Piéron's Law holds in conditions of response conflict

Tom Stafford (t.stafford@shef.ac.uk)

Kevin Gurney (k.gurney@shef.ac.uk)

Leanne Ingram (leanne.ingram@gmail.com)
Department of Psychology, University of Sheffield, Western Bank, Sheffield, S10 2TP UK

Abstract

We show that the effect of stimulus colour-intensity on reaction times in the Stroop task is additive with the effect of Stroop condition. This establishes that the psychophysical relationship between stimulus intensity and response time known as Piéron's Law holds for colour processing, and that it can hold in cognitive tasks under conditions of response conflict, not, as hitherto demonstrated, merely for simple and perceptual choice tasks. Computational modelling shows that existing single stage models of the Stroop task provide an adequate account of these results, in contradiction to the assumption that additive factors imply independent processing stages. **Keywords:** Decision making; Piéron's Law; Response conflict; Stroop task; Colour saturation; Additive Factors.

Introduction

Modelling decision making

All behaviours, however simple, must be the result of a decision process. Looking at decision making and building mathematical models which account for choice reaction times and error rates has a long history within psychology (Luce, 1986). The diffusion model (Ratcliff, 1978; Ratcliff & McKoon, 2008) is perhaps the preeminent example of a model from this tradition. Ratcliff has suggested that, for two alternative forced choice (2AFC) tasks, the diffusion model is the only account which can explain the distribution of reaction times (RTs) for correct responses, as well as the speed and distribution of incorrect responses, under different urgency-accuracy conditions (Ratcliff & Rouder, 1998). More recently the optimality of the diffusion model for making urgency-accuracy trade-offs in two-choice situations under conditions of uncertainty has been demonstrated (Bogacz, Brown, Moehlis, Holmes, & Cohen, 2006). A notable feature of the diffusion model, in common with many mathematical models of decision making, is that it combines the *evidence* favouring the possible choices into a single term — termed 'the drift' in the diffusion model framework. There are obvious benefits of combining evidence into a common metric which determines which response is made and after how long. As well as bringing analytic tractability to the model, it should be apparent that optimal decision making requires the trade-off of different factors influencing the decision. A common metric facilitates an optimal trade-off in the same way that a common currency facilitates economic communication. Gold & Shadlen (2002) outline the theoretical underpinnings of such a 'weight of evidence' metric and discuss its neural instantiation.

Stroop processing

The Stroop task (Stroop, 1935) is a paradigmatic selection task in which a failure of attentional control leads to response conflict (MacLeod & MacDonald, 2000). Participants are presented with a coloured word stimulus and must respond to the stimulus colour. The word can be neutral with respect to the colour (e.g. the word 'HORSE' in red ink), or itself be the name of a colour. Thus this cognitive element of the stimulus (the meaning of the word) can be conflicting (e.g. the word 'GREEN' in red ink) or congruent (e.g. the word 'RED' in red ink) with the perceptual element of the stimulus (the physical colour).

In the conflict condition, selection involves the resolution of a contradiction between one response based on the colour aspect of the stimulus and a different response based on the word aspect of the stimulus. The resolution of this conflict causes delays in responding and errors — the *interference* typically seen in the conflict condition of the Stroop task. Traditionally interest in the Stroop task has focussed on the demonstration it provides of the power of overlearnt behaviours (in this case word-reading) to interfere with response selection. Here we are more concerned with the use of the Stroop task as a thoroughly investigated example of decision making under conditions of stimulus conflict, in which elements of different kinds (words and physical colours) are involved in affecting the response.

Piéron's Law

Piéron (1952) demonstrated that the physical intensity of a stimulus is systematically related to the simple reaction time, so that the average reaction time, RT, is given by

$$RT = R_0 + kI^{-\beta} \quad (1)$$

where R_0 is the asymptotic RT, a fixed component of response which cannot be reduced, I is the physical intensity of the stimulus and k and β are constants. This relation is known as *Piéron's Law*.

