New Empirical Tests of a Quantum Model for Question Order Effects

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Abstract

Recent findings show that human inferences and decisions interfere in ways analogous to incompatible quantum observables, and conceptual judgments are inseparable in ways similar to entangled quantum states. This discovery has led a group of physicists and psychologists to form a new field called “quantum cognition,” which uses mathematical principles of quantum theory to explain human cognitive behavior. The power of this new theoretical approach is illustrated here by testing an a priori and precise prediction derived from quantum theory regarding question order effects commonly observed in survey research. The test of quantum theory was statistically satisfied across a set of 26 national surveys on presidential job approval and country satisfaction in past 10 years. These results suggest that quantum theory, initially invented to explain order effects on measurements in physics, provides a powerful prediction for measurement order effects in social and behavioral sciences too.

The human brain is a powerful and massively complex neural system. It provides the biological substrate for an emergent mind capable of producing highly intelligent cognitive behaviors, such as inferences and decisions. How this happens remains a topic of intense investigation in cognitive neuroscience. The possibility that the brain’s tremendous power arises from parallel computations of quantum physical neuronal interactions has been raised (Hameroff & Penrose, 1996; Hagan, Hameroff & Tuszynski, 2002) but strongly criticized (Tegmark, 2000; McKemish, Reimers, McKenzie, Mark & Hush, 2009). However, what if it is our behavior – rather than our brains – that follows quantum rules?

Supporting this idea, latest evidence shows that human inferences and decisions interfere in ways analogous to incompatible quantum observables (Pothos & Busemeyer, 2009; Busemeyer, Wang & Lambert-Mogiliansky, 2008), and conceptual judgments are inseparable in ways similar to entangled quantum states (Aerts & Sozzo, 2011). Formal principles that quantum theorists invented to deal with properties of complex physical systems provide a powerful mathematical description of human behavior (Busemeyer & Bruza, 2012; Khrennikov, 2010). This discovery has led a group of physicists and psychologists to work together and form the new field of “quantum cognition,” which uses mathematical principles of quantum theory to explain human cognitive behavior. It has successfully accounted for various puzzling findings in psychological literature, ranging across perception (Atmanspacher, Filk & Romer, 2004), associative memory (Bruza, Kitto, Nelson & McEvoy, 2009), conceptual reasoning (Aerts, 2009), probability judgments (Busemeyer, Pothos, Franco & Trueblood, 2011), decision making (Yukalov & Sornette, 2011), and strategic game behavior (Lambert-Mogiliansky & Busemeyer, 2012). It is plausible that the underlying neural systems follow classical dynamic laws, but the emergent cognitive behaviors are coarse “quantized” descriptions (Atmanspacher & Graben, 2007). In fact, more than half a century ago, founding fathers of quantum theory speculated that fundamental quantum principles have implications outside of physics to human cognitive behavior (Pauli, 1950; Bohr, 1958).

Here we tested a new, a priori and precise prediction derived from quantum theory regarding question order effects commonly observed in survey research. This type of exact prediction is rare in social and behavioral sciences. The prediction was statistically supported across a set of 26 national surveys in past 10 years on two important public opinion questions in the U.S.: presidential job approval and country satisfaction. This surprisingly accurate test illustrates the theoretical power of our new approach to use quantum theory as a mathematical tool to explain and predict human cognitive behaviors. We show that quantum theory, initially invented to explain order effects of measurements in physics, provides a powerful prediction for order effects of measurements in psychology.

Measurement Order Effects

One of the prime paradoxes of physics explained by quantum mechanics is that the order of measurements affects the observed statistics. For example, when testing the direction of spin ½ particles, the results depend on whether the “up-down” direction is tested before versus after the
“left-right” direction (Sakurai, 1994). In the terminology of quantum theory, observables like these are defined as incompatible, and the theory was built on a non-commutative algebra of operators (Von Neumann, 1932).

Order effects of measurements are not unique to physics. It has long been recognized that the order of questions can influence human judgments and decisions (Schuman & Presser, 1981; Sudman & Bradburn, 1974). For example, the Pew Research Center conducted a telephone survey experiment during June 10-14, 2009 with a nationally representative sample of 1,502 U.S. adults. A random half of the sample was asked, “Do you approve or disapprove of the way Barack Obama is handling his job as President?” followed by “All in all, are you satisfied or dissatisfied with the way things are going in this country today?” The other half was asked the exact same questions but in the opposite order. It turns out that the presidential job approval rate was 63.38% when it was asked first and dropped to 58.58% when asked second.

