Placing Numbers in Behavioral Space: Activity-Specific Interactions between Number and Space with a Single Response Button

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Abstract
How are we able to reason about abstract concepts that lie resolutely beyond the reach of perception? One strategy is to ground understanding in space. Numbers, for instance, are known to interact with egocentric space during rapid numerical judgments. A range of experimental results have demonstrated that, among literate Western people, this “mental number-line” goes from left to right, with smaller numbers associated with left space, and larger numbers with right space. But what is the nature of this “space”? Previous work has conflated multiple possible egocentric frames of reference—head-based, eye-based, action-based—leaving it unclear which space is interacting with number. In the present paper, two studies investigated whether a single centrally-located button, stationary in hand- and eye-based coordinates, can nevertheless exhibit different spatial properties in virtue of task-specific activity. In a go/no-go paradigm, participants judged the magnitude (Exp. 1) and parity (Exp. 2) of single-digit numbers. Crucially, they responded only with the index or middle finger of a single hand. While judging magnitude (Exp. 1), participants were faster to respond to smaller numbers with the more leftward finger, and larger numbers with the more rightward finger, regardless of the hand being used. This effect disappeared when judging parity (Exp. 2), replaced by finger-specific associations on the left hand only. In sum, in a task-sensitive way, participants associated numbers with egocentric space—but a behavioral space defined relative to embodied interaction rather than head- or eye-based reference frames. We discuss implications for number representation and the nature of “space” in embodied activity.

Keywords: number; space; action; SNARC; embodiment; go/no-go; frames of reference; Merleau-Ponty

Introduction
“[The body’s] spatiality is not, like that of external objects or like that of ‘spatial sensations’, a spatiality of position, but a spatiality of situation.”- Merleau-Ponty (1962, p. 114).

How are we able to reason about abstract concepts that lie resolutely beyond the reach of perception? One strategy is to ground understanding in space. Numbers, for instance, are tightly linked with space across human activity. We recycle the language of space to talk about numbers, counting up to higher numbers and down to lower numbers, and use space to reason about numbers as abstract concepts (Lakoff & Núñez, 2000; Núñez & Marghetis, to appear). Mathematical diagrams often associate numbers with particular locations. And number interacts with space in less explicit ways during the online performance of mathematical activities. In a seminal study, Dehaene and colleagues (1993) asked participants to judge the magnitude (greater or less than 5?) or parity (even or odd?) of single digit numbers. Participants were reliably faster to respond to smaller numbers when responding with a button in left space, and to larger numbers when responding in right space—as if they were spontaneously thinking of numbers along a left-to-right “mental number line.” This interaction between numerical magnitude and spatial location has been dubbed the “SNARC” effect.

In the two decades since, the literature on such number-space associations has exploded (Hubbard et al., 2005; Wood et al., 2008). Similar effects have been found with bipedal responses (Schwarz & Müller, 2006) and saccades (Fischer et al., 2003; Schwarz & Keus, 2004), further reinforcing the genuinely spatial nature of this effect. While the particular direction of this “mental number-line” is quite flexible, shaped by such factors as habitual reading direction (Shaki, Fischer, & Petrusie, 2009) and recent experience (Fischer, Mills, & Shaki, 2010), there is a growing consensus that number and space are intimately related.

But doubts remain. A number of authors have suggested that the effects have less to do with a stable spatial representation of number, and more to do with flexible or non-spatial associations (e.g., Fischer et al., 2010; Gevers et al, 2010; Santens and Gevers, 2006). Others have pointed out that most studies force participants to respond spatially, using buttons that are distinguished by their spatial location, and thus implicitly inject space in virtue of the experimental setup (Núñez, Doan & Nikouлина, 2011). Indeed, when other response modalities are used—e.g., responding with higher or lower pitch—participants exhibit interactions that are SNARC-like but non-spatial (Marghetis et al., 2011). Number-space associations, therefore, may be more flexible and context-sensitive than first assumed.

Beyond these concerns, one additional question has been largely unaddressed: the nature of the “space” that sometimes, undeniably, interacts with number. Previous work has conflated the multiple egocentric frames of reference that we use to encode space, which include head-,
eye-, object-, and action-based frames of reference (cf. Cohen and Anderson, 2002). For instance, in the classic SNARC paradigm, the response buttons are placed to the left and right of the participant’s body, and thus are distinguished similarly in multiple frames of reference: they are lateralized relative to participants’ heads, eyes, and motor responses. When space is forced on the subject in this way, through a forced-choice paradigm where responses are spatially distinguished, it remains unclear as to which “space” is interacting with number.