Piéron's Law has been shown to hold for simple RTs across different sensory modalities (Luce, 1986). It holds for luminance of white light, for amplitude of pure tones and even for taste with respect to the concentration of a substance diluted in water (Bonnet, Zamora, Buratti, & Guirao, 1999). Pins & Bonnet (1996) have demonstrated that Piéron's Law can hold in choice RTs. Their experiments showed that the exponents of the Piéson's Law function that fit the stimulus-intensity to

RT data are consistently similar within a particular modality whatever the task complexity. From this they infer that, firstly, luminance processing continues to some critical level which remains constant and that, secondly, the duration of post-luminance processing must have also have been constant and, third and finally, that the two components of processing combine additively.

Stafford & Gurney (2004) showed that Piéron's Law arises naturally from a number of models of response selection. Thus if stimulus intensity, I, is proportional to the evidence in favour of selection, D, and RT is characterised by a rise-to-threshold process then

$$RT \approx R_0 + kD^{-\beta} \quad (2)$$

where k and β are constants which depend on the critical threshold and the particular properties of the stimulus. D is equivalent to the momentary weight of evidence; this is the drift rate in the diffusion model and equivalent to the input in a simple neuron model. They go on to show that Piéron's Law-like functionality will result from any choice process that relies on the linear rise of a signal to threshold. To understand this consider Figure 1. This geometry, which is obeyed by the diffusion model, the LATER model and simple single-neuron decision models, produces a rise-to-threshold time which is determined by the height of the threshold and the rate of signal rise.

So these results show both empirically (Pins & Bonnet, 1996) and theoretically (Stafford & Gurney, 2004) that we would expect that perceptual detection can be an important determinant of choice response times, and will follow a Piéron's Law-like regularity. However, because Stafford & Gurney (2004) demonstrate that a Piéron's Law-like relation is a necessary consequence of any rise-to-threshold selection process it is not clear whether perceptual detection and subsequent decision are integrated within a single stage, as Ratcliff (2001) implies, or whether perceptual detection is a separate stage which is purely additive to subsequent decision processing as implied by Pins & Bonnet (1996) and explicitly claimed by Carpenter (2004). This is the context for the present work, which involves investigating how choice RTs are affected when both perceptual and cognitive elements of a task are manipulated simultaneously.

Analysis of selection at a single stage

The current experiment involves manipulating the cognitive and perceptual elements in a standard Stroop task. The Stroop condition — either control, conflict or congruent — determines the putative *cognitive* element of the task. This is the factor which influences whether the response decision based on the colour of the stimulus is slowed ('interference') or speeded ('facilitation'). The *perceptual* element is manipulated by changing the percentage colour saturation of the stimulus while keeping the absolute level of light constant. The relationship between simple RT and colour-saturation has not, to our knowledge, been directly tested before. It is strongly predicted that this relation will follow Piéron's Law in the Stroop control condition, where no word is presented.

The question of interest is whether colour saturation and Stroop condition will interact. If the perceptual and cognitive elements of the task are represented independently in separate detection and decision stages then RTs in the three Stroop conditions should be separated by a constant amount across different saturation levels — in other words Piéron's Law should hold for each condition, and the functions that fit the data in each condition should have similar exponents. In the terms of the Additive Factors Method (Sternberg, 1998) the two factors would be additive. If, on the other hand, the representation of the perceptual and cognitive elements of the task are combined within a single metric at a single common stage of decision making — as in the diffusion model — then the difference between the Stroop conditions will vary according to saturation level. In this case, although RTs might follow Piéron's Law within conditions, the functions would have different exponents between conditions. There would be an interaction of factors.

Within the terms of the diffusion model, we can assume that the drift rate will be larger if colour saturation is higher. If the congruency condition affects the total drift rate by a constant amount then we can predict that, if the perceptual and cognitive evidence is combined into a single common metric, the difference between RTs in the control and conflict conditions will differ at different saturation levels; and, for similar reasons, between these conditions and the congruent condition. To see why this is, simply note that the core relationship between RT and drift is given by equation 2. Because RT is a non-linear function of D, decreases in D will have larger effects if D is smaller. If drift is increased due to higher colour saturation, the RT difference between the control and conflict conditions will decrease, despite the constant change in evidence contributed by the cognitive aspect of the stimulus. Therefore the size of the interference effect — the difference in RT between the control and conflict conditions — will be larger if total drift is smaller, e.g., when colour saturation is less. Geometrically, this is shown in Figure 2. In other words, this analysis predicts that if the colour saturation and word condition of the stimulus contribute directly and independently to the strength of evidence for a decision, and if these evidence values, although independent, are combined into a common metric, then the two factors will have an interactive effect on RTs.

