vamvakoussi & Vosniadou, 2004, 2007; Vosniadou et al., 2008; see also Smith et al. 2005). For example, children consider that numbers are answers to the 'how many' question, obey the successor principle (in the sense that when a number is given its unique successor can always be found), and are ordered by means of their position on the count list, with longer numbers being always bigger.

According to the framework theory, students' initial concept of number constrains their interpretation of new information regarding rational number causing persistent misconceptions (Vosniadou, Vamvakoussi & Skopeliti, 2008). Misconceptions such as 'multiplication always makes bigger' and 'the bigger the terms the bigger the fraction' reveal the interference of rational number reasoning on rational number tasks. The framework theory suggests that misconceptions are often caused as students add the new, incompatible information to their initial concept. Such misconceptions represent 'synthetic' attempts that can be thought of as evidence of a progression toward a scientific model and therefore as a part of the learning process (Vosniadou, Vamvakoussi & Skopeliti, 2008).

Other researchers have also noticed that in the process of building a scientific understanding, elements of the initial concept can be very difficult to overcome and may remain intact. Inagaki & Hatano, (2008) found evidence for this idea in the domain of biology and Dunbar, Fugelsang and Stein (2007) in the domain of physics. Dunbar et al. (2007) looked at the presence of "impetus theory" conceptions in students during an fMRI task. They found that while some participants exhibited an understanding of the Newtonian theory, there was evidence that they still maintained elements of the impetus theory conception. Physics' students have to go through the difficult process of re-representing their initial concept of force. The Dunbar et al. (2007) findings indicate that even after transitioning to the scientific concept, remnants of the initial theory can still be maintained, interfering at times with the scientific concept.

It appears that something similar also happens in the case of rational number. When learning about rational numbers in any form (fraction or decimal) students often use the properties of whole numbers to interpret rational numbers (Ni & Zhou, 2005; Smith, Solomon, & Carey, 2005; Vamvakoussi & Vosniadou, 2007). Vamvakoussi and Vosniadou (2010) found that students categorize fractions and decimals as countable and discrete like natural numbers. Natural numbers are countable because the numbers have a specific order that is linked to magnitude. However, no such countable relation exists for fractions because of the infinite number of ways to express a single magnitude. Vamvakoussi and Vosniadou (2010) found that students used their prior knowledge as a framework for interpreting the properties of fractions and decimals. They applied what they knew about natural numbers to fractions and decimals when asked about the countability and discreteness of fractions and decimals.

According to Stafylidou and Vosniadou (2004) children have three main explanatory frameworks for understanding what fractions are: 1) fractions as two independent numbers, 2) fractions as parts of a whole, 3) a ratio relationship between numerator and denominator that can be bigger, smaller, or equal to a whole. The third explanatory framework closely represents the integrated magnitude representation of fractions that would characterize the scientific model of fraction magnitude. However, Stafylidou and Vosniadou (2004) found that most children have understandings of fractions that are more closely related to the first two explanatory frameworks.

There is evidence that adults widely maintain an integrated magnitude representation of fractions. Schneider and Siegler (2010) have found that on magnitude comparison tasks that elicit processing of fractions as parts in relation to the whole, adults show a distance effect such that when the two fractions in the comparison have a greater magnitude difference, participants are faster to indicate which fraction is larger than when the pairs have lesser magnitude difference. This implies that adults may have a mental number line representation of fractions (Schneider and Siegler, 2010). However, in situations where comparisons can be made that do not require processing of the fraction as a whole magnitude, the distance effect is not present (Bonato, Fabbri, Umilta & Zorzi, 2007). This suggests that strategy use on fraction magnitude comparisons is dependent on the fractions that are going to be compared.

Further, the results of Schneider and Siegler (2010) and Bonato et al. (2007) suggest that when adults are able to compare fractions using only the whole number parts of the fractions, they use this method instead of looking at the integrated magnitudes of the fractions. This would suggest that adults first look at fractions in terms of their whole number parts. If they can make a judgment about their magnitudes based only on their whole number parts then adults will do this. Nevertheless, they are able to think of fractions as integrated magnitudes when their whole number parts do not yield correct answers. This suggests that adults still maintain elements of the whole number bias they develop in childhood.

Consistent with the framework theory approach to conceptual change, adults have developed a scientific model of fractions but the strength and intuition of whole number ordering may interfere with this scientific model. The current study seeks to understand the relationship between whole number ordering and judgments about fraction magnitude among adults with a fraction magnitude comparison task.

