Serial Subtraction Errors Revealed

Sue E. Kase (skase@ist.psu.edu), Frank E. Ritter (frank.ritter@psu.edu)
College of Information Sciences and Technology, Pennsylvania State University
University Park, PA 16802 USA

Michael Schoelles (schoem@rpi.edu)
Cognitive Science Department, Rensselaer Polytechnic Institute
Troy, NY 12180 USA

Abstract

A fine-grained analysis of errors and their frequency during the performance of a mental multi-digit serial subtraction task reveals the cognitive processes most prone to failure. Example serial subtraction problems from the experimental problem set are utilized in illustrating different types of errors and their probable causes. A list of considerations is presented for future descriptive and computational modeling of the errors committed during performance of the task.

Keywords: Cognitive arithmetic, Serial subtraction; Errors

Introduction

Mental multi-digit serial subtraction under time pressure, performed and evaluated in front of an audience, can easily overwhelm the working memory resources of human subjects. A fine-grained analysis of the human performance produced several categories of errors: subtracting a value more than the subtrahend, subtracting a value less then the subtrahend, and adding instead of subtracting. These error types were decomposed by probable causes—combinations of retrieval and procedural failures. Dissection of example serial subtraction problems associated with the error types revealed the cognitive processes in the solution process prone to failure. Lessons learned from this detailed level of error analysis can inform the construction of descriptive and computational cognitive models of serial subtraction.

Serial Subtraction

Serial subtraction is the mental arithmetic stressor portion of the Trier Social Stressor Test (TSST, Kirschbaum, Pirke & Hellhammer, 1993). The TSST has been used to provide an acute physiological stress response in human subjects in 100's of studies since the 1960's. The serial subtraction task consists of four 4-minute blocks of mentally subtracting by 7 and 13 from 4-digit starting numbers. Figure 1 illustrates the serial subtraction task with the four starting numbers for each subtraction block shaded in gray. The task is performed mentally with no visual or paper clues. An experimenter gives the subject the starting number; from then on, the subject speaks the answer to each subtraction problem.

Before the task begins, the experimenter explains that the subject's performance is going to be voice recorded and reviewed by a panel of psychologists. Subjects sit in a chair directly in front and near the experimenter who is holding a time keeping device and clipboard of the correct subtraction answers that she checks off as the subject performs the task. Immediately before the task begins the experimenter emphasizes that the task should be preformed as quickly and as accurately as possible. The subjects' subtraction answers are scored against the list of correct answers from the starting number. For each subject the number of subtraction problem attempts are recorded and a percentage correct score is calculated by dividing the total number of correct attempts by the total number of attempts for each block of the serial subtraction.

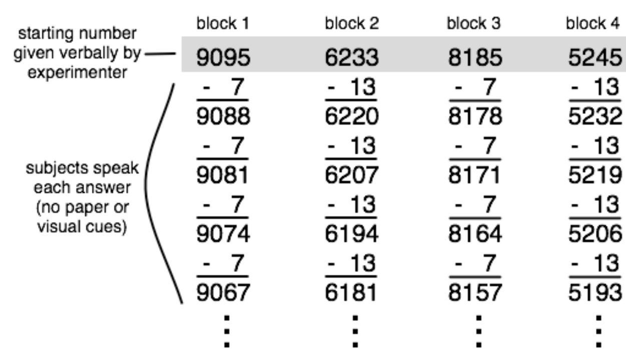

Figure 1: An illustration of the four blocks of the serial subtraction task as in the experiment; subjects perform the task mentally without paper or visual cues.

Human Performance

In this study the serial subtraction performance data from 15 subjects (age range, 18-30) in the control group were analyzed at two levels: as a group and individually (Kase, 2008). The statistics of primary interest were number of attempts and percentage correct for each block of serial subtraction. Additionally, the audio recordings of each subject were transcribed to obtain data on subtraction pace and detailed information about errors. Table 1 shows the subtraction rates averaged across subjects' performance on the two 4-minute blocks of subtracting by 7, and Table 2 for subtracting by 13. The large standard deviations indicate that there is a wide range of performance on this task.