Gauging public opinions is an enormously important task in any democracy. Among many challenges that survey researchers must manage, question order effects are one of the most important (Schuman & Presser, 1981; Moore, 2002). A common practice is rotating question orders between randomly-split samples to balance out question order effects. Whether the order of two questions produces significant effects can be easily tested. Denote \( p(\text{AyBn}) \) as the probability of agreeing (“yes”) to question A and then disagreeing (“no”) to question B, and \( p(\text{BnAy}) \) as the probability of the same answers when the questions were asked in the opposite order. Similarly, probabilities of the remaining response combinations, \( p(\text{AnBy}) \) and \( p(\text{ByAn}) \), are defined. The two order conditions produce a pair of 2x2 contingency tables, which, according to the null hypothesis, should be equivalent except for sampling error (e.g., \( p(\text{AyBn}) = p(\text{BnAy}) \)). Discrepancy from the null hypothesis is measured by \( \chi^2 \). If the null hypothesis is correct, the \( \chi^2 \) statistic should have a \( \chi^2 \) distribution.

Table 1 shows \( \chi^2 \) results for two Gallup survey experiments reported in a seminal article on question order effects (Moore, 2002). Each sampled around 1,000 U.S. adults using the split sample paradigm. In the first poll, people were asked whether Bill Clinton was honest and trustworthy, and whether Al Gore was honest and trustworthy. In the second poll, people were asked whether white people dislike black people, and whether black people dislike white people. Each 2x2 contingency table in Table 1 summarizes the observed proportions for the four response combinations in one question order. As shown by the \( \chi^2 \) test on the order effects, both experiments produced large order effects with strikingly different patterns. Now we come back to the presidential job approval and country satisfaction questions. Is there a robust order effect for this pair of important public opinion questions? To examine this, we obtained from the Pew Research Center all its survey experiments that included this pair of questions in past 10 years. There are 26 surveys in total, with a nationally representative sample between 815 and 3,006 U.S. adults (\( M = 1,644, SD = 422.24 \)). Of each sample in each survey, a random half was asked the presidential job approval question first while the other half was asked the country satisfaction question first. The \( \chi^2 \) test indicates significant question order effects across the 26 surveys (see Figure 1).

### A Quantum Model for Question Order Effects

It would be a speculative leap, however, to think that quantum theory can be applied to human behavior simply because the behavior displays measurement orders effects. Indeed, quantum models of cognition need to be rigorously tested. A precise and empirically testable prediction has been derived from a quantum model for the question order
experimental paradigm (Busemeyer & Bruza, 2012; Wang & Busemeyer, in press). The model is simple, intuitive, but general. First, as illustrated in Figure 2, a person’s prior belief state is represented by a unit length vector (denoted by S) within an N-dimensional vector space. This use of feature vectors to represent belief or knowledge is consistent with many other cognitive models of memory. Second, each answer to a question is represented by a subspace within the vector space. Each subspace corresponds to a projector (see Figure 2). Denote $P_X$ as the projector corresponding to agreement to a question, and $I-P_X$ is the projector corresponding to disagreement to the question, where $I$ is the identity operator. Third, how to compute response probabilities in quantum models? For example, following quantum probability rules, the probability of agreeing to question $A$ and then disagreeing to question $B$ equals the squared length of the result obtained by sequentially projecting the prior belief state on the subspace for agreeing to $A$ and then on the subspace for disagreeing to $B$, that is, $p(AyBn) = ||(I-P_B)P_A S||^2$. If the subspaces for the two questions are incompatible (i.e., not spanned by a common basis), then their projectors are non-commutative (i.e., $P_B P_A \neq P_A P_B$), and question order effects are predicted to occur.

This model makes an a priori and precise prediction, named the Quantum Question (QQ) equality (see Appendix for proof): $[p(AyBn) + p(AnBy)] - [p(ByAn) + p(BnAy)] = 0$. Intuitively, this means, the probability of having different responses to the two questions (e.g., saying “yes” to one and “no” to the other) should remain the same across the two question orders. As shown in the proof, this equality must hold for any belief state and any pair of projectors in any high-dimensional vector space. This precise prediction can be easily tested empirically: if it holds, the difference in observed proportions on the left hand of the QQ equality, defined as $q$, should not statistically differ from zero as tested by $\chi^2$ for difference in proportions.