Experiment 1

Methods

Participants: 20 University of Sheffield undergraduate students (15 female, average age $= 20.05, s.d. = 3.85$) participated in exchange for partial fulfillment of a course requirement.

The Task: As per the conventional single-trial Stroop task, participants were instructed to name the ink colour, not the

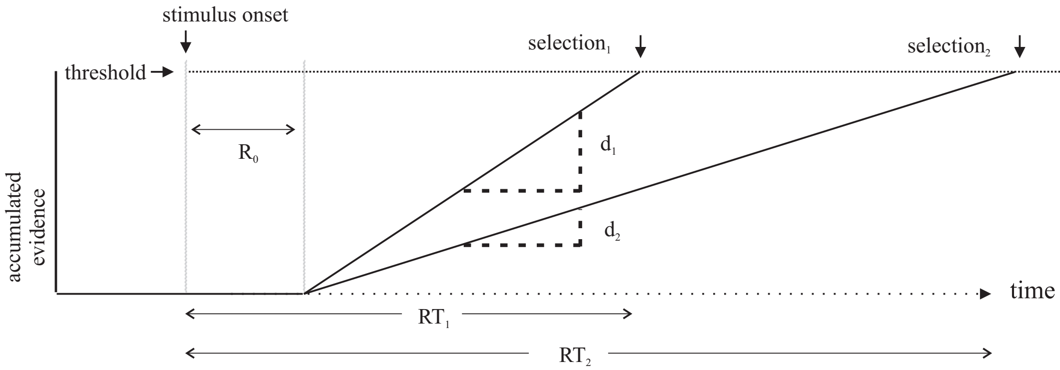

Figure 1: Geometry of selection by linear rise to threshold. Drift rates d_1 and d_2 produce selection times RT_1 and RT_2 respectively.

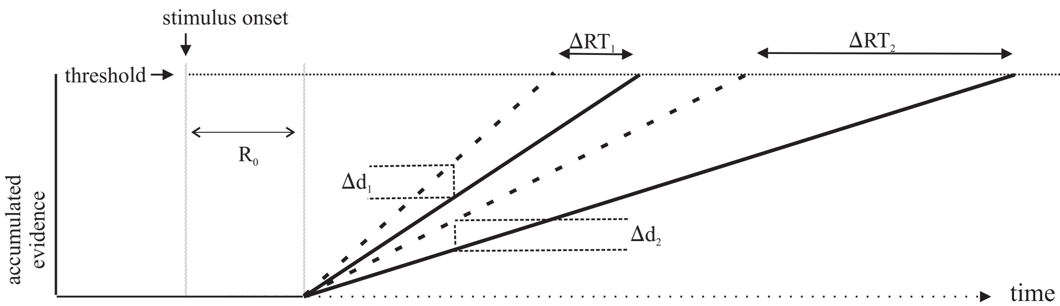

Figure 2: Although the increases in drift rate Δd_1 and Δd_2 are of the same magnitude, ($\Delta d_1 = \Delta d_2$), they result in different increases in reaction time, so that $\Delta RT_1 < \Delta RT_2$.

word, of a coloured word stimulus.

Materials: The stimulus were created using Corel Graphics Suite (version 11.0), in 24pt AventGarde Bk BT typeface. There were three colours of stimuli — red, yellow and blue, and three conditions; either the word was congruent with the colour, conflicted with the colour or was neutral with respect to the colour (this condition used the letters 'XXXX' instead of colour word). Each colour of stimuli, in each condition, was presented at five different levels of colour saturation; 55%, 45%, 32%, 22% and 15%. Hue was kept constant within each colour-set, and brightness was constant across all stimuli.