One possibility is that numbers interact with behavioral space, i.e. the space defined relative to task-specific embodied activity. If this is the case, then observation of the SNARC effect may not depend on the presence of spatial responses distinguished externally in head- or eye-based coordinates, but rather on some juxtaposition of task-relevant actions within the specific context of the task. Testing this possibility requires an experimental design that can isolate effects of behavioral space from those due to head- or eye-based coordinates. We took up this challenge by investigating whether SNARC-like effects could be elicited using only a single, centrally-placed response button, which remained in the same location relative to head- and eye-based spatial coordinates. By requiring participants to press the button with different fingers of the same hand, the stationary button can move relative to task-relevant embodied activity—that is, in behavioral space.

In two studies, participants judged the magnitude (Exp. 1) and parity (Exp. 2) of single-digit Arabic numerals. Crucially, and in contrast to previous studies, we used a go/no-go paradigm in both tasks: participants responded by pressing a single centrally-located button with only their index or middle finger. The finger used was manipulated between blocks, as was the “go” response criterion, while response hand was varied between subjects. By using a central button, kept stationary relative to head- and eye-based frames of reference, and by using a go/no-go paradigm in which different response fingers were never juxtaposed spatially within a block, but only temporally across blocks, we were able to focus on one particular spatial frame of reference: behavioral space, the space defined by the possible embodied actions within the task as a whole. Moreover, by manipulating the response finger, identified only by name (e.g. “index finger”), we were able to avoid any explicit spatial instructions.

If the SNARC effect is driven entirely by visual head- or eye-based frames of reference, then participants should not exhibit any SNARC-like effects here, since at no point are two response options spatially juxtaposed relative to head or eye. If, on the other hand, number interacts with space as enacted by task-specific interactions—that is, behavioral space—then we may find numbers systematically associated with space relative to activity-based frame of reference, with the more leftward finger faster for smaller numbers, and the more rightward finger faster for larger numbers.

**Experiment 1: Magnitude Task**

**Participants**

Undergraduate students (n=32, mean age = 20, 19 females) from a major research university participated in exchange for partial course credit.

**Procedure**

In a go/no-go paradigm, participants judged the relative magnitude of visually-presented single-digit numerals, responding only if the number presented was greater than/less than 5. Participants responded by pressing a single, centrally-located button on a Serial Response Box placed at a comfortable distance in front of them, using either the index finger or middle finger. Response finger (index or middle) and response criterion (greater or less than 5) were fully crossed within participants, so each run had four blocks. Block order was counterbalanced, except that no two consecutive blocks used the same response finger (to avoid muscle fatigue). Response hand (left, right) varied between participants, so each participant maintained the same response hand throughout the experiment. Each participant, therefore, responded with their index and middle fingers for two blocks each.

Each trial began with a central fixation cross (500 ms.), followed by a centrally-presented single-digit number between 1 and 9 (excluding 5). If participants responded, the number would disappear; otherwise it would remain on the screen for 3 s., after which the trial would end automatically. See Figure 1. Each block began with 8 practice trials, followed by 80 experimental trials.

**Results**

One participant was removed for failing to complete the experiment. Accuracy for the remaining 31 participants was high (M=.99, SD=.004). Mean accuracy was analyzed with a 2 x 2 x 2 x 2 mixed ANOVA, with Magnitude (greater or less than 5), Parity (even, odd), and Response Finger (left-finger, right-finger) as within-subjects factors, and Response Hand (left or right) as a between-subjects factor. There were no significant effects on accuracy.

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2 Initial analyses found no effects of participants’ handedness (Dehaene et al, 1993); it was thus removed from further analyses.

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![Figure 1: Paradigm for Experiments 1 and 2. Here, the "Go" criterion would be to respond with the index finger of the left hand, if less than 5 (Exp. 1) or even (Exp. 2).](image-url)
Before analyzing response time, incorrect trials were removed, followed by trials with reaction times that were slower than three standard deviations above each participant’s mean response time, or faster than 200ms (n=83, 1.7% of total trials).