The QQ equality prediction was tested using the aforementioned two Gallup data sets and was supported with high accuracy (see Table 1). To generalize the results, it was further tested using the 26 Pew national survey experiments. If it holds, the observed frequency distribution of $\chi^2$ (shown as the green bars in Figure 1) should be distributed according to a $\chi^2(1)$ distribution. Indeed as predicted, the observed distribution is not significantly different from the expected distribution (see Figure 1). In summary, although the 26 Pew studies exhibit significant questions order effects, there are not significant deviations from the predicted QQ equality.

**Can a Classical Brain Give Rise to Quantum Cognitive Behaviors?**

The surprisingly accurate predictions generated by the quantum model for question order effects is one example of an accumulating body of evidence supporting the general applicability of quantum theory for explaining a wide range of human cognitive behavior findings that are paradoxical from a classical probability perspective (Busemeyer & Bruza, 2012). This, however, leaves a question: can a classical brain give rise to behavior that follows quantum principles? Recently, mathematical physicists have provided a mathematical answer to this puzzle. Essentially, coarse measurements of a classical dynamic system typically generate incompatible observables that result in unresolvable uncertainty relations and entangled correlations (beim Graben & Atmanspacher, 2006; beim Graben, Filk & Atmanspacher, in press). According to quantum theory, order effects occurs for incompatible observables.

A key idea is to distinguish “ontic” states (e.g., states of a dynamic neural network) in a classical phase space from “epistemic” states (e.g., discrete choices or judgments across time) obtained from an observable. The mapping from ontic to epistemic states usually is many to one, where the epistemic states generated by an observable form a partition of the ontic phase space. Knowing the epistemic state does not completely determine the ontic state, but a sequence of measurements across time refines the partition of the phase space. In the limit, the partition reaches a “finest dynamic refinement,” denoted by $\rho$. Now suppose two observables ($f, g$) produce different finest dynamic refinements ($\rho_f \neq \rho_g$) and neither converge to the identity

![Figure 2: An illustration of basic quantum principles used in the question order model. The figure illustrates a simple 3-dimensional vector space, but the space can be arbitrarily high-dimensional. The probability of agreeing to question X is the squared length of the projection of $P_X S$ obtained by projecting the belief state S to the X-Y plane representing the subspace for agreeing to question X. If question X was asked after another question, the belief state would have already been changed by answering the preceding question, and the probability of agreeing to question X (conditioned on the preceding answer) becomes the squared length of the result obtained by projecting the adjusted belief state on the subspace for question X.](image)
partition, as illustrated in Figure 3. This means that no ontic state is accessible by a sequence of measurements from either observable. Then there exists an epistemic state (a set of ontic states) $F_j \in \wp_j$ that determines the value $a$ produced by the observable $f$, but the value of the observable $g$ must remain dispersive. Likewise, there exists an epistemic state $G_j \in \wp_j$ that determines the value $a$ produced by the observable $f$, but the value of the observable $g$ must remain dispersive.

(a set of ontic states) $G_j \in \wp_j$ that determines the value $v$ produced by the observable $g$, but the value of the observable $f$ remains dispersive. It is impossible to simultaneously determine the value $a$ from observable $f$ and the value $v$ from the observable $g$ with arbitrary precision, so that the two observables are incompatible. Consequently, the partitions generated by the two incompatible observables produce incompatible Boolean algebras of events, and the entire collection forms a partial rather than a complete Boolean algebra. Quantum theory is specifically suitable to assign probabilities to events defined on a partial Boolean algebra.

Figure 3. An illustration showing how uncertainty relations are generated by coarse descriptions of classical dynamic systems. The underlying classical phase space $X$ is inconsistently partitioned by two different observables, $f$ and $g$. The cell within $X$ that always assigns a value $a$ to the observable $f$ assigns a range of different possible values $(w,u,z)$ to the observable $g$. In this case, there exists an epistemic state that determines the value $a$ produced by the observable $f$, but the value of the observable $g$ must remain dispersive.