Design: There were two factors, Stroop condition (control, conflict and congruent) and stimuli intensity (at five levels of saturation). After 9 practice trials with the lowest saturation stimuli, the participants completed 4 blocks of trials in which they saw all 60 possible stimuli in a random order. The total of 240 trials was divided into two halves by a rest break.

Stimuli and Responses: All testing was conducted under constant levels of illumination. Stimuli were presented on a black background on a PC running Windows 98 using E-Prime (version 1.1) and a ADI GD910T monitor. Subjects were positioned 45cm from the monitor, upon which stimuli subtended at a visual angle of up to 19°. Trials began with the presentation of a fixation point, which was replaced by the appearance of the stimulus after 1000ms. The stimuli remained for up to 3000ms or until a response was registered, whichever was shorter. The interval between stimulus onset and the onset of the participant's vocal response was recorded by a microphone triggering the voice key of a PST Serial Response Box (SRBox).

Results

Incorrect responses, those faster than 300ms (deemed to be due to an irrelevant sound triggering the voice key) and those slower than 1500ms were removed from the analysis. These invalid trials were only 6% of the total. The mean RTs for the colour-naming experiment are shown in Figure 3. A two factor within-subjects ANOVA was performed. There was a clear effect of condition (F(2,18)= 106.97 $p < 0.0001$). As expected the conflict condition was slower than the control condition and the congruent condition was fastest of all. The effect of stimulus intensity was also highly significant (F(4,16)= 19.16 $p < 0.0001$); higher colour saturations were associated with faster RTs in all three Stroop conditions. There was, however, no interaction of colour saturation and Stoop condition (F(8,12)= 0.433, *n.s.*) — as can be seen from the graph (Figure 3) the difference between the conditions remains constant at all levels of stimulus intensity.

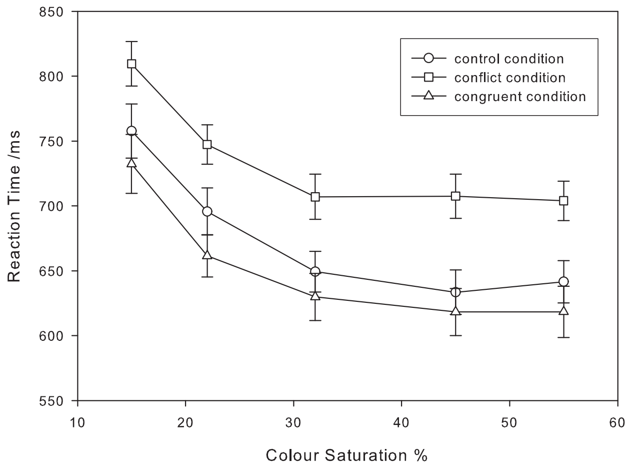

Figure 3: Mean reaction times for different stimulus colour saturations in all three Stroop conditions. Standard error bars shown (n = 20).

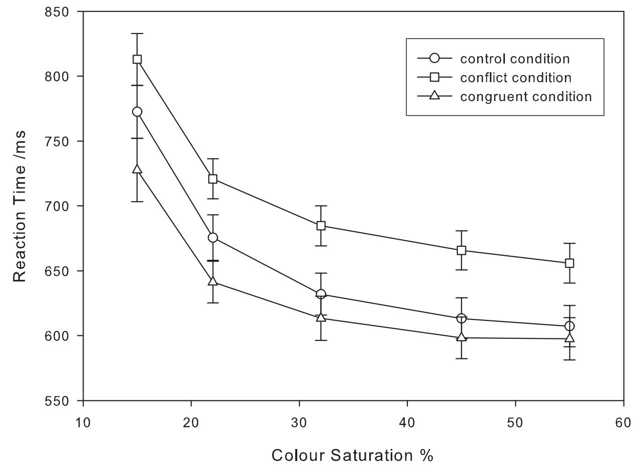

Figure 4: Separated Stroop Task. Mean reaction times for different stimulus colour saturations in all three Stroop conditions. Standard error bars shown (n = 28).

Table 1: Fit data for Piéron's Law against the colour-saturation — reaction time functions in all three Stroop conditions, experiment 1.