In Tables 1 and 2 number of errors are represented as number of incorrect attempts. Subjects sometimes answered a particular subtraction problem incorrectly more than once. In this case each of these incorrect attempts was counted as an error even though the incorrect attempts were associated with the same subtraction problem. The next section describes the serial subtraction solution process in general, as well as, working memory's role in mathematical processing.

Table 1: Human subject (N=15) mean performance and standard deviation for serial subtraction on the two 4-minute blocks of subtracting by 7.

	7s – 1st block	7s – 2nd block
Number of Attempts	47.3 (15.2)	56.9 (21.7)
Percent Correct	82.0 (10.0)	88.0 (7.0)
Number of Errors	7.1 (2.2)	5.7 (2.5)

Table 2: Human subject (N=15) mean performance and standard deviation for serial subtraction on the two 4-minute blocks of subtracting by 13.

	13s – 1st block	13s – 2nd block
Number of Attempts	41.9 (16.0)	47.8 (19.2)
Percent Correct	82.0 (12.0)	84.0 (10.0)
Number of Errors	6.0 (2.6)	6.3 (3.0)

Solving the Problem

Fluent performance on complex cognitive tasks, such as the serial subtraction task, relies on the ability to coordinate and integrate stored information with ongoing processes. This requires efficient organization and maintenance of intermediate results that can be accessed for use at the appropriate time. For example, to solve the problem 5964 – 7 requires the subject to mentally retain both operands in memory while processing the following steps. Compare the units-column minuend (4) with the subtrahend (7) to determine if a borrow operation is required; if so, decrement the tens-column minuend by one (i.e., from 6 to 5) and retain this decremented value in memory while performing the units-column operations. The units-column operations involve encoding the values, performing an addition by adding 10 to the units-column minuend (i.e., 4 + 10 = 14) calculating the units-column difference (i.e., 14 – 7 = 7); and then retaining this partial solution in memory. The final answer is re-constructed from the thousands- and hundreds-column values retained from the original minuend (i.e., 59) concatenated with the two previous partial solutions, the decremented value of the tens-column (i.e., 5), and units-column solution (i.e., 7).

For this investigation we adopt the view that mathematical cognition involves working memory—referenced as "memory" in the previous example. Working memory is generally defined as the preservation of information while simultaneously processing the same or other information (Salthouse & Babcock, 1991). An important concept of working memory is that working memory capacity is limited (Baddeley, 1986). Researchers have emphasized the importance of working memory for understanding mathematical processing (Ashcraft & Kirk, 2001; DeStefano & LeFevre, 2004). When a cognitive task places extreme demands on working memory, accuracy and processing speed may decrease (Ashcraft, 1992). The errors subjects make while performing complex arithmetic, such as serial subtraction, are critical evidence about the underlying cognitive processes required to perform the task. The errors committed by the subjects during their performance of the serial subtraction task are discussed next.

Frequency of Errors

A novel approach is utilized in the categorization of the errors. The subjects are attempting to subtract by 7 or 13. When subjects give an incorrect answer we categorize the error as to mathematical operator (– or +) and subtrahend value that would have produced the erroneous answer. Table 3 shows two example problems with their error categorization. The first column shows a serial subtraction problem. The second column contains a subject's incorrect answer to the problem. The third column describes the error category, for example, "subtraction by 17 error"; because the incorrect answer would be produced by subtracting 17 from the minuend (9039). It is important to remember that *this is a method to categorize and discuss particular types of errors* associated with serial subtraction, and does not mean that the subject intended to subtract by 17 instead of 7. Likewise, the second row in Table 3 describes the error category "addition by 3 error" where the incorrect answer would be produced by adding 3 to the minuend (9046).

Table 3: Method of error categorization.

Problem Subject Attempts to Solve	Subject's incorrect answer	Error Category
9039 – 7	9022	Subtraction by 17 Error 9039 – 17 = 9022
9046 – 7	9049	Addition by 3 Error 9046 + 3 = 9049