Discussion

Scientists are still far from understanding how mental states emerge from the neural substrates. It is too early to conclude whether or not quantum physics plays a significant role in neural processing. Nevertheless, even if the brain is classical, the ubiquitous nature of incompatible observables provides a good reason to consider using quantum theory as a mathematical tool for predicting human behavior (Busemeyer & Bruza, 2012; Khrennikov, 2010). As our quantum question order model encapsulates and illustrates, at least four motivations drives the development of this new field of quantum cognition. (a) Judgments and decisions are not simply read out from memory, but rather, they are constructed from the cognitive state for the question at hand; and (b) drawing a conclusion from one judgment or decision changes the context and disturbs the cognitive system, which then (c) affects the next judgment or decision, producing order effects, so that (d) human judgments and decisions do not obey the commutative rule of Boolean logic. If we replace “judgments or decisions” with “physical measurements” and replace “cognitive system” with “physical system,” then these are exactly the same reasons that forced physicists to develop quantum theory in the first place. Traditionally, quantum theory has rarely been applied outside of physics, but now a growing number of researchers are successfully using it to explain human cognitive behavior.

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Reference


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Yukalov, V. & Sornette, D. (2011). Decision theory with projectors for saying yes to asking “yes” to question C:

\[
\begin{align*}
|P_C \cdot S|^2 &= |P_C \cdot I \cdot S|^2 = |P_C \cdot (P_G + (I-P_G)) \cdot S|^2 = |P_C \cdot P_G \cdot S + P_C \cdot (I-P_G) \cdot S|^2 \\
&= |P_C \cdot P_G \cdot S|^2 + |P_C \cdot (I-P_G) \cdot S|^2 \\
&= |P_C \cdot P_G \cdot S|^2 + |P_C \cdot (I-P_G) \cdot S|^2.
\end{align*}
\]

An order effect for question C when G was asked first expressed as

\[
C_C = TP_C - |P_C \cdot S|^2 = -2 \cdot \text{Re}(S P_G \cdot P_C \cdot (I-P_G)).
\]

Immediately we see that if \( P_G \) and \( P_C \) commute so that \( P_G \cdot P_C = P_C \cdot P_G \) then \( P_G \cdot (I-P_G) = P_C \cdot (I-P_G) = 0 \) and we predict NO order effect. Thus non-commuting projectors are a necessary condition for order effects. Now let us re-examine

\[
C_C = TP_C - |P_C \cdot S|^2 = -2 \cdot \text{Re}(S P_G \cdot P_C \cdot (I-P_G)).
\]

In general, the inner product is a complex number which always can be expressed as \( S P_G \cdot P_C \cdot S \cdot |\cos(\theta) + i \sin(\theta)| \). The real part equals \( \text{Re}(S P_G \cdot P_C \cdot S \cdot |\cos(\theta)|) \). By defining the ratio

\[
R = \frac{|\langle S P_G \cdot P_C \rangle|}{|\langle P_C \cdot S \rangle|},
\]

then according to the Cauchy-Schwarz inequality, \( 0 \leq R \leq 1 \).

Finally we can express

\[
C_C = TP_C - |P_C \cdot S|^2 = 2 \cdot |P_C \cdot P_G \cdot S|^2 - 2 \cdot |R \cdot \cos(\theta)| \cdot |P_C \cdot |P_G \cdot S\rangle|^2,
\]

with \( \theta = \text{Re}(\cos(\phi)) + i \leq +1 \), which is the similarity index referred to in the main text. Similarly, the order effect for question G when C was asked first equals

\[
C_G = TP_C - |P_G \cdot S|^2 = 2 \cdot |P_G \cdot P_C \cdot S|^2 - 2 \cdot \text{Re}(S P_C \cdot P_G \cdot S),
\]

so that

\[
C_G = 2 \cdot |P_G \cdot S|^2 - 2 \cdot |P_G \cdot S|^2 + 2 \cdot |P_C \cdot S|^2 - 2 \cdot \text{Re}(S P_C \cdot P_G \cdot S),
\]