Stroop condition	Correlation Coefficient	R_0	β
Control	0.992 *	621	1.83
Conflict	0.994 *	697	2.37
Congruent	0.999 *	611	2.34
* significant $p < 0.0001$			

Fitting data to Piéron's Law Following the procedure described in Stafford & Gurney (2004) we can fit Piéron's Law curves to the colour-saturation — RT curves in all three Stroop conditions. Table 1 shows the Pearson's r correlation coefficients between the empirical data and Piéron's Law curves of best fit, as well as the asymptotes and exponents.

Inspection of the exponents of the functions fitted to the data averaged across individuals shows them to be very similar. It is also possible to fit Piéron's Law functions to individual's data in each of the three conditions. An ANOVA on the β values of these fitted functions shows that there is no significant difference between the conditions ($F(2,38) = 2.151, p = 0.130$).

Discussion of experiment

The colour-naming experiment demonstrates that a colour-saturation to RT Piéron's Law holds in all three basic Stroop conditions. As far as we are aware this is the first demonstration that Piéron's Law holds for colour-saturation. It is also a confirmation of the finding of Pins & Bonnet (1996) that Piéron's Law holds for choice RTs well as simple RTs, but using a different task. It is the first demonstration of Piéron's Law in a Stroop task, and in so much is the first demonstration in a response conflict situation where the choice is based on a single stimulus.

The lack of interaction between Stroop condition and colour-saturation appears to suggest that perceptual salience and cognitive salience are not combined into a single 'response salience' value. Additionally we can note that these data suggest that perceptual detection is a significant factor compared to decision, even in a cognitive task such as the Stroop task.

Experiment 2: Separated Stroop

It is possible that the additivity of experiment 1 is due to the integration of the word and colour stimuli; note that in the conventional Stroop the word is formed from the colour patch that varies in saturation. A version of the Stroop task, the 'separated Stroop', exists whereby the word and colour stimuli are not integrated, but are presented adjacent to each other. A diminshed but reliable Stroop effect persists in these conditions (MacLeod, 1998). We conducted a second experiment, identical in method to experiment 1 bar that the word and the colour were presented separately immediately above and below fixation (randomly alternating). The results are shown in Figure 4.

These data are well fitted by Piéron's Law functions, as shown in Table 2. An ANOVA on the exponents of the Piéron's Law functions fitted to the individual data shows that there is no significant difference between the exponents for each condition ($F(2,54) = 1.628, p = 0.206$).

Discussion of experiment 2

As with experiment 1, the perceptual factor (colour saturation) and cognitive factor (Stroop condition) did not interact, instead combining additively. This suggests that the results of experiment 1 were not due to the nature of stimulus presentation but are instead fundamental to the processing demanded by the Stroop task.

Extending existing models of the Stroop task

Although a pattern of additive factors in the experimental data can be trivially fitted with a simplistic two-stage model it is also appropriate to ask if existing, continuous processing — i.e. 'single stage' — models of the Stroop task can fit the data. We show that they can. Our starting point is the Cohen, Dunbar & McClelland (1990) model of the Stroop task. This model uses the diffusion model (Ratcliff, 1978) as a response mechanism to arbitrate between competing responses. The 'front-end' of the model is built out of the simple artificial neurons of the PDP tradition (Rumelhart, McClelland, & PDP Research Group, 1986). This early part of the model performs stimulus-response translation, reconciling the word and colour information of the stimulus, the attentional demands of the task instruction ("Respond based on the word, ignore the ink colour") and providing a locus for learning. The front-end weights these factors in converting the stimulus inputs into the common metric of evidence in favour of the responses. For a detailed discussion of this model of the Stroop task and the response mechanism see Stafford & Gurney (2007).

This model is a continuous processing model. Inputs to the response mechanism are continuously updated as the front-end adjusts to the presentation of inputs. In this respect, then, it is considered a 'single stage' model; although it may have many architectural stages, they are a functional unit.