Reaction times were analyzed with a 2 x 2 x 2 x 2 mixed ANOVA, with Magnitude (greater or less than 5), Parity (even, odd), and Response Finger (left-, right-finger) as within-subjects factors, and Response Hand (left, right) as a between-subjects factor. There was a main effect of Parity, with responses to odd numbers significantly faster than responses to even numbers (M_{odd} = 382ms, M_{even} = 398ms, F(1,29) = 54.8, p < 0.001; cf. Dehaene et al, 1993, who found no effect of Parity). There was also a marginally significant—but difficult to interpret—interaction between Parity and Response Hand (F(1, 29) = 4.1, p = .053).

Crucially, the only other significant effect was an interaction between Magnitude and Response Finger (F(1.29)=4.95, p=0.034, $\eta^2_p=.015$). Responses with the left-finger (i.e. index finger of right hand, or middle finger of left hand) were faster for numbers less than 5, while responses with the right-finger were faster for numbers greater than 5 (Fig. 2). There was no three-way interaction between Magnitude, Response Finger, and Response Hand (F(1, 29) = 0.5, p=0.46), suggesting that the effect is due not to finger-specific associations, but to the location of the response fingers in each participant’s behavioral space. The interaction between Magnitude and Response Hand, notably, was not significant (F(1,29)=2.94, p=0.10).

Discussion

When judging the magnitude of single-digit numbers, participants systematically associated smaller numbers with the left, and larger numbers with the right, even though the response button did not change location relative to head- or eye-based coordinates. Rather, magnitude was associated with locations in behavioral space, defined by the possible actions within the task: responding with one of two possible fingers. This was confirmed by the lack of a three-way interaction with Response Hand. In other words, numbers were not associated with particular fingers, as we might expect if participants were using a body-based frame of reference (e.g. DiLuca et al, 2006). Instead, they were associated with particular actions relative to task behavior: responses with the leftmost finger compared to responses with the rightmost finger. Moreover, this interaction arose despite the fact that left- and right-fingered responses were never juxtaposed within a single trial or block, but only manipulated between blocks and thus juxtaposed within the experiment as a whole.

Notably the interaction between Response Hand and Magnitude was not significant, contra the results of studies that have used bimanual responses (e.g. Dehaene et al, 1993; for review, see Wood et al, 2008). We attribute this to the fact that we manipulated response hand between subjects, not within. Responses with the left- and right-

Experiment 2: Parity Task

Participants

Undergraduate students (n=32, mean age = 21, 22 females) from a major research university, who had not participated in the first experiment, participated in exchange for partial course credit.

Procedure

The procedure was identical to Experiment 1, except participants had to respond based on the parity (even vs. odd) of each number, rather than the magnitude. For a given block, therefore, participants would only respond if the number was even / odd (Fig. 1). All nine numbers from 1 to 9 were used as stimuli, so each block began with 9 practice trials followed by 90 experimental trials.

Results

Accuracy was high (M=.99, SD=.01). Mean accuracy was analyzed with a 2 x 2 x 2 x 2 mixed ANOVA, with Magnitude (greater or less than 5), Parity (even, odd), and Response Finger (left-finger, right-finger) as within-subjects factors, and Response Hand (left or right) as a between-subjects factor. The only significant effect was a main effect of Parity (F(1,30)=10.1, p<0.01), with responses to odd numbers more accurate than those to even numbers (M_{odd}=98.6%, M_{even}=97.6%).
Once again, before analyzing response times, incorrect trials were removed, followed by trials with reaction times that were slower than three standard deviations above each participant’s mean response time (n=85, 1.7% of trials). Finally, trials where the target numeral was 5 were also removed, so we could include Magnitude (greater or less than 5) as a factor in our analysis.

In contrast with Experiment 1, numbers were not reliably associated with response side. Instead, while there was a marginal interaction between Magnitude and Finger Side ($F(1,30)=3.3$, $p=.08$), this was complicated by a significant three-way interaction with Response Hand ($F(1,30)=9.3$, $p=0.005$). There was a similar effect for Parity: while there was no two-way interaction between Parity and Finger Side ($F(1,30)=2.1$, $p=0.16$), there was a significant three-way interaction with Response Hand ($F(1,30)=4.9$, $p=0.035$). For both Magnitude and Parity, the effect was driven by the left hand, where the index finger was faster for odd or smaller numbers, and the middle finger was faster for even or larger numbers (Fig. 3). The only other significant effects were main effects of Parity and Magnitude ($F(1,30)=9.0$, $p=0.005$, and $F(1,30)=5.05$, $p=0.03$, respectively), and a hard-to-interpret three-way interaction between Magnitude, Parity, and Hand ($F(1,30)=4.2$, $p=0.05$).