The hypothesis is that when the larger fraction in the comparison has larger whole number parts than the other fraction and is thus consistent with whole number ordering, participants will be significantly more accurate and will have significantly faster reaction times than when the larger fraction in the comparison has smaller whole number parts than the other fraction and thus inconsistent with whole number ordering.

Methods

Participants

Twenty eight undergraduates from Carnegie Mellon University taking introductory psychology courses participated in the study to fulfill a requirement for their course. The average age of students was 19.6 (*SD*= 1.34). There were equal numbers of males and females.

Design

The study was a within-subject design with 2 conditions (consistent vs. inconsistent).

In the consistent condition, the fractions were designed so that the fraction with the larger overall magnitude was made up of larger numbers compared to the opposing fraction, for example, 2/5 and 5/7 where 5/7 is larger. Because the fraction's whole number parts and the actual magnitude of the fraction was larger, these fractions were consistent with whole number ordering.

The fractions in the inconsistent condition were designed in the opposite way. The larger fraction in the comparison was designed with smaller whole number parts than the opposing fraction, for example, 3/7 and 2/3 where 2/3 is larger (see table 1 for more examples).

Overall, there were 40 fraction pairs. 20 of the pairs were in the consistent condition and 20 in the inconsistent condition. The fractions were carefully counterbalanced in each condition to account for the following features. The

Table 1: Sampling of Stimuli Used

Description	Consistent		Inconsistent	
Single Digits Opposite sides of 1/2	2/5	**7/8**	3/8	**2/3**
Single Digits Same side of 1/2	6/8	**7/9**	5/8	**2/3**
Double Digits Opposite sides of 1/2	4/11	**13/18**	8/19	**6/11**
Double Digits Same sides of 1/2	9/16	**17/19**	13/18	**11/13**
Greater than 1	11/8	**18/12**	18/14	**17/11**

fractions were designed to have pairs that were on different sides of ½, the same side of ½, have only single digits or have double digits, both fractions greater than 1 or less than 1, and be either in completely reduced form or be reducible. The two sets of 20 pairs were equated for each of these properties. Additionally, the fraction pairs were counterbalanced so that half of the time the larger fraction appeared on the right side of the screen and the other half of the time the larger fraction appeared on the left side of the screen.

Measures

Accuracy and reaction time (ms) were recorded using E-Prime for each of the fraction comparisons. Participants were instructed that speed and accuracy were both important.

Procedure

Participants completed a computerized magnitude comparison task designed with E-Prime. After obtaining informed consent, participants read the instructions on the computer screen, which explained that they should complete the comparisons as fast as possible and as accurately as possible. For each participant, the order of the fraction comparisons was completely randomized. Followed by the presentation of a 500 ms fixation cross, participants saw two fractions on the screen. They were instructed to press a key on the right side of the keyboard to indicate that the fraction on the right was larger. Conversely, the participants had to press a key on the left side of the keyboard to indicate that the fraction on the left was larger.

Results

Accuracy

Participants completed the comparisons that were consistent with whole number order more accurately. The mean accuracy of the comparisons completed in the consistent group was 86% (*SD* = 13%) and the mean accuracy of the inconsistent group was 77% (*SD*= 12%) ($t(27) = 2.92$, $p<.01$) (see table 1 and figure 1).

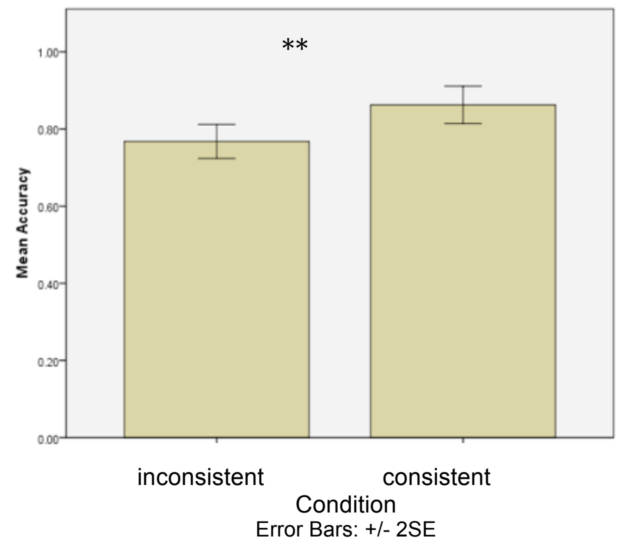

Figure 1: Mean Accuracy

Reaction Time

Reaction times (ms) for the consistent condition were significantly shorter than reaction times for the inconsistent condition including inaccurate trials. The mean reaction time for the consistent condition was 3378 (*SD* = 1525) and the mean reaction time for the inconsistent condition was 3665 (*SD* = 1625) ($t(27)=2.22$, $p=.03$) (see table 1 and figure 2).