Using the above error categorization, frequencies of errors by the subjects when subtracting by 7 and 13 are broken down in Tables 4 and 5. Table 4 lists the errors and frequencies when subjects subtracted by 7. The table is divided into three main sections. The center section lists summary frequencies. When subtracting by 7 for two 4-minute blocks the subjects made a total of 1511 subtraction attempts, 1346 correct responses, and 165 errors. The error percentage is 10.9% with 56 of the errors categorized as additions (see second example, Table 3), and 107 of the errors categorized as subtractions other than by 7 (see first example, Table 3). Two errors occurred when the minuend of a subtraction problem was given as the answer. The leftmost section of Table 4 categorizes errors of subtraction that are more than 7. Errors with a frequency greater than 3 are listed individually; remaining errors with lower frequency are grouped under Other. For example, on the left side of Table 4 under the Value Subtracted column, the value –17 is listed with a frequency of 7 and 0.5%. This means that the subtraction by 17 error occurred 7 times which is 0.5% of the total errors. The rightmost section of Table 4 lists subtraction errors that are less than 7, and addition errors, both with frequencies greater than 3. For example, the first value in the right Value Subtracted column shows the subtraction by 6 error occurred 25 times accounting for 1.7% of the total errors. Midway down in the right Value Subtracted column, 12 errors are categorized as the addition by 93 error (+93) for 0.8% of the total errors.

Table 5 is similar in format to Table 4 and lists the subtraction by 13 errors and frequencies. When subtracting by 13 for two 4-minute blocks the subjects made a total of 1306 subtraction attempts (205 less than the subtraction by 7 attempts), with 1141 correct responses, and 165 errors (same number of errors as subtraction by 7). The error percentage is 12.6% compared to 10.9% when subtracting by 7. Of the 165 errors 131 of the errors were categorized as subtractions other than 13, and 33 of the errors were categorized as additions (a smaller percentage than subtracting by 7, 13.9% less).

Table 4: Frequency of errors when subtracting by 7s for two 4-minute blocks.

Subtracting More Than 7			Summary	Frequency	Percent	Subtracting Less Than 7 or Adding		
Value Subtracted	Frequency	Percent				Value Subtracted	Frequency	Percent
-907	4	0.3	Total Attempts	1511		-6	25	1.7
-207	4	0.3	Correct	1346	89.1%	-5	18	1.2
-17	7	0.5	Errors	165	10.9%	+3	12	0.8
-13	4	0.3				+93	12	0.8
-10	4	0.3	Error Breakdown			+893	5	0.3
-9	4	0.3	Subtracting			Other	30	2.0
-8	27	1.8	Other Than 7	107	64.8%	Total	102	6.8
Other	7	0.6	Additions	56	33.9%			
Total	61	4.4	Duplicates	2	1.3%			

Table 5: Frequency of errors when subtracting by 13s for two 4-minute blocks.

Subtracting More Than 13			Summary	Frequency	Percent	Subtracting Less Than 13 or Adding		
Value Subtracted	Frequency	Percent				Value Subtracted	Frequency	Percent
-113	5	0.4	Total Attempts	1306		-12	19	1.5
-23	15	1.1	Correct	1141	87.4%	-11	11	0.8
-17	4	0.3	Errors	165	12.6%	-10	6	0.5
-14	7	0.5				-3	34	2.6
Other	25	2.4	Error Breakdown			+87	15	1.1
Total	56	4.7	Subtracting			+987	7	0.5
			Other Than 13	131	79.4%	Other	23	1.6
			Additions	33	20.0%	Total	108	8.6
			Duplicates	1	0.6%			

Error Types

The performance of cognitive arithmetic tasks requires accessible representations of information specific to the problem (i.e., elementary subtraction or addition facts) as well as efficient procedures enabling the problem solving process (i.e., counting, carry, and borrow) (Ashcraft, 1992; Cocchini, Logie, Della Sala, MacPherson, & Baddeley, 2002). The next two sections discuss the errors listed in Tables 4 and 5 in reference to their retrieval and procedural causes.

Fact Retrieval Errors

Generally, errors were most frequently clustered around the intended subtrahend. For example, during subtraction by 7 over one third of the total errors resulted from subtraction by 8, 6, and 5 errors (see Table 4). These types of errors accounted for 70 of the 165 total errors, or 42%. Likewise a similar pattern is observed during subtraction by 13, subtraction by 14, 12, and 11 errors, accounted for 37 of the 165 errors, or 22%, almost a quarter of the total errors.