That the effect was driven by the left hand was confirmed by separate repeated-measures ANOVAs for each hand, with Response Finger (index, middle), Parity, and Magnitude as factors. For the right hand, Response Finger did not interact with Parity ($F(1,15)=0.2$, $p=0.90$) or with Magnitude ($F(1,15)=0.02$, $p=0.88$). But for the left hand there were significant interactions between Response Finger and Magnitude ($F(1,15)=5.07$, $p=0.039$), and between Response Finger and Parity ($F(1,15)=7.1$, $p=0.02$).

**Discussion**

When tasked with determining the parity of single-digit numbers, participants no longer exhibited the SNARC-like effect found in Experiment 1. Instead, we found finger-specific associations with both parity and magnitude, but only on the left hand. A change in task, therefore, induced new associations with numbers, tied to specific fingers rather than to a more general behavioral space. We return to the possible origins of these finger-specific associations in the General Discussion.

**General Discussion**

What space do numbers inhabit? We conducted two studies to investigate the possibility that the form of “egocentric” space with which number interacts is *behavioral* space, the space of possible embodied interaction with the world. Indeed, contrary to what we would expect if number-space interactions are driven entirely by head- or eye-based coordinates, we found that numbers interacted with space even when participants responded with only a single, centrally-located button. This effect, moreover, was not driven entirely by body-based representations, since they were not specific to particular fingers or hands. Instead, responses to smaller numbers were faster with the more leftward finger of either hand, while responses to larger numbers were faster with the more rightward finger—a left-to-right “mental number line” defined entirely in terms of the embodied interaction between finger and apparatus.

This effect, however, was task-dependent, and disappeared when magnitude was not task-relevant. Instead, when participants were attending to parity, they associated specific fingers of the left hand with parity and magnitude: the index finger with odd or small numbers, and the middle finger with even or large numbers. The space with which numbers interacted, therefore, was flexibly tied to body and activity in a task-specific way.

**Task differences and finger-based representations**

What might account for the different results of Experiments 1 and 2? One possibility is that parity and magnitude tasks require participants to attend to different information. Parity tasks, for instance, may activate linguistic and categorical representations, while magnitude tasks may activate analog visuospatial representations (van Dijk, Gevers & Fias, 2009). It may be that the rather subtle spatial difference between fingers is sufficiently small that an interaction between magnitude and space requires the explicit activation of analog visuospatial representations of magnitude. Alternatively, since the classic bimanual Parity task explicitly distinguishes the response options by their positions on the left and the right, those linguistic labels may interact with categorical representations of numerical magnitude (cf. Proctor and Cho, 2006), explaining why the SNARC effect is seen in these types of Parity tasks but not in our deliberately modified setup.

Why, then, did magnitude and parity interact with specific fingers during the Parity task? One possibility is that the associations exhibited in Experiment 2 originate in culturally-specific gestures for numbers. In Quentin Tarantino’s film *Inglourious Basterds*, an American spy
posing as a German is exposed when he orders two beers with the American rather than the German gesture: index and middle finger extended, instead of thumb and index finger. The participants at our American university, therefore, may have finger-specific associations as a result of the fingers they use to gesture for numbers: a single extended index finger for one, adding the middle finger for two. Within our task, these index-one and middle-two associations may have been extended to the rest of the numbers, with smaller or odd numbers associated with the index finger, and larger or even numbers associated with the middle finger—much like the relation between one (smaller and odd) and two (larger and even). This is only speculation, of course, although it does make a specific prediction: German participants should behave differently on our Parity task, responding faster with the thumb for smaller or odd numbers, and faster with the index finger for larger or even numbers, if we test these two fingers instead.

The results of Experiment 2 are illuminated further by recent research on finger-based representations of numbers. Fischer and colleagues, for instance, have suggested that stable finger-counting routines may explain cross-cultural variability in the direction of the SNARC; native English speakers, for instance, may count from left-to-right on their fingers, and also exhibit a left-to-right SNARC (Fischer, 2008; Lindemann, Alipour, and Fischer, 2011). Others, however, have found right-handed native English speakers to be ten times more likely to start counting on their right hand than on their left (Tschantzcher et al., 2012), a pattern that we have also observed in pilot studies. Additionally, associations have been found (in the form of response-time facilitation effects) between specific numbers and the fingers used for those numbers in a habitual finger-counting routine (e.g. di Luca et al., 2006). The current study, by contrast, found categorical (i.e. magnitude and parity) rather than number-specific associations with finger. It may be that specific finger-number associations are only salient when multiple fingers are spatially juxtaposed at a single time.