The reaction times were also significantly shorter for the consistent condition compared to the inconsistent condition for only accurate trials. The mean reaction time for the consistent condition was 3240 (*SD* = 1467) and the mean reaction time for the inconsistent condition was 3619 (*SD* = 1579) ($t(27) = 2.67$, $p=.01$) (see table 2 and figure 3).

Table 2: Mean Accuracy and Reaction Time (ms)

Condition	Accuracy **	Reaction Time (ms) all trials *	Reaction Time (ms) only accurate trials **
Consistent	86%	3378	3240
Inconsistent	77%	3665	3619
Overall	82%	3521	3430

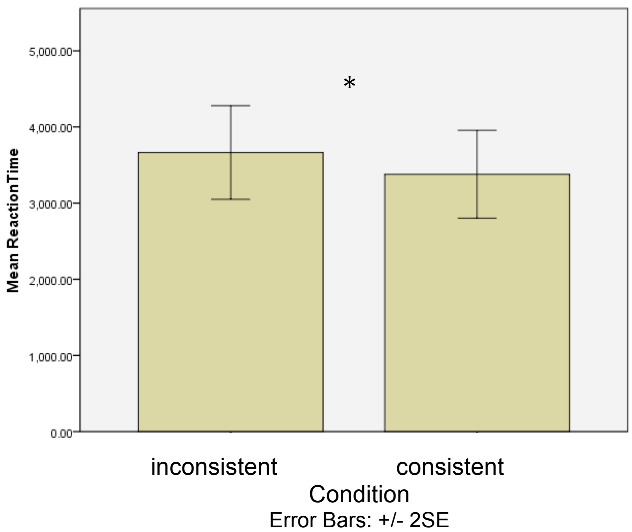

Figure 2: Mean Reaction Time (ms) all trials

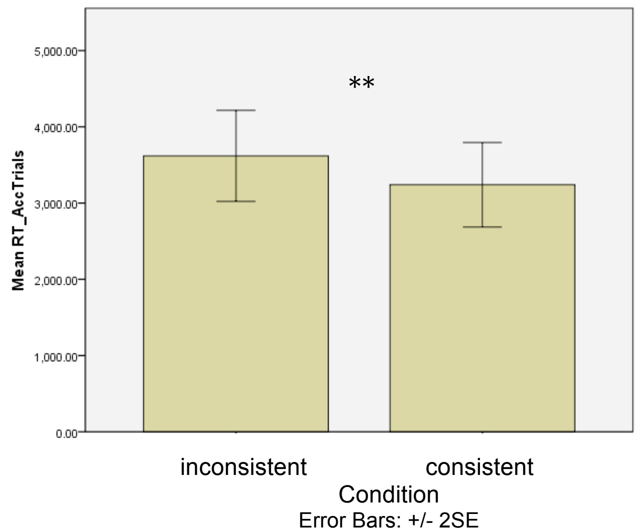

Figure 3: Mean Reaction Time (ms) accurate trials only

Analysis of Errors

There were a number of errors made in both the consistent and inconsistent conditions. The mean reaction time of the errors of the consistent condition was 4380 ms; this was 1140 ms slower than the reaction time for the correct trials for the consistent condition (*M* = 3240). 60% of the comparisons that had errors in the consistent condition were comparisons that involved double digit numbers. This suggests that these problems may be more difficult overall. In fact, 67% of the comparisons that had errors in the inconsistent condition were also comparisons that involved double digit numbers. There was a significant interaction between condition and double digits for accuracy, $F(1,27) = 21.263$, $p<.001$, and for reaction time, $F(1,27) = 6.585$, $p=.01$. The mean reaction time of the errors in the inconsistent condition was 3485 ms; this was 134 ms faster than the correct trials for the inconsistent condition (*M* = 3619) (see table 2).

Individual Differences

Some individual participants had mean accuracies and reaction times in directions opposite of the main findings. An analysis of the mean accuracy (*M*= 86% consistent, *M*= 89% inconsistent) showed no significant difference between the means, $F(1,6)=.96$, $p=.33$. The same is true of an analysis of mean reaction time (*M* = 3998 consistent, *M* = 3651, $F(1,9)=.9$, $p=.34$). This indicates that there was no difference in the reaction times and means for these participants between the two conditions. Of the 6 participants that exhibited higher accuracy for the inconsistent condition than the consistent condition, 3 had mean accuracies of 85% for the consistent condition and 90% for the inconsistent condition. This is a difference of 1 more trial incorrect in the consistent condition than the inconsistent condition. The other three participants had average overall accuracies of 71% which was well below the overall accuracy of all participants, 82%.