In the cognitive arithmetic literature, errors involving answers that would be correct if one of the operands were changed by ±1 are typically called 'near errors' (Ashcraft, 1992). This type of error is thought to be caused by a fact retrieval failure. For example, when subtracting by 7, the units-column operation involves a simple number fact (i.e., 9 – 7). These simple subtractions, would rely heavily on fact retrieval from long-term memory (Ashcraft & Battaglia, 1978; Widaman, Geary, Cormier, & Little, 1989). Subjects skilled in arithmetic would have memory representations of arithmetic facts that specify the result of applying a mathematical operator (i.e., – or +) to particular operands (e.g., the fact 9 – 7 = 2). The problem elements, operator and operand symbols, serve the role as retrieval cues

(Siegler & Shrager, 1984). The retrieval process might begin only when a subject has encoded all of the problem's elements. Alternatively, the retrieval process might begin earlier on the basis of partial cues provided by individual problem elements (Campbell, 1994). Near errors may be the result of an incorrect partial match on one of the operands near in the counting sequence to the intended subtrahend. For example, when subtracting by 7 under time pressure, the subject might easily retrieve the 9 – 6 = 3 fact instead of the 9 – 7 = 2 fact.

Another explanation of fact retrieval errors is the phenomenon called error priming. Campbell (1991) found that errors frequently match the correct answer to a problem solved earlier in the experimental session. In some cases a recently used subtraction fact might be more active in working memory than the fact that needs to be used in the current subtraction problem resulting in an incorrect fact retrieval.

In general, direct retrieval of arithmetic facts is thought to utilize little working memory as the operands and operator are encoded and the solution quickly retrieved. This is not the case for more complex subtractions requiring a borrow operation, or multi-digit minuends and subtrahends where a mixture of fact retrieval and procedural operations are needed to solve the problem.

Procedural Errors

Research on multi-digit arithmetic problems supports the view that working memory is related to the number of steps required to solve problems—more steps require more working memory resources (Ashcraft & Kirk, 2001; Fürst & Hitch, 2000). These studies found that errors, especially those attributable to working memory failures, were most frequent at points in the problem solving process where the demands for retaining intermediate results were highest. Salthouse (1992) and Hitch (1978) described the borrow operation as an in-context working memory manipulation. Therefore, we would expect working memory demands to be greater for serial subtraction problems requiring a borrow (e.g., 8101 – 7) than for non-borrow problems (e.g., 8157 – 7).

Over half (57%) of the serial subtraction problems used in the experimental problem set required a borrow operation in the calculation of the solution. Broken down by subtrahend: 70% of the subtraction by 7 problems required a borrow, and 37.4% of the subtraction by 13 problems required a borrow. Several of the serial subtraction problems required more than one borrow as shown in Table 6. The 1 Borrow column in Table 6 shows the number of problems that required one borrow operation in calculating the solution. The 3 Borrow column shows that two of the subtraction by 7 and three of the subtraction by 13 problems required three borrow operations to solve.

Figure 2 confirms that a borrow operation increases both error rate and mean reaction time when subtracting by 7 and 13 during the four 4-minute blocks of serial subtraction. The No Borrow and Borrow problem categories are shown on the x-axis. The top two plots compare the error rate when subtracting by 7 (upper left) and subtracting by 13 (upper right). The bottom two plots compare the mean reaction time in seconds when subtracting by 7 (bottom left) and 13 (bottom right). The upward sloping lines in all the plots indicate that the procedural complexity of performing a borrow operation increases both the error rate and mean reaction time most likely by placing a greater demand on working memory during the problem solving process.

Table 6: Frequencies by number of borrow operations required on a per problem solution for subtraction by 7 and subtraction by 13 problem sets.

	Number of Borrow Operations per Problem		
Number of Problems	1 Borrow	2 Borrows	3 Borrows
Subtraction by 7	189	18	2
Subtraction by 13	60	8	3