**Which spaces?**

The current results do not rule out the possibility that other spatial frames of reference also contribute to known interactions between number and space. Head- and eye-based coordinates may also play a role, and future studies should contrive to situate response-buttons in ways that tease apart the contributions of head- and eye-based coordinates, both from each other and from an action-based frame. Indeed, the classic SNARC effects may have been so pronounced exactly because they conflated multiple complementary frames of reference, which conspired to produce particularly strong effects.

Moreover, number may be associated with still other “spaces,” including distinctions between peripersonal and distal space, although interactions between these spaces are still under-theorized and starkly under-explored. One study on the relation between peripersonal space and number bisection found an interaction between distance in peripersonal space and number bisection, perhaps related to biases in lateral spatial attention (Longo and Lourenço, 2010), although the precise mechanism for this interaction is still unknown. In a study that contrasted finger-based and space-based representations of number, Riello & Rusconi (2011) examined the possibility of a unimanual SNARC using a Two Alternative Forced Choice paradigm. Participants responded with the index and middle finger of the same hand, pressing buttons on either side of participants’ midline. Response hand and orientation (face up or down) were also manipulated. They found co-existing hand-based and space-based representations of number, which were either complementary or incompatible depending on the hand and its orientation.

Two conditions in Riello & Rusconi (2011) are of particular interest: responses with downward-facing left and right hands. In contrast with the results of the current studies, Riello and Rusconi only found a classic left-to-right SNARC effect on the right downward-facing hand. They explained this by positing an interaction between hand-based (from thumb to little finger) and space-based (left to right) representations of number, which would be in conflict on the left hand when facing downward, but in accord on the right hand. The difference between our results and theirs may be due to a number of factors. For one, participants in Riello and Rusconi (2011) responded with two adjacent buttons—pressed by the index and middle finger of the same hand—that were placed on either side of the participants’ midline, and thus were distinguished in multiple frames of reference (head-, eye-, hand-, and action-based). Additionally, the simultaneous spatial juxtaposition of the two response options—unlike our design, in which different response options were only juxtaposed temporally between blocks—may have highlighted hand-based representations. Our results suggest that, when fingers are not spatially juxtaposed within a single block, behavioral space interacts spontaneously with number during magnitude judgments. Moreover, unlike previous studies (e.g. Dehaene et al, 1993), we did not find an interaction between hand side and magnitude. We attribute this to the fact that hand side was not contrasted within the task, but only manipulated between subjects, and thus this was not a salient distinction for the individual. Living organisms, after all, “enact a world as a domain of distinctions” (Varela, Thompson & Rosch, 1993, p.140).

Previous attention to space, furthermore, has eclipsed attention to time. Existing studies have juxtaposed spatial responses within a single block, so that on any given trial there were always multiple spatial responses available. In our studies, which used a go/no-go paradigm, the different response options were contrasted across the experiment as a whole, rather than within a block, and so any particular trial involved only one possible spatial response (or lack of response). Said otherwise, the different possible spatial responses in our tasks were not juxtaposed at any particular slice of time in which a response was made. In spite of this, reliable associations between number and space emerged.
The relevant units of analysis, we conclude, are the behavioral contrasts within the temporally- and corporeally-extended cognitive ecology circumscribed by the task.

**Conclusion**

In two experiments, we found that number interacts with space even when responses are not distinguished by their location in head- or eye-based spatial frames of reference. Numbers were associated with locations in behavioral space, enacted by the participant within the context of the task. This association between number and behavioral space, however, was task-specific, appearing only when numerical magnitude was directly task-relevant. While space is a ubiquitous and powerful cognitive resource (e.g., Kirsh, 1995; Tversky, 2011), it is neither fixed nor monolithic. The spaces of human activity are multiple, defined relative to varied frames of reference, and we deploy them flexibly during abstract thought. As Merleau-Ponty (1962) argued a half century ago, the space that we inhabit is not pre-given, but constituted by the motility of the body. Our reasoning about abstract concepts is not grounded in some single, static, or stable representation of space, but in the space we enact through embodied activity.

**Acknowledgments**

Thanks to Anthony Chan, Sarah Saturday, and Luke Eberle for data collection, and to Rafael Núñez for discussion.

**References**