Of the 10 participants whose reaction times were slower for the consistent condition than the inconsistent condition, 4 participants also had mean accuracies that were the reverse of the main findings.
The overall mean reaction time of these 10 participants was 3610, which is slightly slower than the overall mean reaction time (*M=3521)*.

Table 3: Analysis of Errors by Condition

Condition	Mean Reaction Time (ms)	Standard Deviation of RT (ms)	Percent of Incorrect Comparisons that had 2 digits
Consistent	4383	1473	60%
Inconsistent	3294	1230	67%

Discussion

Consistent with the hypothesis, participants were more accurate and had faster reaction times for the magnitude comparisons that were consistent with whole number ordering compared to the magnitude comparisons that were inconsistent with whole number ordering. Participants were 9% more accurate and 287 ms faster over all trials, 379 ms faster for only accurate trials, in the consistent condition compared to the inconsistent condition. While the reaction times are very close, the difference may reflect the extra processing demanded by the inconsistent condition.

These reaction time data support the hypothesis that participants exhibit some interference from their initial concept of whole number, which they have to inhibit. The accuracy results indicate that the participants are not always successful in inhibiting the response consistent with whole number ordering – they often just use the whole number parts to assess magnitude. This conclusion is also consistent with the reaction time data for errors in the two conditions. Reactions times were faster for erroneous responses in the inconsistent condition- presumably because the participants incorrectly used whole number ordering - but slower when the errors were committed in the consistent condition - presumably because the participants had to override the (correct in this case) habitual response consistent with whole number ordering.

Overall these results support the framework theory approach in showing that even in the case of adults whole number ordering can still interfere and delay responses or cause errors in responses. In previous work we have shown that the whole number bias can be the source of a number of misconceptions often found in research investigating the development of primary and secondary students' understanding of rational number (Stafylidou & Vosniadou, 2004; Vamvakoussi & Vosniadou, 2010). The present findings indicate that the whole number bias can persist even in the case of college students who have presumably achieved a more sophisticated understanding of fractions and are not likely to commit the kinds of misconceptions found in younger students.

These results are consistent with the findings of Dunbar et al. (2007) in which participants exhibited interference from their initial concepts of mechanics and physics. In the current study, the results support the idea that participants had interference from their concept of whole number. The participants were slower to respond and less accurate in their responses when these responses required the inhibition of a competing response. The results also consistent with the Dunbar et al. (2007) argument that adults must change from one knowledge representation to another during the process of conceptual change.

Additionally, these results can contribute to the discussion between Schneider and Siegler (2010) and Bonato et al. (2007) about how adults represent fraction magnitudes. First, it is clear that adults do not represent fractions only as whole number parts. If they did they would never be able to succeed in the present task. On the other hand, it is also clear that adults do not always use an integrated magnitude representation. Rather the present study provides more support to the idea that adults may have different representations of fractions (whole number parts, integrated magnitudes, or others) and may use whichever representation is simplest and most readily available for a particular stimulus pair. In the current study, participants may have selected a strategy that was quick and usually correct in order to maximize accuracy and minimize response time. Thus, adults may have different strategies that are context dependent.

The finding that the difference between reaction times increases when examining only accurate trials suggests that even when participants were able to generate the correct judgment about magnitude, it is probable that they may have started out with a whole number parts model that was later modified. Alternatively, the increase in difference may indicate that participants were trying to be more methodical for these questions by using the integrated magnitude representation. The finding that the inaccurate trials were faster offers more evidence supporting the interpretation that participants were not successful in using the integrated magnitude model.

Overall, this study supports the idea that it is fruitful to examine the development of fraction understanding from a conceptual change point of view (Vosniadou & Verschaffel, 2004). There is little doubt that children start with a concept of number which is more consistent with the mathematical concept of natural number and that this initial concept constrains the development of fraction understanding and can be the cause of persistent misconceptions. The results of the present study indicate that this whole number bias is so strong that it persists in its interference with fraction magnitude tasks even into adulthood. Although adults are able to overcome this whole number bias it affects the accuracy and speed of their responses.

It appears that the nature of the initial concepts children create may have important implications for the development of the more advanced scientific and mathematical concepts of adulthood. The overgeneralization of the properties of whole numbers, like counting, to fractions needs to be addressed in order to decrease the amount of interference the whole number bias has in adulthood. More importantly, we can learn more about adults' difficulties in understanding scientific concepts by examining their initial and synthetic constructions. The failure to reconcile entrenched presuppositions related to initial concepts in childhood can have lasting negative effects for adult understandings of scientific concepts.

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