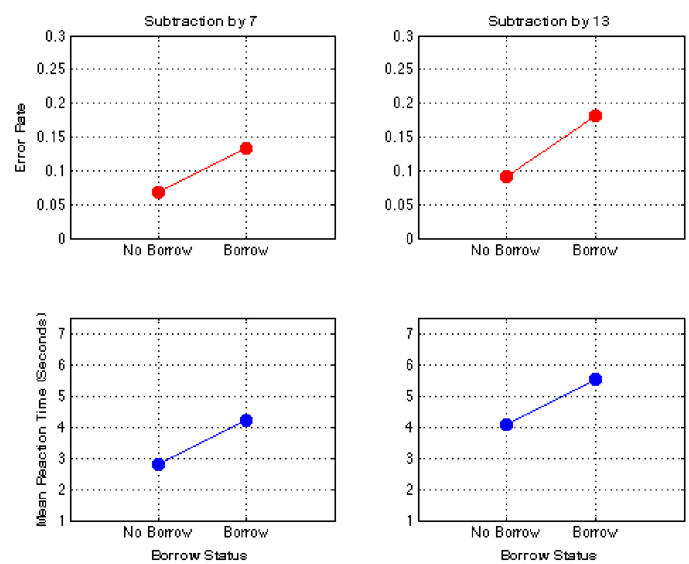

Figure 2: Borrow operation effects on error rate and mean reaction time for subtraction by 7 and 13.

The borrow operation and its utilization of working memory capacity appears to contribute to the misreporting of intermediate results and digits in the minuend yet to be processed. Borrow-inducted errors may involve, for instance, forgetting the thousands- or hundreds-column values during the processing of a lower-order borrow; or forgetting the tens-column digit of the subtrahend while adding 10 to the units-column as first step in the borrow operation; or forgetting to complete the last step of the borrow operation with the decrement of one. Misreporting intermediate results may also stem from placekeeping failures such as retrieving the wrong intermediary result at specific points in the problem calculation, or recalculating the same column twice, or skipping over a column calculation. Specific problem instances related to these misreporting errors are discussed in the next section.

Example Errors

Tables 4 and 5 listed categories of errors resulting from subtractions other than the subtrahend and from additions. The retrieval and procedural causes discussed in the previous sections are applied to individual example problems pulled from these tables.

In Table 4, the addition-of-93 error and the addition-of-893 error occurred 17 times across subjects. With a 4-digit minuend, the value of the thousands- and hundreds-columns must be maintained in working memory waiting processing. These values can be forgotten during the processing of the tens- and units-column calculations. Figure 3 shows example problems associated with the misreporting of the hundreds-column resulting in the addition-of-93 error, and example problems associated with transposing the hundreds- and thousands-column values resulting in the addition-of-893 error.

Addition-of-93 Error

Frequency Missed:	**2**	**2**		**2**	**3**	
Problem:	8899	8892	8885	8794	8787	8780
	- 7	- 7	- 7	- 7	- 7	- 7
	8892	8885	8878	8787	8780	8773
Sequence:	▬▬	▬▬	▬▬	▬▬	▬▬	▬▬
Error:	**8992**	**8985**	**8978**	**8887**	**8880**	**8873**

Addition-of-893 Error

Frequency Missed:		**2**			
Problem:		8983	8976	8962	8101
		- 7	- 7	- 7	- 7
		8976	8969	8955	8094
Sequence:		▬▬	▬▬		
Error:		**9876**	**9869**	**9855**	**8994**

Figure 3: Example subtraction by 7 problems associated with the addition-of-93 and addition-of-893 errors.

The addition-of-93 error occurred with seven different subtraction problems. The top of Figure 3 shows six of the seven problems. Four of these problems were missed more than once across subjects (labeled with Frequency Missed). Interestingly, six of the seven problems appear as part of a 3-problem sequence in serial subtraction (labeled Sequence, denoted with a wide gray line). Four of the seven problems required a units-column borrow operation. The subjects' erroneous answers (labeled Error) commonly misreport the hundreds-column value by plus one with the increment either from 7 to 8, or 8 to 9, nearby in the counting sequence to the subtrahend (near error). Surprisingly, nearly all of the minuends contained at least two 8s usually accompanied by a 9 or 7. This type of error could result from a combination of retrieval and procedure causes: high operand activation for values near in sequence to the subtrahend with low-order borrow-induced forgetting of the higher-order digits.

The bottom of Figure 3 shows the four problems generating the addition-of-893 error. In most of these problems the thousands- and hundreds-column values of 8 and 9 were erroneously transposed. The problems all required a units-column borrow operation. The addition-of-893 error also appeared when the hundreds-column value of 1 was involved in a borrow. In this case, the hundreds-column value of the erroneous answer was misreported as 9 instead of the correctly decremented value of 0 after two borrow operations.