To address the current experiments the Cohen model (Cohen et al., 1990) is modified in this simple way only — in the original model the word and colour inputs were represented by 0 or 1 values. To simulate the present experiment intermediate intensity values between 0 and 1 were used for both colour and word inputs. This reflects the corresponding variation in the strength of the inputs with varying colour saturation. Because both the word and colour inputs to the model begin at the same time and have identical values we denote this the 'single stage model with locked inputs'.

Figure 5 shows the match to the experimental data. Piéron's Law holds for all Stroop conditions and the interference and facilitation effects are constant across saturation conditions (see Table 3).

Table 2: Fit data for Piéron's Law against the colour-saturation — reaction time functions in all three Stroop conditions, experiment 2.

Stroop condition	Correlation Coefficient	R_0	β
Control	1.000 *	595	2.07
Conflict	0.999 *	646	2.02
Congruent	0.999 *	593	2.61

* significant $p < 0.0001$

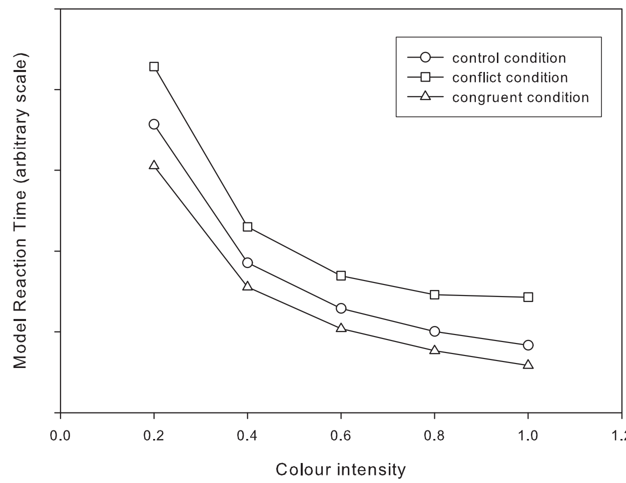

Figure 5: Single stage model with locked inputs: simulated reaction times for the standard Stroop task across a range of colour intensities

Discussion of Modelling

The present results provide a specific instance of the claim that continuous processing models can mimic discrete processing models (McClelland, 1979). This weakens our confidence that inference about the underlying architecture can be made from data patterns consistent with factor additivity. These result would appear to strongly contradict the assumption that word information remains unaffected by the colour saturation.

Discussion

Primarily these data show that Pièron's Law holds for colour saturation and in conditions of response conflict. This demonstrates an extension of the relevance of this function beyond the mere perceptual and into the 'cognitive realm'. At first glance, the data also seems to support the view that simple decision making *can*, at least in some conditions, consist of two stages, detection and decision, which combine additively. Furthermore an analysis of a single stage response mechanism which linearly combines stimulus evidence into a common metric suggests that it cannot account for the pattern of data in tasks like this. However the extension of existing

Table 3: Fit of Piéron's Law against simulation data for one stage model with locked inputs.

Stroop condition	Correlation Coefficient	β
Control	1.00 *	1.01
Conflict	1.00 *	1.40
Congruent	1.00 *	0.90

* significant $p < 0.0001$

models of Stroop processing shows that continuous, 'single-stage', models can match the empirical results. Although the Cohen model of Stroop processing incorporates a diffusion model response mechanism, it does this in combination with a PDP-based stimulus processing 'front-end' which transforms the inputs before they enter the response mechanism. Taken together this model of Stroop processing matches the pattern of empirical data, just as a two-stage model with discrete processing of detection and decision stages would. This is an illustration of the phenomenon of model-mimicry (Townsend & Wenger, 2004) and more generally of the dangers of judging models solely by the goodness of fit to data (Roberts & Pashler, 2000). The reason this model defies the analysis presented in the introduction is because the PDP front-end performs a non-linear transformation on the stimulus information before it is combined as 'evidence' in the response mechanism.

The data does not allow us to choose between continuous or discrete models as more favourable, but it does seem to suggest strongly that the representation of the word and colour elements of the stimuli, in this paradigm, are locked. We hypothesise that in Stroop conditions subjects' focus of attention is such that the representations of the colour and word are bound together and hence colour-saturation has a *de facto* influence on word-recognition — as is reflected in the locking on the stimuli inputs in the simulations which successfully replicate the empirical data.