These two order effects share the same term, \( 2 \cdot \text{Re}(S P_C \cdot P_G \cdot S) \), and therefore together they imply the relation

\[
0 = (2 \cdot |P_G \cdot P_C \cdot S|^2 - C_C) - (2 \cdot |P_G \cdot P_C \cdot S|^2 - C_G)
\]

\[
= (2 \cdot |P_G \cdot S|^2 - 2 \cdot |P_G \cdot P_C \cdot S|^2) + 2 \cdot |P_C \cdot S|^2 - (2 \cdot |P_G \cdot P_C \cdot S|^2 - 2 \cdot |P_G \cdot P_C \cdot S|^2).
\]

Appendix

1. Proof of the QQ equality.

Here we briefly introduce the basic axioms of quantum theory and then derive the QQ equality. We use the Dirac bracket notation so that \( \langle S \mid T \rangle \) represents the inner product between two vectors. According to quantum theory, events represented as subspaces of a Hilbert space. Corresponding to each event A there is a orthogonal projector \( P_A \). The state of a quantum system is represented by a unit length vector \( S \) within the Hilbert space. The probability of event A equals the squared length of the projection \( p(A) = |P_A \cdot S|^2 \). If event A is observed, then the state is updated according to Lüder’s rule \( S_A = P_A \cdot S || P_A \cdot S || \).

Define \( S \) as the initial state. Denote the projector for saying yes to question C as \( P_C \) and denote \( P_G \) as the
The log likelihood for the model constrained involves 4 free parameters and the constrained model involves 2–1 = 1 free parameter, and so the χ² statistic has df = 1.

3. χ² tests used in Figure 1.

First we describe the χ² test for order effects. The χ² statistic for testing an order effect for each of the 26 data sets was computed using Equations 1a and 1b defined above, producing 26 observed χ² values. If the null hypothesis is correct, these should be distributed according to a χ² distribution with df = 6. Ten categories were constructed by computing the 9 category bounds: .5844 equals the 10th percentile, 1.0052 equals the 20th percentile, 1.4237 equals the 30th percentile, 1.8692 equals the 40th percentile, 2.3660 equals the 50th percentile, 3.9462 equals the 60th percentile, 3.6649 equals the 70th percentile, 4.6416 equals the 80th percentile, and 6.2514 equals the 90th percentile. (For example, Pr(χ²(6) < 6.2514 | H₀) = .90.) These category bounds divide the expected frequency distribution (under the null hypotheses) into two 10 equally likely categories, with 2.6 expected frequency within each of the 10 categories using these cutoffs. Then frequency of the 26 observed χ² values were counted for each category. Denote fᵢ as the observed frequency for category i = 1, 10. The log likelihood for the unconstrained model equals

\[ G_U = \sum f_i \ln(f_i/26). \] (3a)

The log likelihood for the expected frequencies according to the null hypothesis equals

\[ G_C = \sum f_i \ln(2.6/26). \] (3b)

The χ² statistic is defined by the difference \( \chi^2 = 2(G_C - G_U) \). The unconstrained model involves 10–1 = 9 free parameters and the constrained model has no free parameters, and so the χ² statistic has df = 9.

Next we describe the χ² test for the QQ equality. The χ² statistic for testing the QQ equality for each of the 26 data sets was computed using Equations 2a and 2b defined above, producing 26 observed χ² values. If the null hypothesis is correct, these should be distributed according to a χ² distribution with df = 1. Ten categories were constructed by computing the 9 category bounds: .0642 equals the 10th percentile, .1485 equals the 20th percentile, .2750 equals the 30th percentile, .4549 equals the 40th percentile, .7083 equals the 60th percentile, 1.0742 equals the 70th percentile, 1.6424 equals the 80th percentile, and 2.7055 equals the 90th percentile. (For example, Pr(χ²(9) < 2.7055 | H₀) = .90.) These category bounds divide the expected frequency distribution (under the null hypotheses) into two 10 equally likely categories, with 2.6 expected frequency within each of the 10 categories using these cutoffs. Then frequency of the 26 observed χ² values were counted for each category. Denote fᵢ as the observed frequency for category i = 1, 10. Then Equations 3a and 3b were used to compute the log likelihoods of the unconstrained and constrained models. Once again, the χ² statistic is defined by the difference \( \chi^2 = 2(G_C - G_U) \). The unconstrained model involves 10–1 = 9 free parameters and the constrained model has no free parameters, and so the χ² statistic has df = 9.