Table 5 contains the subtraction-of-3, subtraction-of-23, and the addition-of-87 error categories. These three types of errors accounted for 64 errors across subjects' performance. Example problems associated with each of these error types are shown in Figure 4.

Subtraction-of-3 Error

Problem:	6090	5921	4972	5050	5258	4595
	- 13	- 13	- 13	- 13	- 13	- 13
	6077	5908	4959	5037	5245	4582
Error:	**6087**	**5918**	**4969**	**5047**	**5255**	**4592**

Subtraction-of-23 Error

Frequency Missed:		**2**	**3**			
Problem:	6233	6077	5076	5063	4998	4790
	- 13	- 13	- 13	- 13	- 13	- 13
	6220	6064	5063	5050	4985	4777
Sequence:			▬▬	▬▬		
Error:	**6210**	**6054**	**5053**	**5040**	**4975**	**4767**

Addition-of-87 Error

Frequency Missed:			**2**		**2**	
Problem:	6207	6194	5895	4894	4881	4868
	- 13	- 13	- 13	- 13	- 13	- 13
	6194	6181	5882	4881	4868	4855
Sequence:	▬▬	▬▬		▬▬	▬▬	▬▬
Error:	**6294**	**6281**	**5982**	**4981**	**4968**	**4955**

Figure 4: Example subtraction-of-13 problems associated with the subtraction-of-3, subtraction-of-23, and the addition-of-87 errors.

The top of Figure 4 shows five example problems associated with the subtraction-of-3 error. This type of error had the highest overall frequency (34) of all the subtraction errors. Generally described, the subtraction-of-3 error is a misreporting of the tens-column value by plus one. The majority of the time (73.5%) the error involved a borrow from the tens-column. After a correct subtraction of the units-column, either the decrement of the tens-column borrow is forgotten or, in the case of no borrow, the 1 in the tens-column of the subtrahend (13) appears to be ignored.

Similar to the subtraction-of-3 error, the subtraction-of-23 error involved the misreporting of the tens-column by minus one. Example problems are shown in the middle of Figure 4. Surprisingly, 14 of the 15 problems causing this error did not require a borrow operation. The units- and tens-column calculations should have been accomplished with simple columnar fact retrievals making for some of the easiest subtraction problems in the experimental series. It is possible that after the units-column subtraction fact was retrieved, that a near error occurred during the fact retrieval for the tens-column processing (i.e., –2 instead of –1 of the subtrahend), or that the simple subtraction of the 1 is performed a second time as if the subject loses his place and performs that column's subtraction again.

The bottom of Figure 4 shows example addition-of-87 errors caused by misreporting the hundreds-column value by an erroneous increment of one. About half of the 15 problems associated with this error were difficult in that they required a hundreds-column borrow, or a tens-column borrow, and sometimes appeared in a sequence. In the case of no borrow, the hundreds-column value only needed to be carried down to the solution. The previous intermediate result from subtracting the 1 in the tens-column of 13 may be activated and still in working memory promoting an erroneous increment of 1 to the hundreds-column value.

Conclusion

Little previous research is available on understanding the cognitive processes required to solve complex multi-digit subtraction problems (e.g., Geary, Frensch, & Wiley, 1993), and no previous research specifically addresses solving mental serial subtraction. The following list summarizes the most important findings of this serial subtraction error analysis. Individually, each of these findings has been reported in various simple arithmetic tasks. Here we see that combinations of retrieval and procedural failures contribute to an incorrect serial subtraction attempt. These findings provide useful information to cognitive arithmetic researchers. We believe that the present analysis represents progress toward a general, predictive theory of serial subtraction and its performance errors by noting several of the most common types.

1. Fact retrieval errors: (a) Retrieving a fact near in value to the subtrahend. (b) Incorrect retrieval based on a partial match to a common value in the minuend. (c) Retrieving an activated fact from a previously calculated column.

2. Errors induced by a borrow: (a) Misreporting of higher-order minuend values. (b) Forgetting the tens-column digit of the subtrahend, after executing a borrow. (c) Errors within the borrow operation itself, depending on location of the borrow, forgetting the tens-column value while executing the unit-columns addition of 10 and fact retrieval, and (d) forgetting to decrement the tens-column value in completing the borrow.

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