Acknowledgments

We thank Sarah Fox for help running the experiments, David Lawrence & David Yates for reading drafts of this article and Marius Usher and Eddy Davelaar for useful discussion of the material.

References

Bogacz, R., Brown, E., Moehlis, J., Holmes, P., & Cohen, J. D. (2006). The physics of optimal decision making: a formal analysis of models of performance in two-alternative forced-choice tasks. *Psychological Review*, *113*(4), 700-765.

Bonnet, C., Zamora, M. C., Buratti, F., & Guirao, M. (1999). Group and individual gustatory reaction times and Piéron's law. *Physiology & Behavior*, *66*(4), 549-558.

Carpenter, R. H. S. (2004). Contrast, probability, and saccadic latency: Evidence for independence of detection and decision. *Current Biology*, *14*(17), 1576-1580.

Cohen, J. D., Dunbar, K., & McClelland, J. L. (1990). On the control of automatic processes - a parallel distributed-processing account of the Stroop effect. *Psychological Review*, *97*(3), 332-361.

Gold, J., & Shadlen, M. (2002). Banburismus and the brain: Decoding the relationship between sensory stimuli, decisions, and reward. *Neuron*, *36*(2), 299-308.

Luce, R. (1986). *Response times: Their role in inferring elementary mental organisation*. New York: Clarendon Press.

MacLeod, C. (1998). Training on integrated versus separated stroop tasks: The progression of interference and facilitation. *Memory & Cognition*, *26*(2), 201-211.

MacLeod, C., & MacDonald, P. (2000). Interdimensional interference in the stroop effect: uncovering the cognitive and neural anatomy of attention. *Trends in Cognitive Sciences*, *4*(10), 383–391.

McClelland, J. (1979). On the time-relations of mental processes: An examination of systems of processes in cascade. *Psychological Review*, *86*, 287-330.

Piéron, H. (1952). *The sensations: Their functions, processes and mechanisms*. London: Frederick Muller Ltd.

Pins, D., & Bonnet, C. (1996). On the relation between stimulus intensity and processing time: Piéron's law and choice reaction time. *Perception & Psychophysics*, *58*(3), 390-400.

Ratcliff, R. (1978). A theory of memory retrieval. *Psychological Review*, *85*, 59-108.

Ratcliff, R. (2001). Putting noise into neurophysiological models of simple decision making. *Nature Neuroscience*, *4*(4), 336-336.

Ratcliff, R., & McKoon, G. (2008). The diffusion decision model: Theory and data for two-choice decision tasks. *Neural Computation*, *20*(4), 873-922.

Ratcliff, R., & Rouder, J. (1998). Modeling response times for two-choice decisions. *Psychological Science*, *9*(5), 347–356.

Roberts, S., & Pashler, H. (2000). How persuasive is a good fit? A comment on theory testing. *Psychological Review*, *107*(2), 358-367.

Rumelhart, D., McClelland, J., & PDP Research Group the. (1986). *Parallel distributed processing: Explorations in the microstructure of cognition*. Cambridge, MA: The MIT Press.

Stafford, T., & Gurney, K. (2004). The role of response mechanisms in determining reaction time performance: Piéron's law revisited. *Psychonomic Bulletin & Review*, *11*(6), 975-987.

Stafford, T., & Gurney, K. (2007). Biologically constrained action selection improves cognitive control in a model of the Stroop task. *Philosophical Transactions of the Royal Society London, Series B*, *362*, 1671-1684.

Sternberg, S. (1998). Discovering mental processing stages: The method of additive factors. In D. Scarborough & S. Sternberg (Eds.), *An invitation to cognitive science: Methods, models, and conceptual issues (2nd edition)* (p. 702-863). Cambridge, MA.: MIT Press.

Stroop, J. (1935). Studies of interference in serial verbal reactions. *Journal of Experimental Psychology*, *18*, 643-662.

Townsend, J. T., & Wenger, M. J. (2004). The serial-parallel dilemma: A case study in a linkage of theory and method. *Psychonomic Bulletin & Review*, *11*(3), 391-418.